Stephan Boltzmann Law and Boltzmann's Constant

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This experiment serves as a quantitative observation of the Stephan Boltzmann Law for a blackbody, which states that the power of thermal radiation emitted by a body is proportional to the temperature of the body raised to the fourth power. The voltage across the Tungsten filament of an electric lamp was varied and measured, and by placing the lamp in series with an ammeter, the power of emitted thermal radiation was determined. An expression for the temperature at any resistance experienced by the filament was also determined. It was observed that the power followed a powerlaw in temperature with the exponent in question equal to 4.0 ± 0.4. For temperatures greater than or equal to 2370 K, the contribution of energy lost by convection and/or conduction was negligibly small, and the contribution of energy absorbed by the filament was also negligible. The Boltzmann constant was estimated to have a value of (7.2 ±1.5) x 10^{-8} W/m^2K^4, which is within two standard deviations of the accepted value, 5.670 x 10^{-8} W/m^2K^4.

INTRODUCTION

Anyone who has ever seen the bright sun or glowing lava has observed electromagnetic radiation emitted by these bodies in heat transfer. Fundamentally, any body not at a temperature of 0 K emits thermal radiation in the form of electromagnetic waves, while no medium is required for heat transfer.[5] In 1879, Stephan Boltzmann discovered that a body emits thermal radiation at a rate that is proportional to the temperature of the surface of the body raised to the fourth power, and he also discovered the proportionality constant for this relationship, σ = 5.670 x 10^{-8} $Wm^{-2}K^{-4}$ (Boltzmann constant).[2]

In this experiment, a range of voltages will be applied across a Tungsten filament in an attempt to verify this relationship for temperatures of the surface of the filament ranging from (323 – 2775) K. Furthermore, a powerlaw for the rate of energy exchange between the filament and the environment as a function of temperature will be determined. Considering only the rate of energy emission by the filament at high temperatures, another powerlaw function will be determined to calculate an experimental value for Boltzmann's proportionality constant.

THEORY

According to Provost's theory of exchanges, "In a state of equilibrium, the amount of energy radiated per unit time from an object is equal to the energy absorbed by it in the form of radiation from surrounding objects."[5] Under ideal circumstances, a perfect absorber is a body that is a perfect radiator, and it is referred to as a blackbody. (As a perfect absorber, the body does not reflect or scatter any radiated energy it intercepts.) The Planck distribution describes the emission spectrum of electromagnetic or blackbody radiation in thermal equilibrium within a cavity, and it also describes the "energy spectrum of lattice vibrations in an elastic solid." [3] A mode refers to an oscillation amplitude pattern in the cavity or solid. Furthermore, a mode of frequency ω can only be excited in quantized units of energy, $\hbar\omega$; therefore, the energy, ε_s, of a state with s quanta in a mode is defined:

$$\varepsilon_s = s\hbar\omega \qquad (1)$$

As shown in Kittel and Kroemer[3], considering all states using the partition function, the Planck distribution function may be determined, resulting in the thermal average number of photons in a mode when the photons are in thermal equilibrium with a reservoir at temperature τ. Considering (1), the thermal average energy per mode may be determined, and then considering all modes, the total energy of the photons in the cavity per unit volume may be determined. The total energy per unit volume may then be determined:

$$\frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4 \qquad (2)$$

Equation 2 is the Stephan Boltzmann law of radiation which states that the radiant energy density for a blackbody is proportional to the fourth power of the temperature, where $\tau = k_B T$. Therefore, the energy flux density, or rate per unit surface area, J_U, at which energy is emitted by the blackbody is defined as:

$$J_U = \sigma T^4 \qquad (3)$$

The experimentally determined proportionality constant σ, or the Boltzmann constant, has a value[3] of 5.670 x 10^{-8} $Wm^{-2}K^{-4}$.

For any body, the net power emitted as a function of the body's surface area, A_s, the temperature of the environment, T_o ,and the temperature of the body's surface, T, respectively is

$$P_a = e\sigma A_s(T^4 - T_o^4) \qquad (4)$$

The constant e is the emissivity or emissive power of the body's surface, and hence for a blackbody, e = 1. Due to the opposite "directions" of absorption and emission, the net power (P), or rate of energy exchange via thermal radiation is defined:[2] $P = P_r - P_a$. For a radiant source such as the Tungsten filament used in this experiment, the rate at which energy, E, is supplied to the lamp containing the filament is just P = IV. The rate at which energy is lost by the filament via radiation and convection at thermal equilibrium is defined:[1]

$$IV = e\sigma A_s(T^4 - T_o^4) + K(T - T_o) \qquad (5)$$

where K is a constant for the lamp. According to Edmonds[1], the contribution of the convection term is negligible, and due to the fact that $T_0^{\ 4}$ is less than 1% of T^4 above 800K, the energy absorbed by the filament from the environment is negligible. Therefore, the power of thermal radiation emitted by the filament is equivalent to Equation 3 at high temperatures.

EXPERIMENT

To perform this experiment, a Hewlett Packard 6282A DC power supply (0-10 V, 0-10 A) is connected to a Pasco Model TD-8555 Stephan Boltzmann Lamp (13 V @ 3 A max,). The lamp is connected to a Keithley 169 Multimeter to measure the voltage across the lamp filament. The circuit is completed by connecting the lamp to a Keithley 197 DMM to measure the current through the lamp. See Figure 1:

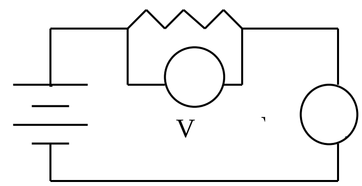

Figure 1. Electrical schematic of experimental setup.

The HP power supply was adjusted such that a maximum of 12 V was established across the lamp filament. The voltage across the filament was decreased by increments of (0.5 ± 0.1) V to a voltage of 0.5 V. At each voltage increment, the current through to the lamp was also read (± 0.02 A) and recorded .

A Gaertner traveling microscope was used to determine the surface area of the Tungsten filament, or radiant source. Considering the fact that the filament is a coiled cylinder, the diameter of the filament, D, was measured to determine the perimeter of a circle on the filaments surface: πD. The length of each loop was determined by measuring the diameter of a loop, D_c, to determine its perimeter: πD_c , and the additional length of filament connecting two loops was determined by measuring the distance, l_o, between the center of each loop. The number of loops, N, was counted and it was assumed that the length of filament connecting the two coils was simply an additional, stretched loop. Therefore, the surface area of the filament was determined using the following expression: $A_s = \pi DN(l_o + \pi D_c)$.

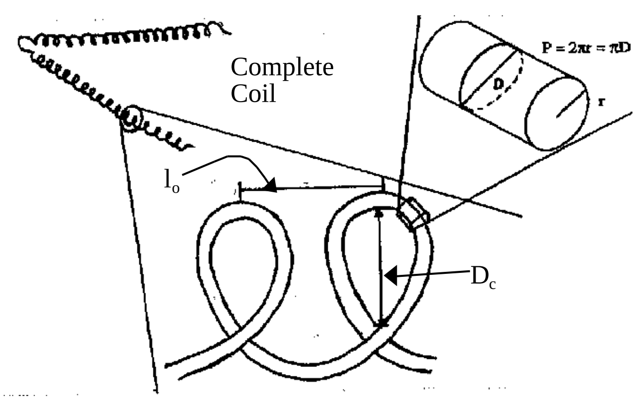

Figure 2. Schematic of filament

Each length was measured three times, and the respective averages were used to determine the surface area of the filament, A_s,: D = (6.5 ± 0.9) x 10^{-3} cm, D_o = (81.7 ± 0.3) x 10^{-3} cm, l_o = (12.9 ± 0.4) x 10^{-3} cm. Therefore, A_s was found to be (1.82 ± 0.29) x 10^{-5} cm^2.

ANALYSIS AND INTERPRETATION

To determine whether the rate of energy emitted was proportional to T^4 and to determine the Boltzmann constant, the temperature of the filament at each voltage step was determined. Provided with the lamp from Pasco was an experimentally determined set of values for the quotient R/R_{300K} , or R/R_o (R = resistance across the filament, R_o = resistance across the filament at room temperature), as a function of the temperature, T (K), of the filament surface. Therefore, these values were plotted T vs. R/R_o using the data manipulation program, Igor Pro (version 3) to determine a fit expression relating any recorded value of R/R_o to the corresponding value for the surface temperature of the filament. The following expression with associated error values was determined for T in K:

$$T = (-1.856 \pm 0.072)x^2 + (206.6 \pm 1.3)x + (118.3 \pm 4.6) \qquad (6)$$

where x = R/R_o. The fundamental expression, V = IR was used to determine the resistance for each voltage step. The value of R_o was determined by setting the voltage at a value close to 0.0 V, (50 ± 2) x 10^{-4} V, immediately after turning the power supply on, and recording the corresponding current, (183 ± 3) x 10^{-4} A. Calculated values for R/R_o were determined, which allowed for the determination of T for each voltage step using Equation 5. The values for P were plotted as a function of T to assess the validity of $P = eA_s\sigma T^4$ and determine the value of the Boltzmann constant. Considering Equation 5, the following expression describes a linear relation of log[P] as a function of log[T]: $\log[P] = 4\log[T] + \log[e\sigma A_s]$.

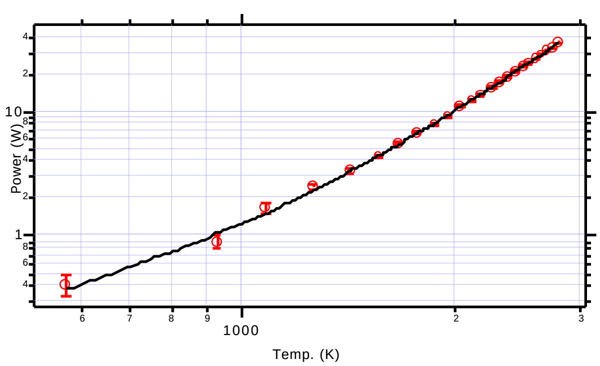

Figure 3. The solid line is a fit by Equation 8

Figure 3 shows a linear relationship between P and T^4 for temperatures greater than or equal to 2000 K, which is contrary to the argument made by Edmonds'[1] lower limit of 800 K. The curve that is evident at the lower end of the temperatures appears to be a function of the radiation absorbed by the bulb and the energy lost via convection described by the additional terms in Equation 6.

To determine the powerlaw for this data, a program was written in Igor Pro to properly fit the data and determine coefficients in the expression:

$$y = c_1 x^{c_2} + c_3 x + c_0 \qquad (8)$$

where x = T. For all non-zero coefficients, this would correspond to Equation 6, which considers the radiation absorbed from the environment and the energy lost via convection.

It is important to note that the precision for measuring the voltage decreased as the voltage was taken to zero; therefore, the fit was properly weighted. The fit was then determined varying all constants to assess the relationship between P and T and is shown as the solid line in Figure 3.

A fit to all the data points held the value of c_2 constant at 4.0, and the remaining constants were permitted to vary. The value of c_1 was determined to be (5.5 ± 0.3) x 10^{-13}, which can later be used to determine an experimental value for σ.

Fits were plotted, holding c_o and c_3 constant at 0, c_2 constant at 4, but varying c_1 to determine the temperature range at which the terms for energy absorbed from the environment and energy lost via convection were negligible. For temperatures greater than or equal to 2370 K the fit describes all data within the error. (See Figure 4.)

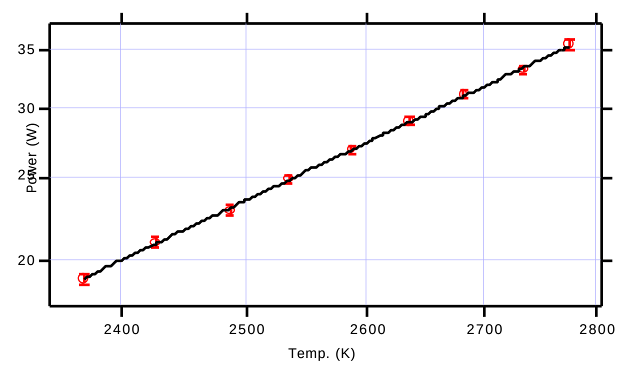

Figure 4. At high temperatures this log-log plot represents a straight of slope 4, which is the power of T.

The narrow range of higher temperatures was considered so that a better approximation of $c_1 = e\sigma A_s$ could be determined as (5.56 ± 0.20) x 10^{-13}.

To further assess the relationship between P and T, values of P were plotted as a function of T^4.

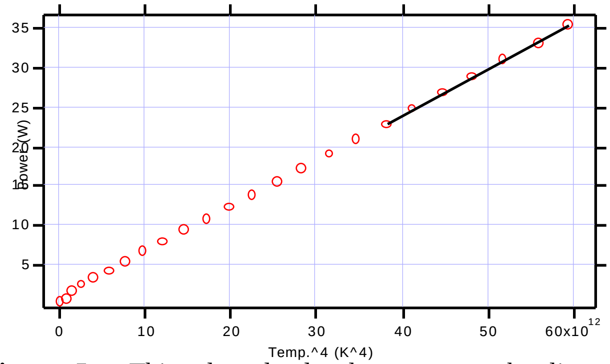

Figure 5. This plot clearly demonstrates the linear relationship between P and T^4 at higher temperatures.

The absorption of radiation and loss of energy via convection by the filament at low temperatures is evident in Figure 5 in the tapering of power values as the plot nears zero. A value of $(5.9 \pm 1.6) \times 10^{-13}$ was determined for c_1 considering the same temperature range as for Figure 5.

The value of σ was then determined using the two fitted values for c_1. The value of the emissivity, e, for the Tungsten filament was determined by considering the accepted values over a range of temperatures. The spectral emissivity, e_λ, for a material relates to a selected region of that surface's heat spectrum.[6] One[6] value of e_λ for Tungsten at $\lambda = 0.65\ \mu$ at 2273 K is 0.43. Edmonds[1] listed e_λ with an average value of 0.41, and the CRC[4] lists the values of $e_\lambda = 0.43$, over a temperature range, (1600 – 2800). Using an average value of $e_\lambda = 0.43$ and the calculated value for A_s previously determined:

$$\bar{\sigma} = (7.2 \pm 1.5) x 10^{-8} \frac{W}{m^2 K^4}$$

It must be noted that considering the inherent error in the manner in which D_c and D were measured, this experimental value for σ may not be considered conclusive or completely reliable. To eliminate any discrepancies, the surface area of the coil could be determined before the construction of the coil; or more practically, the general dimensions of the type of filament used to manufacture the TD-8555 electric lamp could be obtained from Pasco Scientific for comparison with the determination of A_s in this experiment. With that information, an experimental value for the emissivity of the Tungsten filament could also be determined using the known value of σ.

CONCLUSION

For the Tungsten filament of the Pasco electric lamp used in this experiment, the rate of energy exchange between the environment and the filament varied as the absolute temperature to the fourth power. At high temperatures (above 2370 K), this filament emitted thermal radiation like a blackbody, as the rate of thermal radiation from the filament was directly proportional to T^4. By not needing the terms describing the energy absorbed by the filament or the energy lost by convection, Boltzmann's relationship was supported by a determination of the Boltzmann constant, $(7.2 \pm 1.5) \times 10^{-8}$ W/m^2K^4, which is within two standard deviations of the known value of 5.670×10^{-8} $Wm^{-2}K^{-4}$.

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