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On the relation between Compromise Programming and Ordered Weighted Averaging operator

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ABSTRACT

This paper will demonstrate a strong relationship between the Compromise Programming (CP) method and the Ordered Weighted Averaging (OWA) operator. We will mathematically prove that the application of the CP method reflects the pessimistic view of the Decision Maker (DM), if the distance from the ideal point is minimized. It will however present an optimistic DM, if the distance from the nadir is maximized. The OWA operator may cover all of the optimistic, neutral and pessimistic views.

A practical relationship will be developed between the power value of the CP method and the optimism degree of the DM based on fuzzy quantifiers. The theoretical results will be illustrated by a real case study of a water resources management problem.

1. Introduction

A large variety of solution methods has been developed for solving multi-criteria decision making (MCDM) problems as it is well demonstrated in the literature [4–7,13,16,25,32]. The most frequently used techniques for solving MCDM problems are the distance based methods, scoring methods, outranking methods, trade-off schemes, value and utility functions, pairwise comparisons and interactive methods. Distance based techniques, especially Compromise Programming (CP) methods [30], have been successfully applied in many areas. The classical scoring method is usually discussed in the literature as a simple additive weighting algorithm. Another important weighting method is the Ordered Weighted Averaging (OWA) method [21].

In this paper we will focus on the relationship between CP and OWA. The power parameter of the CP method has a very important effect on the results, however there is no precise mathematical method known from the literature to define this parameter. We will however introduce a strong relationship between the power parameter of the CP method and the optimism degree of the DM in the OWA method. A guideline will be then obtained for the easy selection of the suitable power parameter of the CP method.

This paper is organized as follows: Section 2 reviews the CP method. Section 3 introduces an improved version of the OWA method (called Revised OWA or ROWA), which offers a more sensitive way in obtaining the weights than by using the well-known version of the OWA method. Section 4 presents and discusses the mathematical relationship between the parameters of CP and ROWA. Section 5 illustrates these theoretical results with an illustrative case study of water resources management. And finally, Section 6 concludes the paper.

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2. CP (Compromise Programming) method

The original CP method attempts to minimize the distances of the evaluation vectors of the alternatives from the ideal point [30]. The main advantage of using this method is due to its simple conceptual structure. A large number of authors extended and used this methodology (e.g. [1–3,12,15,27]). Initially, CP calculates a combined distance for each alternative as

$$F_{CP} = \left[\sum_{i=1}^{n} a_i^p\right]^{1/p}, \quad 1 \leqslant p < \infty, \tag{1}$$

where a_i is the normalized difference between component i of the ideal point and that of the evaluation vector of an alternative $(1,2,\ldots,n)$. F_{CP} is a combined n-dimensional distance of any alternative from the ideal point, and so the alternative with the smallest combined distance is accepted as the most preferred solution.

Another version of the CP method maximizes the distance of the alternatives from the worst point (nadir). In this case the structure of the CP method will remain the same as before. In the first case, the objective is to minimize the distance from the ideal point while in the second case, maximizing the distance from the nadir is the objective.

The value of p expresses how strongly each difference is emphasized. The case of $p = 1$ corresponds to simple average; the case of $p = 2$ implies a simple squared weighting, while in the case of $p = \infty$, only the largest deviation is considered [2]. Tecle [15] gives a nice description: "Varying the parameter p from 1 to infinity, allows one to move from minimizing the sum of individual regrets (i.e., having a perfect compensation among the objectives) to minimizing the maximum regret (i.e., having no compensation among the objectives) in the decision making process. The choice of a particular value of this compensation parameter p depends on the type of problem and desired solution. In general, the greater the conflict between players, the smaller the possible compensation becomes".

The choice of the best alternative therefore depends very much on the selected value of p. There is however no precise mathematical method known from the literature to obtain this parameter. The focus of this study is to present a relationship between this parameter and the optimism degree of the DM in the OWA method. Based on this relationship we will be able to obtain a clear and straightforward approach to select the value of parameter p in the CP method.

3. Ordered Weighted Averaging operator

There are three basic types of aggregation operators known from the fuzzy set literature: operators for the intersection of fuzzy sets (t-norms, being the Min operator an example), operators for the union of fuzzy sets (t-conorms, being the Max operator an example), and averaging operators. Yager [21] introduced a special aggregation technique based on the OWA operator, which is a common generalization of the three basic aggregation functions. Since its introduction, the OWA operator has been used in many fields including neural networks, database systems, fuzzy logic controllers, expert systems, market research, linguistic quantified propositions, mathematical programming, lossless image compression, and also in solving MCDM problems [18].

An n-dimensional OWA operator assigns a combined measure for each alternative in a MCDM problem as

$$F_{OWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n, \tag{2}$$

where $F_{OWA} : R^n \mapsto R$, and b_j is the jth largest element in the set of the normalized inputs, a_i $(i = 1,2,\ldots,n)$. The coefficients, w_j, are the order weights such that $w_j \geqslant 0$ and $\sum_{j=1}^{n} w_j = 1$. Similarly to the CP method, the objective is to minimize F_{OWA}, if a_i represents the difference between the evaluation of an alternative with respect to criterion i and component i of the ideal point. Otherwise, if the a_i values represent the differences between the evaluations of an alternative and the components of the nadir, then the objective is to maximize F_{OWA}.

Notice that the components of the evaluation vector have been ordered before multiplying them by the order weights. The OWA method has a large variety by the different selections of the order weights [10]. The order weights depend on the optimism degree (well-known as Orness degree) of the DM [8,24]. In the case of a maximization problem larger weights at the beginning of the weight vector show a higher optimism degree (risk acceptance) of the DM. This is the case when the inputs are distances from the nadir components. Similarly in the case of minimization problems smaller weights at the beginning of the weight vector also indicate a higher optimism degree (risk acceptance). This is the case if the inputs are distances from the ideal point components. A DM is considered optimistic if he/she is willing to accept risk and a risk-averse DM is considered pessimistic, since he/she is afraid to take risk. For maximization problems, in his pioneering paper, Yager [21] defined the optimism degree, θ, based on these weights as follows:

$$\theta = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j. \tag{3}$$

Yager [22] and Xu [19,20] later gave a general overview of the different methods for determining the order weights.

The three major states of θ and the relevant optimism views of the DM are as follows:

1. The pessimistic view: $0.0 \leqslant \theta < 0.5$,
2. The neutral view: $\theta = 0.5$,
3. The optimistic view: $0.5 < \theta \leqslant 1.0$.

We will next introduce an improved version of the OWA method. In this method we will use linguistic terms such as most, few, many, and about half. These linguistic variables express impression of spatial information and human cognition over the evaluation criteria and feasible alternatives which are assessed, evaluated or ranked in the decision making problem [3]. Zadeh [26] recommended modeling these linguistic quantifiers by using fuzzy sets. For example, in the study of Nasibov and Kinay [11] different ways of weighting fuzzy opinions are proposed for the evaluation of student performances. The fuzzy sets however are not always easy to be obtained from the linguistic terms, therefore Truck and Akdag [17] chose to keep the words instead of going through the fuzzy modeling.

In this study the linguistic quantifiers are used to characterize the inputs of an OWA operator. These quantifiers are modeled by Regular Increasing Monotonic (RIM) quantifiers, Q, in which the more objects are included, the higher is the satisfaction of the DM. That is, $Q(r_1) \geqslant Q(r_2)$ as $r_1 \geqslant r_2$. We also assume that $Q(0) = 0$ and $Q(1) = 1$.

Yager [21] recommended obtaining the weights of an n-dimensional OWA operator by relation

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right), \quad j = 1, 2, \ldots, n. \tag{4}$$

For large values of n Yager [22] approximated dQ/dr at $r = j/n$ as

$$\left.\frac{dQ}{dr}\right|_{r=j/n} \approx \frac{Q(j/n) - Q((j-1)/n)}{1/n} = \frac{w_j}{1/n},$$

so the OWA weights can be approximated as follows:

$$w_j \approx \frac{1}{n}\left.\frac{dQ}{dr}\right|_{r=j/n}. \tag{5}$$

The accuracy of the approximation can be estimated by using the error formula for the linear Taylor's polynomial. Notice first that with some $r \in \left(\frac{j-1}{n}, \frac{j}{n}\right)$,

$$Q\left(\frac{j-1}{n}\right) = Q\left(\frac{j}{n}\right) - \frac{1}{n}Q'\left(\frac{j}{n}\right) + \frac{Q''(r)}{2n^2}$$

implying that

$$\left|w_j - \frac{1}{n}Q'\left(\frac{j}{n}\right)\right| \leqslant \frac{1}{2n^2}M_2, \tag{6}$$

where $|Q''(r)| \leqslant M_2$. Therefore, in the case of bounded second derivative of the quantifier Q the error of approximation (6) converges to zero as n tends to infinity.

Based on this approximation we can introduce a revised method to obtain the weights by using the exact derivative with the term j/n being replaced by b_j if the inputs are the distances from the ideal point, and by $1 - b_j$ if the inputs are the distances from the nadir. The main purpose of these selections is that the DM can make his/her choice based on the actual evaluation measures rather than by their relative orders.

The sum of the weights (4) equals 1, and since the weights (5) are only approximations of these weights, their sum is only approximately unity. We can ignore the error of this approximation by not normalizing the weights into unit sum.

We have to consider therefore two cases:

Case (A) If a_i is the distance from the ideal point components then

$$w_j = \frac{1}{n}\left.\frac{dQ}{dr}\right|_{r=b_j}; \tag{7a}$$

Case (B) If a_i is the distance from the nadir components then

$$w_j = \frac{1}{n}\left.\frac{dQ}{dr}\right|_{r=1-b_j}, \tag{7b}$$

where b_j is the jth element of the ordered set of the inputs a_i with $b_1 \geqslant b_2 \geqslant \cdots \geqslant b_n$. The reason that we use the terms $(1 - b_j)$ instead of b_j, in the second case, is due to the opposite ordering of the criteria in Eq. (5) in comparison to the ordering of the input values in the case of RIM quantifiers. It can be easily shown from Eq. (7b) that by using a concave quantifier the w_j values will become larger for larger inputs rather than for smaller inputs, which represents the optimistic view of the DM. Similarly the w_j values are larger for smaller inputs rather than for larger inputs by using a convex quantifier representing the pessimistic nature of the DM. For example, the quantifier shown in Fig. 1 is convex, which characterizes a pessimistic

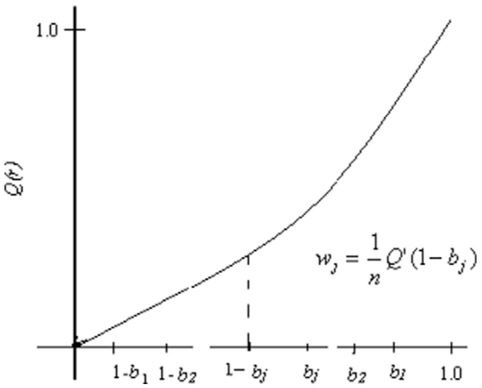

Fig. 1. Determining the OWA weights by using the derivatives of the RIM quantifiers.

DM. Its derivative is increasing; therefore larger value of $1 - b_j$ generates larger weights. In the case of concave quantifiers, their derivatives are decreasing, so larger value of $1 - b_j$ generates smaller weights. In the case of Eq. (7a) we have opposite conclusions.

This method of selecting the weights is based on the exact derivatives of the quantifier and the actual evaluation measures, so we will call this method the Revised OWA or ROWA [29]. The ROWA operator with weights (7a) and (7b) and with any fuzzy linguistic quantifier is a neat operator because the combined measure, F_{ROWA}, is independent of the ordering of the inputs. We have two variants of the ROWA operator depending on the types of the inputs.

In case A, we minimize the following objective function:

$$F_{\text{ROWA}}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j = \frac{1}{n} \sum_{j=1}^{n} Q'(b_j) b_j = \frac{1}{n} \sum_{i=1}^{n} Q'(a_i) a_i. \tag{8a}$$

In case B, we maximize the objective

$$F_{\text{ROWA}}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j = \frac{1}{n} \sum_{j=1}^{n} Q'(1 - b_j) b_j = \frac{1}{n} \sum_{i=1}^{n} Q'(1 - a_i) a_i. \tag{8b}$$

Comparing this procedure to the original version of OWA we see that an additional advantage of using neat OWA operator is that the DM makes his/her choices based on the context of the problem (e.g. to the b_j values). It is however a disadvantage that the weights have to be calculated individually for each alternative.

4. Comparison of the two methods

We will next explain the role of the p parameter in the CP method by using the semantics of the order weights of the ROWA method. Consider the CP method given in Eq. (1). The exponent $1/p$ does not change the ranking of the alternatives. We can also normalize the evaluation values into the unit interval $[0, 1]$ without changing their orders. The combined distance of each alternative from the ideal point or from the nadir in the CP method can be written as

$$F_{\text{CP}}(a_1, a_2, \ldots, a_n) = \frac{1}{n} \sum_{i=1}^{n} w_i a_i, \tag{9}$$

where $w_i = a_i^{p-1}$ and n is the number of criteria.

For any constant value of p, w_i is context based since it is related directly to the input value, a_i. For a larger input value, the corresponding weight,$w_i = a_i^{p-1}$, also becomes larger. As p increases, higher and higher weight is given to the larger input values. Therefore, with $p > 1$, the weights of a particular distance depend on the magnitudes of the inputs [31], which is similar to the basic idea of using the order weights in the OWA method. If the inputs are the distances from the ideal point, then a pessimistic DM will assign larger weights to larger inputs. Indeed this type of the CP method considers a pessimistic DM. The other type of the CP method (maximizing distance from nadir) considers the DM to be optimistic, since larger weights are assigned to larger values in the ordered set of the inputs as it was already discussed in Section 3. We will next mathematically prove these findings.

An MCDM problem with n criteria and m alternatives is considered. If it is solved by the CP and the ROWA methods, the resulting ranks of the alternatives by the two methods are the same for all problems if the corresponding weights in Eqs. (8a) and (8b) (for the ROWA method) and Eq. (9) (for the two types of CP) are the same.

In case A,

$$Q'(a_i) = a_i^{p-1} \tag{10}$$

which can be obtained from the quantifier

$$Q(r) = \frac{r^p}{p}. \tag{11}$$

In case B,

$$Q'(1 - a_i) = a_i^{p-1} \tag{12}$$

which is based on the quantifier

$$Q(r) = \left[1 - \frac{(1-r)^p}{p}\right]. \tag{13}$$

Eqs. (11) and (13) show a very strong relationship between the CP and OWA methods. Some particular quantifiers representing the CP model are shown in Fig. 2.

The optimism degree θ of the DM can be calculated by using fuzzy quantifiers. Yager [23] showed that for RIM quantifiers

$$\theta = \int_0^1 Q(r)dr, \tag{14}$$

which can be obtained from Eqs. (3) and (4) as n tends to infinity. Liu [9] generalized and extended Eq. (14). By substituting Eqs. (11) and (13) into (14) we get the following results.

In case A,

$$\theta = \int_0^1 \frac{r^p}{p} dr = \frac{1}{p(p+1)} \tag{15}$$

implying that $0.0 \leqslant \theta \leqslant 0.5$, since $p \geqslant 1$.

In case B:

$$\theta = \int_0^1 \left[1 - \frac{(1-r)^p}{p}\right] dr = 1 - \frac{1}{p(p+1)}, \tag{16}$$

implying that $0.5 \leqslant \theta \leqslant 1.0$.

The optimism degree is clearly related to the value of p. If the value of p is 1, then the optimism degree becomes 0.5, which represents the neutral view of the DM. In the case of distance minimizing CP method increasing value of p corresponds to the decreasing value of the optimism degree of the DM. However, the optimism degree increases for the CP method if the distance from the nadir is maximized as shown in Fig. 3. Consequently, the selection of Case A in the CP method has a pes-

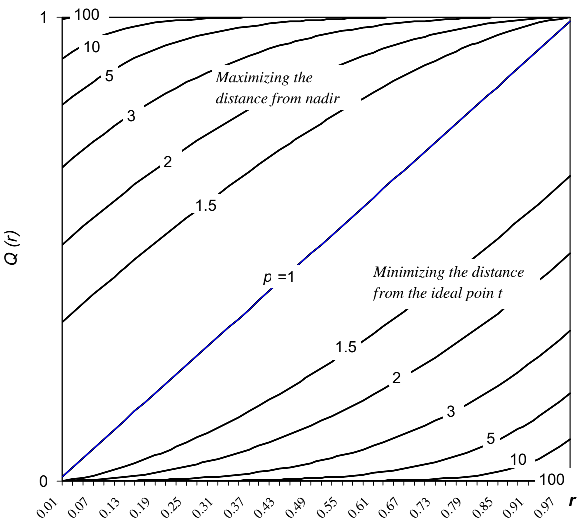

Fig. 2. Some particular quantifiers representing the CP method.

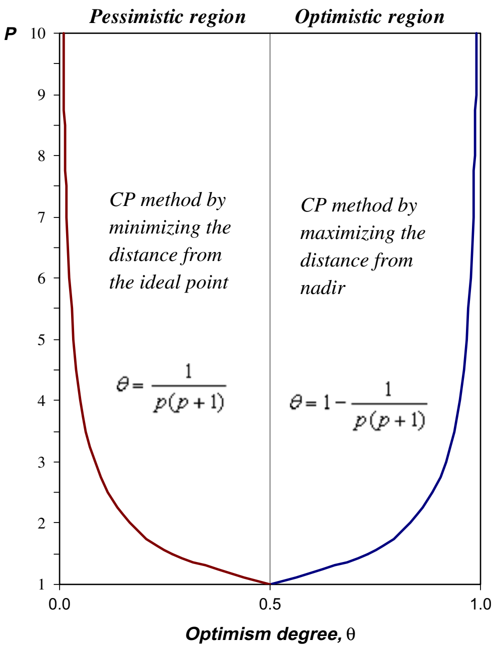

Fig. 3. The relation between the power parameter of the CP method and the optimism degree of the DM.

simistic bias without the attention to the real risk taking attitude of the DM and the choice of Case B of the CP method has an optimistic bias.

These results can also be explained based on regular human behavior. For example, consider a long distance runner: when he/she considers the remaining distance from the goal, he/she is pessimistic and tries to increase his/her effort. The same person becomes optimistic if he/she sees the increasing distance from the starting point.

The value of $p > 1$ in the CP method plays a similar role as θ in the OWA method. It models however only the pessimistic or the optimistic views. By selecting $p = 1$ both the CP and the ROWA methods are reduced to simple additive weighting (SAW).

A mathematical method to obtain the power parameter of the CP method is as follows. The DM should first select the number of criteria that have to be considered in the combined distance measure. Then based on the fuzzy quantifiers presented in Fig. 2, we are able to find the specific p value to be used in the CP model. The suggested linguistic variables and the relevant p values (from Eqs. (11) and (13)) and the θ values (from Eqs. (15) and (16), respectively) are listed in Table 1.

5. Case study

The focus of this case study is the inter-basin water transfer (IBWT) projects to the Zayanderud basin of Iran. The Central zone and the Southeast region of Iran are facing a high degree of water shortage. The socio-economic condition of the people can be significantly improved by water transfer from the neighboring basins. In order to meet the increasing water demand of the Zayanderud basin four possible IBWT projects (Gukan, Cheshmelangan, Kuhrang III and Behestabad) have been recently considered to transfer water from the Large Karun River to this region. However, due to the high growth potential of the Karun basin, there are various concerns about water transfers to other basins, which might generate conflicts among the stakeholders. Therefore, it is very important to carefully evaluate each possible IBWT project before even considering its implementation.

Table 1
Guideline to select fuzzy quantifiers and equivalent p and θ values.

Linguistic quantifier	Type of the CP model	p	θ	Situation
All of the criteria	(A) Minimizing the distance from the ideal point	10	0.01	Pessimistic
Most of the criteria		2	0.17	
Many of the criteria		1.5	0.27	
Half of the criteria		1	0.50	Neutral
Some of the criteria	(B) Maximizing the distance from nadir	1.5	0.73	Optimistic
Few of the criteria		2	0.83	
At least one criteria		10	0.99	

The evaluation of the four projects with respect to seven criteria is presented in Table 2. These data were obtained from a group of experts from the DM's company, which is responsible for these projects. The uncertainty of the data is represented by triangular fuzzy numbers (TFN) or linguistic variables. The following linguistic variables are used: Very Low (VL), Low (L), Slightly Low (SL), Medium (M), Slightly High (SH), High (H), and Very High (VH).

Many earlier studies promoted the CP method since it is considered the most suitable approach for solving MCDM problems in the field of water resources management (e.g. [12,14]). We will next calculate the combined distances from the ideal point and from the nadir by using both the CP and the ROWA methods. Before applying these methods, the original data of Table 2 are synthesized as follows:

Step 1. The evaluations of the projects with respect to the criteria are either linguistic or fuzzy. The linguistic data are transformed into fuzzy numbers according to the scales shown in Table 3, and then these fuzzy numbers are defuzzified by using the centroid method.

Step 2. The defuzzified evaluations are normalized into the unit interval [0,1] by using the following linear transformations.
In case A,

$$a_i = \frac{f_i^* - f_i}{f_i^* - f_{wi}}, \tag{17}$$

and in Case B,

$$a_i = 1 - \frac{f_i^* - f_i}{f_i^* - f_{wi}} = \frac{f_i - f_{wi}}{f_i^* - f_{wi}}. \tag{18}$$

Here f_i is the defuzzified evaluation of an alternative with respect to criterion i, f_i^* is the desired value of this criterion and f_{wi} is its worst value. For positive criteria they are selected as the maximum and the minimum values of the inputs, respectively. For negative criteria, they are selected in the opposite order.

Step 3. In the original version of OWA the criteria weights are considered to be equal, however in this case they are different (Table 2), so the normalized evaluations values are multiplied by their weights. The final evaluation matrix showing the distances from the ideal point is shown in Table 4, and based on these values the combined distances can be computed by using the CP method.

Case (A) *CP method with minimizing the distance from the ideal point*
The results of applying this CP method are shown in Fig. 4. They are the same as those obtained by using the ROWA method with the specific quantifier presented in Eq. (11). The results are robust since the ranking of the projects remains the same in the entire region of $p \geqslant 1.0$ (as well as for $0.0 < \theta \leqslant 0.5$). Kuhrangh III has the smallest combined distance, and therefore it is the most preferred alternative. The Cheshmelangan and the Gukan projects are the second and third most preferred ones while Beheshtabad has the lowest preference.

Table 2
Evaluation matrix of the IBWT projects (based on [28]).

Alternatives	Criteria						
	Range of environmental impacts	Benefit/cost	Consistency with policies	Public participation	Resettlement of people	Diversification of financial resources	Allocation of water to prior usages
	Weights of criteria						
	SL (negative)	M	H	L	VH (negative)	M	VH
1 Gukan	L	(1.5,0.1,0.1)	H	SH	(0, 0, 0.1)	(5,1,1)	SH
2 Cheshmelangan	M	(1.4,0.3,0.3)	VH	M	(0,0,0.1)	(0,0,0.2)	VH
3 Kuhrang III	SL	(1.1,0.3,0.3)	VH	H	(200,50,50)	(3,1,1)	VH
4 Beheshtabad	SH	(1.6,0.3,0.3)	H	VH	(4000,50,50)	(4,1,1)	VH

Table 3
Linguistic variables and equivalent TFNs.

Linguistic variables	Number
Very Low	(0,0,0.1)
Low	(0.2,0.1,0.2)
Slightly Low	(0.35,0.2,0.2)
Medium	(0.5,0.2,0.2)
Slightly High	(0.65,0.2,0.2)
High	(0.8,0.2,0.1)
Very High	(1.0,0.1,0)

Table 4
The weighted normalized inputs (distances from the ideal point).

Seven criteria							
Gukan	0.000	0.100	0.800	0.133	0.000	0.000	0.950
Cheshmelangan	0.233	0.200	0.000	0.200	0.000	0.500	0.000
Kuhrang III	0.117	0.500	0.000	0.067	0.047	0.204	0.000
Beheshtabad	0.350	0.000	0.800	0.000	0.950	0.102	0.000

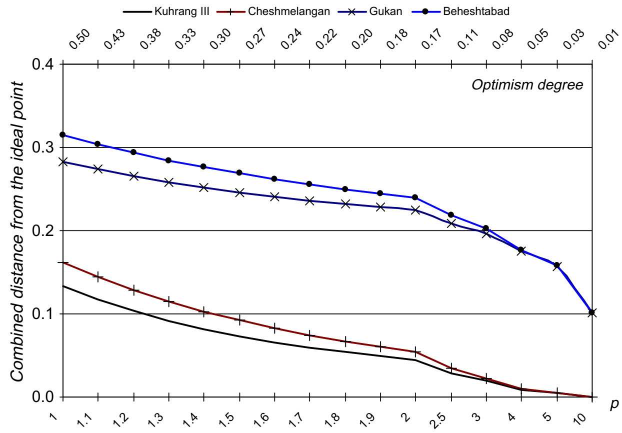

Fig. 4. The combined distances from the ideal point by using the CP (Case A) and the corresponding ROWA methods.

Case (B) *CP method with maximizing the distance from the nadir*The combined distances from the nadir by using the CP method (Case B) are shown in Fig. 5. They are the same as the results of the ROWA method with the specific quantifier presented in Eq. (13). The most preferred project has the largest combined distance from the nadir and the smallest distance refers to the least preferred alternative. The ranking of the projects, in the interval $1.0 < p < 1.8$, is similar to the result obtained previously (Fig. 4), however for $p > 1.8$ and $0.8 < \theta < 1.0$, Cheshmelangan becomes the most preferred project.

Table 5 shows the rankings of the alternatives based on the two types of the CP method, and in the second column we present the current states of the projects which are already decided by the DM. Cheshmelangan is in operation (rank 1), Kuhrang III is under construction (rank 2), Gukan is in the stage of final study (rank 3) and Beheshtabad is under investigation (rank 4).

According to Table 5, the ranks of Gukan and Beheshtabad are the same in the current state of the projects as those obtained by using both types of CP and the corresponding ROWA methods. However, Cheshmelangan and Kuhrang III are either

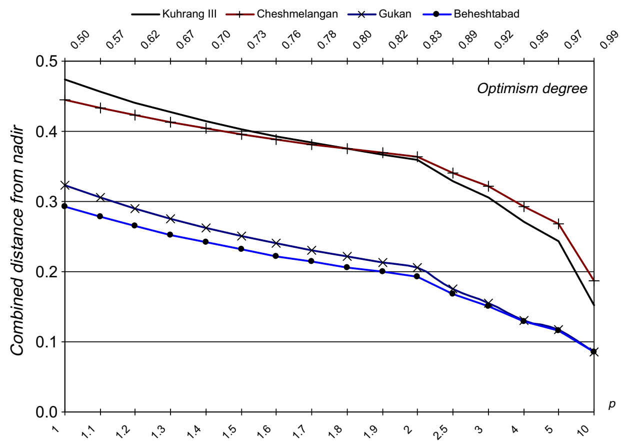

Fig. 5. The combined distances from the nadir by using the CP (Case B) and the corresponding ROWA methods.

Table 5
Rankings of the IBWT projects.

Projects	Current state of the projects	CP method		
		(A) Minimizing distance to the ideal point	(B) Maximizing distance from the nadir	
		$p > 1$	$1 < p < 1.8$	$p > 1.8$
		Corresponding ROWA method		
		$0 < \theta < 0.5$	$0.5 < \theta < 0.8$	$0.8 < \theta < 1$
Gukan	3	3	3	3
Cheshmelangan	1	2	2	1
Kuhrang III	2	1	1	2
Beheshtabad	4	4	4	4

Table 6
General comparison of CP and ROWA.

Evaluation criteria	CP	ROWA
Modeling the complete range of the optimism degree	No	Yes
Using fuzzy quantifiers	No	Yes
Context based model	Yes	Yes
Existing tools for obtaining the parameters	Low	High
Time needed by the DM	High	Low
Linear objective function	No	Yes

in rank 1 or in rank 2. Notice that the initial decisions of the DM (shown by the current state of the projects) are the same as the results shown in the last column of Table 5. These results are obtained by the CP method with $p > 1.8$ and the corresponding ROWA method with $0.8 < \theta < 1.0$. These results were obtained with the optimistic aggregation of the inputs, so the DM was originally risk-prone about the IBWT projects. Water managers are usually risk-averse persons. However in this case, the DM is water receiver not water supplier and therefore the objective is to bring as much water as possible to the Zayanderud basin, which explains his/her optimistic view.

Based on the theoretical results and the case study, a general comparison of the CP and the ROWA methods is presented in Table 6. It is clear that ROWA is more suitable overall than distance based models like CP. Since each variant of the CP method models pessimistic or optimistic DM, it is not appropriate to be used for solving MCDM problems with uncertain and unknown optimism degrees of the DM. This is however the case in many water resources management problems.

6. Conclusions

This paper demonstrated a strong relationship between the CP method and OWA operators. The CP method actually follows a specific fuzzy quantifier which provides a method for the suitable selection of the power parameter of the CP method. We also developed a strong mathematical relationship between the power value and the optimism degree of the DM. It was shown that the CP method models a pessimistic DM, if it minimizes the combined distance from the ideal point and it produces optimistic results if the distance from the nadir is maximized. However, ROWA can cover both cases since it models the entire scale of optimistic and pessimistic views of the DM by using a quantifiable measure. The theoretical results were successfully illustrated by using a real case study of inter-basin water transfer projects.

Finally we note that another alternative way of revising the OWA method is based on the weights

$$\bar{w}_j = Q(b_j) - Q(b_{j-1}), \quad (b_0 = 0) \tag{19}$$

which also uses the evaluation measures instead of the rankings of the criteria and does not include the errors of the derivative approximations. The sum of these weights is $Q(b_n)$, so after normalizing the weights we have the modified revised OWA weights as

$$w_j = \frac{Q(b_j) - Q(b_{j-1})}{Q(b_n)}, \tag{20}$$

and the resulting OWA is also a neat operator; however the weights have to be recalculated for each alternative. The theoretical properties of the weight selection (20) and its applications will be the subject of our future research.

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