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DIFFEOMORPHISMS WITH VARIOUS C^1 STABLE PROPERTIES*

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Abstract Let M be a smooth compact manifold and Λ be a compact invariant set. In this article, we prove that, for every robustly transitive set Λ, $f|_\Lambda$ satisfies a C^1-generic-stable shadowable property (resp., C^1-generic-stable transitive specification property or C^1-generic-stable barycenter property) if and only if Λ is a hyperbolic basic set. In particular, $f|_\Lambda$ satisfies a C^1-stable shadowable property (resp., C^1-stable transitive specification property or C^1-stable barycenter property) if and only if Λ is a hyperbolic basic set. Similar results are valid for volume-preserving case.

Key words Specification property; hyperbolic basic set; topologically transitive; shadowing property

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1 Introduction

In the studies of dynamical systems, the pseudo-orbit shadowing property usually plays an important role in the investigation of stability theory and ergodic theory. Wen, Gan and Wen [13] proved that C^1-stably shadowable chain component is hyperbolic. Lee, Morivasu, Sakai [6] showed that a chain recurrent set has C^1-stable shadowing property if and only if the system satisfies both Axiom A and the no-cycle and also proved that a chain component containing a hyperbolic periodic point p has C^1-stable shadowing property if and only if it is the hyperbolic homoclinic class of p. Moreover, Tajbakhsh and Lee [12] proved that a homoclinic class has C^1-stable shadowing property if and only if it is hyperbolic. Recently, Sakai, Sumi, and Yamamoto showed that a closed invariant set satisfies C^1-stable specification property if and only if it is a hyperbolic elementary set. Because the specification property

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there naturally implies topologically mixing, they gave a characterization of the hyperbolic (mixing) elementary sets. Specification property is due to Bowen and Sigmund and holds for every mixing compact set with shadowing property. In this article, we show that, for every transitive compact set with shadowing property, a version of transitive specification property is true. Furthermore, we also discuss a notion called barycenter property due to Abdenur, Bonatti, Crovisier [1], weaker than transitive specification property. Here, we are mainly to characterize the diffeomorphsims satisfying C^1-generic-stable shadowable property, transitive specification property, or barycenter property (for particular case, C^1-stable shadowable property, transitive specification property or barycenter property). More precisely, for a robustly transitive set, it has one of the above properties if and only if it is a hyperbolic basic set.

Let (M, d) denote a compact metric space and $f : M \to M$ be a homeomorphism. Let Λ be a compact and f-invariant set and $f|_\Lambda$ be the restriction of f on the set Λ. Now, we start to introduce the notion of shadowing, specification, and barycenter properties. A sequence $\{y_n\}_{n=a}^b \subseteq \Lambda$ is called a δ-pseudo-orbit ($\delta \geq 0$) of f if $d(f(y_n), y_{n+1}) \leq \delta$ for every $a \leq n \leq b$. A system $f|_\Lambda$ is said to have the shadowing property if for every $\varepsilon > 0$ there exists $\delta > 0$ such that, for a given δ-pseudo-orbit $y = \{y_n\}_{n=a}^b \subseteq \Lambda$, we can find $x \in \Lambda$, which ε-traces y, that is, $d(f^n(x), y_n) < \varepsilon$ for every $a \leq n \leq b$. $f|_\Lambda$ is said to satisfy the transitive specification property if the following holds: for any $\varepsilon > 0$, there exists an increasing sequence of integers $M_0(\varepsilon) = 0 < M_1(\varepsilon) < M_2(\varepsilon) < \cdots$ tends to $+\infty$ such that, for any $k \geq 1$, $n \geq 1$, any k points $x_1, x_2, \cdots, x_k \in \Lambda$, and any integers n_1, n_2, $\cdots$, n_k, there exists a point $z \in \Lambda$ and a sequence of integers $c_1 = 0 < c_2 < \cdots < c_k$ with $c_{j+1} - c_j - n_j \in [M_{n-1}(\varepsilon), M_n(\varepsilon)]$ $(j = 1, 2, \cdots, k-1)$, such that $d(f^{c_j+i}(z), f^i(x_j)) < \varepsilon$, $0 \leq i \leq n_j$, $1 \leq j \leq k$. Now, we begin to recall the barycenter property (a little different to [1]). Let $P(f|_\Lambda)$ be the set of periodic points of f in Λ. In particular, set $P(f) = P(f|_M)$. Given two periodic points $p, q \in P(f|_\Lambda)$, we say p, q have the barycenter property, if for any $\varepsilon > 0$, there exists an integer $N = M(\varepsilon, p, q) > 0$ such that, for any two integers n_1, n_2, there exist a point $z \in \Lambda$ and an integer $X \in [0, N]$, such that $d(f^i(z), f^i(p)) < \varepsilon$, $-n_1 \leq i \leq 0$, and $d(f^{i+X}(z), f^i(q)) < \varepsilon$, $0 \leq i \leq n_2$. $f|_\Lambda$ is said to satisfy the barycenter property if the barycenter property holds for any two periodic points p, $q \in P(f|_\Lambda)$.

Obviously, the barycenter property is weaker than the transitive specification property. The transitive specification property means that whenever there are k pieces of orbits they may be approximated up to ε by one orbit, provided that the time for switching from the forward piece of orbit to the afterward and the time for switching back are bounded between two integers $M_{n-1}(\varepsilon) \leq M_n(\varepsilon)$, these integers $M_n(\varepsilon)$ being independent of the lengths of the k pieces of orbits. Here, this notion of transitive specification property is weaker than the usual (mixing) specification property defined in Sigmund [11].

Let Λ be as before. Λ is transitive if there is some $x \in \Lambda$ whose forward orbit is dense in Λ. A transitive set Λ is trivial if it consists of a periodic orbit. Note that transitive specification property implies that Λ is topologically transitive. Λ is locally maximal in some neighborhood $U \subseteq M$ of Λ if $\Lambda = \bigcap\limits_{k\in\mathbb{Z}} f^k(U)$. A set Λ is a basic set (resp. elementary set) if Λ is locally maximal and $f|_\Lambda$ is transitive (respectively, topologically mixing). In a Baire space X, we call $\mathcal{R} \subseteq X$ a residual set if it contains a dense G_δ set.

Let M be a closed C^∞ manifold and let $\mathrm{Diff}(M)$ be the space of diffeomorphisms of M

endowed with the C^1-topology. Denote by d the distance on M induced from a Riemannian metric on the tangent bundle TM. Given $f \in \mathrm{Diff}(M)$, denote by $\mathcal{O}(p)$ the periodic f-orbit of $p \in P(f)$. If $p \in P(f)$ is a hyperbolic saddle with period $\pi(p) > 0$, then, there are the local stable manifold $W^s_\varepsilon(p)$ and the local unstable manifold $W^u_\varepsilon(p)$ of p for some $\varepsilon = \varepsilon(p) > 0$. It is seen that if $d(f^n(x), f^n(p)) \le \varepsilon$ for any $n \ge 0$, then $x \in W^s_\varepsilon(p)$ (a similar property also holds for local unstable manifold $W^u_\varepsilon(p)$ with respect to f^{-1}). The stable manifold $W^s_\varepsilon(p)$ and the unstable manifold $W^u_\varepsilon(p)$ of p are defined as usual. The dimension of the stable manifold $W^s_\varepsilon(p)$ is called the index of p, and denoted by index(p).

An f invariant compact set Λ is robustly transitive in some neighborhood U if Λ is locally maximal in U and there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$(called a continuation of $\Lambda_f(U) = \Lambda$) is transitive.

Now, we state our main result as follows.

Theorem 1.1 Let Λ be an f invariant compact set and assume that Λ is robustly transitive in some neighborhood U. Then, the following conditions are equivalent:

(1) $f|_\Lambda$ is C^1-generic-stably shadowable, that is, there is a C^1-neighborhood $\mathcal{U}(f)$ of f and a residual set $\mathcal{R} \subseteq \mathcal{U}(f)$ such that, for any $g \in \mathcal{R}$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has shadowing property;

(1′) $f|_\Lambda$ is C^1-stably shadowable, that is, there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has shadowing property;

(2) $f|_\Lambda$ satisfies the C^1-generic-stable transitive specification property, that is, there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a residual set $\mathcal{R} \subseteq \mathcal{U}(f)$ such that, for any $g \in \mathcal{R}$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has transitive specification property;

(2′) $f|_\Lambda$ satisfies the C^1-stable transitive specification property, that is, there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has transitive specification property;

(3) $f|_\Lambda$ satisfies the C^1-generic-stable barycenter property, that is, there are a C^1- neighborhood $\mathcal{U}(f)$ of f and a residual set $\mathcal{R} \subseteq \mathcal{U}(f)$ such that, for any $g \in \mathcal{R}$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has barycenter property;

(3′) $f|_\Lambda$ satisfies the C^1-stable barycenter property, that is, there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ has barycenter property;

(4) there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a residual set $\mathcal{R} \subseteq \mathcal{U}(f)$ such that, for any $g \in \mathcal{R}$, any two periodic hyperbolic saddles p, $q \in \Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$, $W^s(\mathcal{O}(p)) \cap W^u(\mathcal{O}(q)) \neq \emptyset$;

(4′) there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, any two periodic hyperbolic saddles p, $q \in \Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$, $W^s(\mathcal{O}(p)) \cap W^u(\mathcal{O}(q)) \neq \emptyset$;

(5) there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, any periodic point of $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ is hyperbolic and has the same index;

(6) there is a C^1-neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, $\Lambda_g(U) := \bigcap_{k\in\mathbb{Z}} g^k(U)$ is a hyperbolic basic set.

Remarks 1. The result in [10] is a particular case of (2′) in Theorem 1.1 (being as the mixing case), because the specification property assumed on $\Lambda_g(U)$ in [10] naturally implies that $\Lambda_g(U)$ is topologically mixing (that is, Λ is robustly mixing). And we point out that the codition (2) in Theorem 1.1 can be also as a generalization of the result in [10], because (2) is weaker than (2′).

2. From [10], the set of transitive Anosov diffeomorphisms is a characterization of the set of diffeomorphisms satisfying C^1-stable mixing specification property. Here, by Theorem 1.1, it is also a characterization of the set of diffeomorphisms satisfying C^1-stable (or generic-stable) transitive specification property. Furthermore, if M is topologically transitive, the set of Anosov diffeomorphisms is also a characterization of C^1-stable (or generic-stable) shadowable property or C^1-stable (or generic-stable) barycenter property.

The equivalence of (5) and (6) is due to Mañé [7], and using this, Sakai, Sumi and Yamamoto [10] proved that $f|_{\Lambda_f(U)}$ satisfies the C^1-stable specification property (mixing case) if and only if Λ is a hyperbolic elementary set. Actually, it is essentially proved $(4') \Rightarrow (5)$ in [10]. More precisely, $(4')$ implies any two hyperbolic periodic saddles $p, q \in \Lambda_g(U)$ have the same index (see the proof of Lemma 2.2 in [10]) and the latter implies that all periodic points in $\Lambda_g(U)$ are hyperbolic (see Lemma 2.4 in [10]). $(6) \Rightarrow (1')$, $(a') \Rightarrow (a)$ $(a = 1, 2, 3, 4)$, and $(2) \Rightarrow (3)$ are obvious and thus it is enough to show $(1) \Rightarrow (2)$, $(3) \Rightarrow (4)$, and $(4) \Rightarrow (5)$.

2 Proof of Our Main Theorem

To prove $(1) \Rightarrow (2)$, $(3) \Rightarrow (4)$, and $(4) \Rightarrow (5)$ in our main theorem, we divide them into three lemmas. First, we show a general lemma which implies $(1) \Rightarrow (2)$.

Lemma 2.1 Let $f : M \to M$ be a homeomorphism on a compact metric space M and Λ be a transitive f-invariant subset. If $f|_\Lambda$ satisfies the shadowing property, then, Λ satisfies the transitive specification property.

Remark If we assume Λ is mixing in Lemma 2.1, then (f, Λ) satisfies the mixing specification property of Sigmund [11].

Proof of Lemma 2.1 For any $\varepsilon > 0$, by shadowing property, there exists $\delta > 0$ such that any δ-pseudo orbit in Λ can be ε shadowed by a true orbit in Λ.

Take and fix for Λ a finite cover $\alpha = \{U_1, U_2, \cdots, U_{r_0}\}$ by nonempty open balls U_i in Λ satisfying $\mathrm{diam}(U_i) < \delta$, $i = 1, 2, \cdots, r_0$. As Λ is transitive, for any $i, j = 1, 2, \cdots, r_0$, there exist a positive integer $X_{i,j}^{(1)}$ such that

$$f^{-X_{i,j}^{(1)}}(U_i) \cap U_j \neq \emptyset.$$

Let

$$M_1 = \max_{1 \le i \neq j \le r_0} X_{i,j}^{(1)}.$$

Similarly, for any $i, j = 1, 2, \cdots, r_0$, we can take a positive integer $X_{i,j}^{(2)} \geq M_1$, such that

$$f^{-X_{i,j}^{(2)}}(U_i) \cap U_j \neq \emptyset.$$

Let

$$M_2 = \max_{1 \le i \neq j \le r_0} X_{i,j}^{(2)}.$$

By induction, for any $i,\, j = 1,\, 2,\, \cdots,\, r_0$, there is a sequence of increasing integers $1 \le X_{i,\,j}^{(1)} < X_{i,\,j}^{(2)} < \cdots < X_{i,\,j}^{(n)} < \cdots < +\infty$, such that

$$f^{-X_{i,\,j}^{(n)}}(U_i) \cap U_j \neq \emptyset$$

and

$$X_{i,\,j}^{(n)} \ge M_{n-1},$$

where

$$M_{n-1} = \max_{1 \le i \neq j \le r_0} X_{i,\,j}^{(n-1)}.$$

Setting $M_0 = 0$, clearly, $\{M_n\}_{n \ge 0}$ is an increasing sequence tending to $+\infty$.

Now, let us consider a given sequence of points $x_1,\, x_2,\, \cdots,\, x_k \in \Lambda$, and a sequence of positive numbers $n_1,\, n_2,\, \cdots,\, n_k$. Take and fix $U_{i_0},\, U_{i_1} \in \alpha$ such that $x_i \in U_{i_0}$, $f^{n_i}(x_i) \in U_{i_1}, i = 1,\, 2,\, \cdots,\, k$. Fixing an integer $n \ge 1$, take $y_i \in U_{i_1}$, such that $f^{X_{(i+1)_0,\, i_1}^{(n)}}(y_i) \in U_{(i+1)_0}$ for $i = 1,\, 2,\, \cdots,\, k-1$. Take $y_k \in U_{k_1}$, such that $f^{X_{(k+1)_0,\, k_1}^{(n)}}(y_k) \in U_{1_0}$. Thus, we get a periodic δ-pseudoorbit in Λ:

$$\{f^t(x_1)\}_{t=0}^{n_1} \cup \{f^t(y_1)\}_{t=0}^{X_{2_0,1_1}^{(n)}} \cup \{f^t(x_2)\}_{t=0}^{n_2} \cup \cdots \cup \{f^t(x_k)\}_{t=0}^{n_k} \cup \{f^t(y_k)\}_{t=0}^{X_{(k+1)_0,\, k_1}^{(n)}}.$$

Hence, there exists a point $z \in \Lambda$ ε-shadowing the above sequence. More precisely,

$$d(f^{c_{i-1}+j}(z), f^j(x_i)) < \varepsilon,\ j = 0,\, 1,\, \cdots,\, n_i,\ \ i = 1,\, 2,\, \cdots,\, k,$$

where c_i is defined as

$$c_i = \begin{cases} 0, & \text{for } i = 0, \\ \displaystyle\sum_{j=1}^{i} [n_j + X_{(j+1)_0,\, j_1}^{(n)}], & \text{for } \ i = 1,\, 2,\, \cdots,\, k. \end{cases}$$

Secondly, we prove a lemma about the relationship of homoclinic related property and barycenter property, which deduces (3) $\Leftrightarrow$ (4) of our main theorem.

Lemma 2.2 Let $f : M \to M$ be a diffeomorphism on a compact manifold M. Then, for two hyperbolic periodic points $p,\, q \in P(f)$, $p,\, q$ have the barycenter property $\Leftrightarrow W^u(\mathcal{O}(p)) \cap W^s(\mathcal{O}(q)) \neq \emptyset$.

Proof of Lemma 2.2

"$\Rightarrow$": Let $\varepsilon(p)$ and $\varepsilon(q) > 0$ be as before with respect to p and q. Take $\varepsilon = \min\{\varepsilon(p),\, \varepsilon(q)\}$, and let $N = N(\varepsilon, p, q) > 0$ be the number of barycenter properties. For any $n \ge 0$, by barycenter property, there exist $z_n \in \Lambda$ and an integer $X_n \in [0, N]$, such that

(i) $d(f^j(z_n), f^j(p)) \le \varepsilon$ for $-n \le j \le 0$,

(ii) $d(f^{j+X_n}(z_n), f^j(q)) \le \varepsilon$ for $0 \le j \le n$.

Take a subsequence $\{n_k\}$ such that $n_k \to \infty$ and $X_{n_k} \equiv X$ for some fixed integer $X \in [0, N]$. Let $z = \lim_{k \to +\infty} z_{n_k}$ by taking a subsequence again if necessary. By (i) and (ii), one has $z \in W^u_{\varepsilon(p)}(p) \subseteq W^u(p)$ and $f^X(z) \in W^s_{\varepsilon(q)}(q) \subseteq W^u(q)$.

"$\Leftarrow$": If $W^u(\mathcal{O}(p)) \cap W^s(\mathcal{O}(q)) \neq \emptyset$, then we can take $z \in W^u(\mathcal{O}(p)) \cap W^s(\mathcal{O}(q))$. So, for any $\varepsilon > 0$, there is $N_1 = N_1(\varepsilon, p, q) > 0$ such that $d(f^j(z), f^j(p)) < \varepsilon$ for all $j \le -N_1$ and

$d(f^j(z), f^j(q)) < \varepsilon$ for all $j \geq N_1$. Moreover, we can assume N_1 to be a common multiple of the period of p and q. Put $x = f^{-N_1}(z)$ and let $N = 2N_1$. Then, for any two integers n_1, n_2, x is needed for barycenter property, that is, $d(f^j(x), f^j(p)) = d(f^{j-N_1}(z), f^{j-N_1}(p)) < \varepsilon$ for all $-n_1 \leq j \leq 0$ and $d(f^{j+N}(x), f^j(q)) = d(f^{j+N_1}(z), f^{j+N_1}(q)) < \varepsilon$ for all $0 \leq j \leq n_2$.

Before proving (4) $\Rightarrow$ (5), we state a lemma which says that (4) $\Rightarrow$ all hyperbolic points have the same index. A diffeomorphism f is said to be Kupka-Smale if the periodic points of f are hyperbolic and for any two periodic points p, q of f, $W^s(p)$ is transversal to $W^u(q)$. It is well known that the set of Kupka-Samle diffeomorphisms is C^1-residual in $\mathrm{Diff}(M)$ (see [9]). Note that, if $\mathcal{U}$ is an open set of $\mathrm{Diff}(M)$, then the set of Kupka-Samle diffeomorphisms restricted on $\mathcal{U}$ is still C^1-residual in $\mathcal{U}$.

Lemma 2.3 Let $f : M \to M$ be a diffeomorphism on a compact manifold M. Then, condition (4) in Theorem 1.1 implies that, for any two hyperbolic saddles p, $q \in \Lambda_g(U) \cap P(g)$ with respect to $g \in \mathcal{U}(f)$, $\mathrm{index}(p) = \mathrm{index}(q)$.

Proof This proof is an adaption of Lemma 2.2 in [10]. Let $\mathcal{U}(f)$ be as in condition (4) of Theorem 1.1. Fix a $g \in \mathcal{U}(f)$, and let $p, q \in \Lambda_g(U) \cap P(g)$ be hyperbolic saddles. Then, there is a C^1-neighborhood $\mathcal{V}(g) \subseteq \mathcal{U}(f)$ such that, for any $\varphi \in \mathcal{V}(g)$, there are continuations p_φ and q_φ (of p and q) in $\Lambda_\varphi(U)$, respectively (As $\Lambda_\varphi(U) = \Lambda \subset \mathrm{int}U$, we can assume that $\Lambda_g(U) \subset \mathrm{int}U$ for any $g \in \mathcal{U}(f)$, reducing $\mathcal{U}(f)$ if necessary).

By contradiction, if $\mathrm{index}(p) < \mathrm{index}(q)$ (the other case is similar), then, we have

$$\dim W^s(p, g) + \dim W^u(q, g) < \dim M,$$

where $W^s(p, g)$ and $W^u(q, g)$ are respectively the stable and unstable manifolds of p and q with respect to g. As the intersection of two residual sets is still residual, then the set of diffeomorphisms restricted on $\mathcal{V}(g)$ satisfying not only Kupka-Smale but also condition (4) of Theorem 1.1 is still residual in $\mathcal{V}(g)$. Take such a diffeomorphism $\varphi \in \mathcal{V}(g)$. Then,

$$W^s(p_\varphi, \varphi) \cap W^u(q_\varphi, \varphi) = \emptyset,$$

because $\dim W^s(p, g) = \dim W^s(p_\varphi, \varphi)$ and $\dim W^u(q, g) = \dim W^u(q_\varphi, \varphi)$. In contrast, because φ is a diffeomorphism satisfying condition (4) of Theorem 1.1, then,

$$W^s(p_\varphi, \varphi) \cap W^u(q_\varphi, \varphi) \neq \emptyset.$$

This is a contradiction.

End of proof of (4)⇒(5) By lemma 2.3, we only need to prove that every periodic point $p \in \Lambda_g(U)$ of $g \in \mathcal{U}(f)$ is hyperbolic. By contradiction, suppose that $p \in \Lambda_g(U)$ of $g \in \mathcal{U}(f)$ is not hyperbolic. Then, by Lemma 2.4 in [10], there is $\varphi \in \mathcal{U}(f)$ possessing hyperbolic points q_1 and q_2 in $\Lambda_\varphi(U)$ with different indices. This is a contradiction to Lemma 2.3.

3 A Remark for Volume-Preserving Version

Let ω be a volume measure on the smooth compact manifold M and $\mathrm{Diff}_\omega(M)$ be the space of diffeomorphisms preserving ω. We point out the statements in Theorem 1.1 may be changed for the volume-preserving diffeomorphisms, because all the main techniques may be replaced by those of volume-preserving version, that is, the set of transitive volume-preserving Anosov diffeomorphisms is a characterization of the set of volume-preserving diffeomorphisms satisfying C^1-stable (or generic-stable) shadowable property, C^1-stable (or generic-stable) transitive or mixing specification property or C^1-stable (or generic-stable) barycenter property. Let us explain it more precisely as follows. The equivalence of condition (5) and (6) (see [7]) in Theorem 1.1 can be replaced by the recent result in [2], Frank's lemma [5] (important to prove Lemma 2.4 in [10], which is needed in our proof of (4)⇒(5)), can be replaced by the pasting lemma for volume-preserving systems (see [3]) and Kupka-Smale property for volume-preserving case can be found in [8]. In particular, we point out that we need not assume the robust transitivity of M when we prove the result that one volume-preserving diffeomorphism satisfying C^1-stable (or generic-stable) shadowable property is Anosov, because generic volume-preserving diffeomorphisms are transitive from [4] (and so that, by Lemma 2.1, generic-stable shadowing implies generic-stable transitive specification for volume-preserving). Moreover, we note that volume-preserving Anosov diffeomorphisms are always transitive from the viewpoint of structurally stable property of Anosov systems and the transitivity of generic volume-preserving diffeomorphisms [4], because every volume-preserving Anosov diffeomorphism has a topologically conjugated diffeomorphism arbitrarily nearby which can also be chosen transitive from [4], and transitivity is an invariant property under conjugation. So, the statements above for volume-preserving case can be directly as a characterization of (not necessarily adding "transitive") volume-preserving Anosov diffeomorphisms.

References

[1] Abdenur F, Bonatti C, Crovisier S. Nonuniform hyperbolicity of C^1-generic diffeomorphisms (to appear in Israel J Math)

[2] Arbieto A, Catalan T. Hyperbolicity in the Volume Preserving Scenario. arXiv: 1004.1664, Preprint, 2010

[3] Arbieto A, Matheus C. A pasting lemma and some applications for conservative systems. Ergodic Theory and Dynamical Systems, 2007, **27**: 1399–1417

[4] Bonatti C, Crovisier S. Recurrence et generecite. Inv Math, 2004, **158**: 33–104

[5] Franks J. Necessary conditions for stability of diffeomorphisms. Trans Amer Math Soc, 1971, **158**: 301–308

[6] Lee K, Morivasu K, Sakai K. C^1 stable shadowing diffeomorphisms. Disc Contin Dyn Syst, 2008, **22**: 683–697

[7] Mañé R. Ergodic Theory and Differentiable Dynamics. Springer-Verlag, 1987

[8] Robinson C. Generic properties of conservative systems. Amer J Math, 1970, **92**: 562–603

[9] Robinson C. Dynamical systems: stability, symbolic dynamics, and chaos. CRC Press LLC, 1999

[10] Sakai K, Sumi N, Yamamoto K. Diffeomorphisms satisfying the specification property. Proc Amer Math Soc, 2010, **138**(1): 315–321

[11] Sigmund K. Generic properties of invariant measures for Axiom-A diffeomorphisms. Invent Math, 1970, **11**: 99–109

[12] Tajbakhsh K, Lee K. Hyperbolicity of C^1-stably shadowing homoclinic classes. Trends in Mathematics-NewSeries, 2008, **2**: 79–82

[13] Wen X, Gan S, Wen L. C^1-stably shadowable chain component is hyperbolic. J Diff Eqns, 2009, **246**: 340–57