

Roland Potthast

**Point sources
and multipoles
in inverse scattering
theory**

CHAPMAN & HALL/CRC

Boca Raton London New York Washington, D.C.

CHAPMAN & HALL/CRC

Research Notes in Mathematics Series

Main Editors

H. Brezis, *Université de Paris*

R.G. Douglas, *Texas A&M University*

A. Jeffrey, *University of Newcastle upon Tyne (Founding Editor)*

Editorial Board

H. Amann, *University of Zürich*

R. Aris, *University of Minnesota*

G.I. Barenblatt, *University of Cambridge*

H. Begehr, *Freie Universität Berlin*

P. Bullen, *University of British Columbia*

R.J. Elliott, *University of Alberta*

R.P. Gilbert, *University of Delaware*

D. Jerison, *Massachusetts Institute of Technology*

B. Lawson, *State University of New York
at Stony Brook*

B. Moodie, *University of Alberta*

S. Mori, *Kyoto University*

L.E. Payne, *Cornell University*

D.B. Pearson, *University of Hull*

I. Raeburn, *University of Newcastle, Australia*

G.F. Roach, *University of Strathclyde*

I. Stakgold, *University of Delaware*

W.A. Strauss, *Brown University*

J. van der Hoek, *University of Adelaide*

Submission of proposals for consideration

Suggestions for publication, in the form of outlines and representative samples, are invited by the Editorial Board for assessment. Intending authors should approach one of the main editors or another member of the Editorial Board, citing the relevant AMS subject classifications. Alternatively, outlines may be sent directly to the publisher's offices. Refereeing is by members of the board and other mathematical authorities in the topic concerned, throughout the world.

Preparation of accepted manuscripts

On acceptance of a proposal, the publisher will supply full instructions for the preparation of manuscripts in a form suitable for direct photo-lithographic reproduction. Specially printed grid sheets can be provided. Word processor output, subject to the publisher's approval, is also acceptable.

Illustrations should be prepared by the authors, ready for direct reproduction without further improvement. The use of hand-drawn symbols should be avoided wherever possible, in order to obtain maximum clarity of the text.

The publisher will be pleased to give guidance necessary during the preparation of a typescript and will be happy to answer any queries.

Important note

In order to avoid later retyping, intending authors are strongly urged not to begin final preparation of a typescript before receiving the publisher's guidelines. In this way we hope to preserve the uniform appearance of the series.

CRC Press UK

Chapman & Hall/CRC Statistics and Mathematics

Pocock House

235 Southwark Bridge Road

London SE1 6LY

Tel: 020 7450 7335

Library of Congress Cataloging-in-Publication Data

Potthast, Roland.

Point sources and multipoles in inverse scattering theory / Roland Potthast.

p. cm. -- (Chapman & Hall/CRC research notes in mathematics series ; 427)

Includes bibliographical references and index.

ISBN 1-58488-252-2 (alk. paper)

1. Scattering (Mathematics) 2. Inverse problems (Differential equations) I. Title. II. Series.

QA377 .P64 2001

515'.353--dc21

2001025323

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials or for the consequences of their use.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage or retrieval system, without prior permission in writing from the publisher.

The consent of CRC Press LLC does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from CRC Press LLC for such copying.

Direct all inquiries to CRC Press LLC, 2000 N.W. Corporate Blvd., Boca Raton, Florida 33431.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

Visit the CRC Press Web site at www.crcpress.com

© 2001 by Chapman & Hall/CRC

No claim to original U.S. Government works

International Standard Book Number 1-58488-252-2

Library of Congress Card Number 2001025323

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

To my parents,
 who supported my way,
and to my wife, Regina,
 for sharing the joy and the work.

Contents

1 Introduction and tools

- 1.1 A survey on inverse scattering theory
- 1.2 Basic definitions and tools

2 Direct scattering problems

- 2.1 Acoustic obstacle scattering
- 2.2 The inhomogeneous acoustic medium
- 2.3 Electromagnetic scattering by a perfect conductor
- 2.4 The electromagnetic inhomogeneous medium
- 2.5 Scattering by orthotropic media
- 2.6 Anisotropic electromagnetic media

3 Uniqueness and stability in inverse scattering

- 3.1 Acoustic scattering
- 3.2 Electromagnetic scattering

4 The case of finite data

- 4.1 Finite data in inverse acoustic scattering
- 4.2 Inverse electromagnetic scattering

5 The point-source method and applications

- 5.1 Reconstruction of acoustic scatterers
- 5.2 Inverse electromagnetic scattering
- 5.3 Reconstruction of the boundary values of scattered fields
- 5.4 Convergence of a regularized Newton method

6 Singular sources and shape reconstruction

- 6.1 Acoustic scattering
- 6.2 Electromagnetic scattering

7 Linear sampling methods

- 7.1 The original linear sampling method
- 7.2 Spectral theory and a modified linear sampling method
- 7.3 Shape reconstruction for orthotropic media
- 7.4 Anisotropic electromagnetic media

References

Preface

Books are written in different places and with various purposes motivating the authors and influencing style and content. Some works are mainly introductions into an area and arise from lectures and courses. Others summarize research and present results in a concise way.

In the present book, an introduction to recent research results is a main task. I consider research as a fascinating story in which many people take part and contribute to the whole of knowledge and technologies. It is important to realize that many different styles and characters are involved in this business. Many things, like the way of presenting results, are a matter of taste and personal strengths. I think that each person is important in his or her contributions, weaknesses, and strengths and should not be neglected by others.

Secondly, research is the story of important open problems and thrilling answers. Today's questions are for example: How can I succeed to simulate or calculate a natural phenomenon? Can I reconstruct a quantity from the knowledge of some others? Is it possible to get insight into the matter by mathematical analysis? Sometimes it is difficult to express what is missing and how to approach a particular problem. Then, it might be helpful to ask: Can we phrase or solve the question in a different way to get new insight into the matter? Often problems which have seemed insolvable with current techniques become accessible and solvable by new influences from technology or new mathematical tools.

For example, when solving inverse scattering problems I always hesitated to apply a general optimization scheme to a large nonlinear problem. I was convinced that it must be possible to get more concrete insight into the matter and to solve the inverse problem without multiply solving the direct problem. But for a long time we had no idea how this should be possible. Then, at the same time, the linear sampling method and the point-source method proved successful in reconstructing the shape of scatterers without using much optimization. Suddenly three or four different new methods (point-source method, singular-source method, linear sampling method, and modified linear sampling method) were alive and gave new input into the area.

A second story is the interpretation and understanding of Newton's

method. During the doctoral dissertation, I spent a lot of time studying Fréchet derivatives. Since it is one main solution method to solve scattering problems, the lack of a differentiability proof via the boundary integral approach was quite unsatisfactory. Then the possibility to prove differentiability via boundary integral operators opened up before us. But the analysis of Newton's method in inverse scattering still remained incomplete: from the viewpoints of geometry and of convergence it was not clear what happened at each iteration step. [Chapter 5](#) will present some recent results on the convergence and interpretation of Newton's method which answer these questions.

These are just examples of points where it was possible to find solutions to open questions. You will find more in this work, and other researchers tell *their* stories and experiences. But many questions are important and *not* answered until now. I think that we are just at the very beginning in inverse problems, remote sensing, image recognition, nondestructive testing, and medical imaging!

A large part of the scientific ideas in this book have been developed in interaction with Prof. Dr. Rainer Kreß, Göttingen, and Prof. Dr. David Colton, Delaware. Thank you for your encouragement and criticism. I also enjoyed many stimulating discussions in London with Dr. Simon Chandler-Wilde at Brunel University. Some work on the implementation of the methods is done in cooperation with Dr. Klaus Giebermann, Bonn. I would like to thank Dr. Tilo Arens for providing his code for both direct and inverse problems. Also, thanks are given to Dr. Imre Paulovits, ORGALOGIC GmbH/Systems Research Laboratories, Köln, for supporting the finish of my habilitation procedure in 1999 when I worked full-time in his company, and to Prof. R. Kress, Dr. S. Chandler-Wilde, Dr. Frank Hettlich and to Dr. Tilo Arens for reading and correcting parts of this book.

I am grateful to *Deutsche Forschungsgemeinschaft* (DFG), that financed my research over four years and paid for several trips to the U.S., including one year at the University of Delaware. I appreciate the hospitality of this institution, of Prof. Dr. Peter Monk and his family and of Heidi and Lothar Jeromin, Delaware.

Finally, I would like to express special thanks to my wife Regina, who strongly supported and encouraged me over the years.

Roland Potthast, Brunel University, London, 2001

Classics

As long as a branch of knowledge offers an abundance of problems, it is full of vitality. *David Hilbert*

Before you generalize, formalize, and axiomatize, there must be mathematical substance. *Hermann Weyl*

Our science, in contrast to others, is not founded on a single period of human history, but has accompanied the development of culture through all its stages. Mathematics is as much interwoven with Greek culture as with the most modern problems in engineering. It not only lends a hand to the progressive natural sciences but participates at the same time in the abstract investigations of logicians and philosophers. *Felix Klein*

Visions

The youth will discover the joy and beauty of mathematics, its power and strength, the clear lines of thoughts, arguments, geometric vision, the joy to go beyond borders and exceed limits of old things, and, at the same time, to study concepts and proofs which are hundreds or thousands of years old and still are as young and amazing as a newborn baby.

Find the faith and courage to try something new and to go into thoughts which have not been thought before. There is the danger to come back with empty hands, to find that you are not gifted enough and not successful enough. But the thrilling presence of the new, the amazement when the fog slowly dissolves or when suddenly the long searched concept or truth opens up before you is worth the stake.

Appreciate the work of everyone, since every being has his or her place. Some rearrange long-known knowledge in a proper way, others go into unknown lands. We deeply need each other, even if we do not know this, to grasp the truth of this creation and to shape our lives. And we all need God, the creator, living spirit, holy love, the beginning and the end.