Group Decision and Negotiation **15:** 593–604, 2006.
DOI: 10.1007/s10726-005-9008-4

A Note on Linguistic Hybrid Arithmetic Averaging Operator in Multiple Attribute Group Decision Making with Linguistic Information

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Abstract

In this paper, we propose a linguistic hybrid arithmetic averaging (LHAA) operator, which is based on linguistic weighted arithmetic averaging (LWAA) operator and extended ordered weighted averaging (EOWA) operator, and study some desirable properties of the LHAA operator. The LHAA operator can not only reflect the importance degrees of both the given argument and its ordered position, but also relieve the influence of unfair arguments on the decision results by weighting these arguments with small values. Based on the LWAA and LHAA operators, we develop a practical approach to multiple attribute group decision making under linguistic environment. The approach first aggregates the individual linguistic preference values into a collective linguistic preference value for each alternative by using the LWAA and LHAA operators (it is worth pointing out that the aggregation process does not produce any loss of linguistic information), and then orders the collective linguistic preference values to obtain the best alternative(s). Finally, an illustrative example is also given to verify the approach and to demonstrate its feasibility and practicality.

Key words: aggregation, multiple attribute group decision making with linguistic information, linguistic weighted arithmetic averaging (LWAA) operator, extended ordered weighted averaging (EOWA) operator, linguistic hybrid arithmetic averaging (LHAA) operator, arithmetic averaging operator

1. Introduction

With the rapid development of computer and communication technology, the world gets smaller and more tightly knit. The socio-economic environment becomes more complex and challenging mainly due to 1) human impacts on the environment, and 2) increasing amount of data about the environment. The increasing complexity of the socio-economic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem (Kim et al. 1999), and usually, a DM can not provide the information with a numerical value, but with a linguistic label (Sugeno 1974; Zadeh 1975a, 1975b, 1976; Delgado 1993; Torra 2001), because 1) a decision should be made under time pressure and lack of data (Kim and Ahn 1999; Xu 2004a), 2) many of the attributes are intangible or non-monetary because they reflect social and environmental impacts (Kim et al. 1999), 3) a decision maker has limited attention and information processing capabilities (Kahneman et al. 1982; Weber 1987; Park et al. 1996). As a result, many decision making processes, in the real world, take place in group settings with linguistic information (Herrera 1995; Herrera et al. 1996; Bordogna et al.1997). These processes may be encountered in such situations as selecting applications for different kinds of scholarships and selecting projects for different kinds of funding policies (Köksalan and Ulu 2003), and evaluating the "speed", "comfort" or "design" for different kinds of cars. For example, as evaluating the "comfort" or "design" of a car, linguistic labels like "good", "fair", "poor" are usually be used, and evaluating a car's speed, linguistic labels like "very fast", "fast", "slow" can be used (Levrat et al. 1997; Bordogna et al. 1997). The importance of group decision making with linguistic information has greatly increased with the rapid expansion of e-commerce and internet related activities (Yager 2001). When a problem is solved using linguistic information, it implies the need for computing with words (Zadeh and Kacprzyk 1999; Lawry 2001). Degani and Bortolan (1988) have developed a method based on the extension principle, which makes operations on the fuzzy numbers that support the semantics of the linguistic terms. Delgado et al. (1993) proposed a symbolic method, which makes computations on the indexes of the linguistic terms. Herrera et al. (1995,1996) presented several group decision making processes using linguistic OWA operator which is based on the ordered weighted averaging (OWA) operator defined by Yager (1988,1993), and on the convex combination of linguistic labels defined by Delgado et al. (1993). Bordogna et al. (1997) proposed a "soft" consensus degree referred to a fuzzy majority (Kacprzyk 1986) of the DMs based on the concept of linguistic quantifier (Zadeh 1983), and developed a linguistic model for evaluating a consensual judgment and a consensus degree on each alternative in group decision making based on the OWA operator. Herrera and Herrera-Viedma (2000) established three steps for solving multiple attribute decision making problem under linguistic information: (1) the choice of the linguistic label set with its semantic in order to express the linguistic preference values according to all the attributes, (2) the choice of the aggregation operator of linguistic information in order to aggregate the linguistic preference values, and (3) the choice of the best alternatives. However, all these methods produce a loss of information and hence a lack of precision (Carlsson and Fullér 2000). Afterwards, Herrera and Martínez (2000a, 2000b, 2000c) developed a fuzzy linguistic representation model for overcoming this limitation, which represents the linguistic information with a pair of values called 2-tuple, composed by a linguistic term and a number. Together with the model, they also presented a computational technique to deal with the 2-tuples without loss of information. However, sometimes, the linguistic labels presented in these three documents may produce a lack of rationality in the decision results in practical applications (Xu, 2004b, 2005a). In this paper, we shall propose a linguistic hybrid arithmetic averaging (LHAA) operator, which is based on linguistic weighted arithmetic averaging (LWAA) operator and extended ordered weighted averaging (EOWA) operator, for aggregating linguistic information, and then by using the LWAA and LHAA operators, we shall develop a practical approach to multiple group attribute decision making with linguistic information, which can relieve the influence of unfair arguments on the decision results by using the normal distribution based method (Xu 2005b) to assign low weights to those "false" or "biased" ones. Finally, a problem of evaluating university faculty for tenure and promotion is also used to illustrate the developed approach.

2. Linguistic Hybrid Arithmetic Averaging (LHAA) Operator

At present, many aggregation operators have been developed to aggregate arguments (Carlsson and Fullér 1995, 1997a, 1997b, 2000; Xu and Da 2003, etc.). Two of the most common operators for aggregating arguments are the weighted arithmetic averaging (WAA) operator (Harsanyi 1955) and the ordered weighted averaging (OWA) operators (Yager 1988, Yager and Kacprzyk 1997), which are defined as follows:

Definition 1 (Harsanyi 1955). Let $WAA : R^n \to R$, if

$$WAA_{\omega}\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \omega_j a_j \tag{1}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the a_j $(j = 1, 2, \ldots, n)$, and $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$, R is the set of all real numbers, then WAA is called the weighted arithmetic averaging (WAA) operator.

From Definition 1, we know that the WAA operator first weights all the given arguments, and then aggregates all these weighted arguments into a collective one.

Definition 2 (Yager 1988). An ordered weighted averaging (OWA) operator of dimension n is a mapping $OWA : R^n \to R$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$. Furthermore

$$OWA_w\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{2}$$

where b_j is the j th largest of the a_i $(i = 1, 2, \ldots, n)$.

The fundamental aspect of the OWA operator is the re-ordering step, in particular an argument a_i is not associated with a particular weight w_i but rather a weight w_i is associated with a particular ordered position i of the arguments. Up to now, many methods have been developed for obtaining the OWA weights. Yager (1988) suggested a way to compute the OWA weights using linguistic quantifiers. O'Hagan (1988) developed a procedure to generate the OWA weights that have a predefined degree of orness and maximize the entropy of the OWA weights. Yager (1993) introduced some families of the OWA weights. Filev and Yager (1998) developed two procedures, based on the exponential smoothing, to obtain the OWA weights. Yager and Filev (1999) suggested an algorithm to obtain the OWA weights from a collection of samples with the relevant aggregated data. Fullér and Majlender (2001) used the method of Lagrange multipliers to solve O'Hagan's procedure analytically. Xu and Da (2002) established a linear objective-programming model for obtaining the OWA weights under partial weight information. Especially, based on the normal distribution (Gaussian distribution), Xu (2005b) developed a simple method for determining the OWA weights as follows:

$$w_i = \frac{e^{-\frac{(i-\mu_n)^2}{2\sigma_n^2}}}{\sum_{j=1}^{n} e^{-\frac{(j-\mu_n)^2}{2\sigma_n^2}}}, \quad i = 1, 2, \ldots, n \tag{3}$$

where μ_n is the mean of the collection of $1, 2, \ldots, n$, and $\sigma_n(\sigma_n > 0)$ is the standard deviation of the collection of $1, 2, \ldots, n$, i.e.,

$$\mu_n = \frac{1}{n}\frac{n(1+n)}{2} = \frac{1+n}{2}$$

$$\sigma_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(i-\mu_n)^2}$$

For example, if $n = 3$, then by (3), we have

$$w = (0.2429, 0.5142, 0.2429)^T$$

which is shown in Figure 1.

The prominent characteristic of the developed method is that it can relieve the influence of unfair arguments on the decision results by assigning low weights to those “false” or “biased” ones).

The WAA and OWA operators, however, have usually been used in situations where the input arguments are the exact values. Xu (2004b) extended the WAA and OWA operators to accommodate the situations where the input arguments are linguistic variables.

Let $S = \{s_\alpha | \alpha = -t, \ldots, -1, 0, 1, \ldots, t\}$ be a finite and totally ordered discrete label set, whose cardinality value is odd, and t is a non-negative integer. Any label, s_α, represents a possible value for a linguistic variable, and it must have the following characteristics:

(1) The set is ordered: $s_\alpha > s_\beta$ if $\alpha > \beta$;

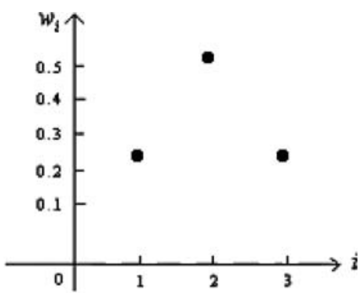

Figure 1. The weights $w_i (i = 1, 2, 3)$.

(2) There is the negative operator: $\text{neg}(s_\alpha) = s_{-\alpha}$.

For example, S can be defined as:

$$S = \{ s_{-3} = \text{none}, \quad s_{-2} = \textit{very low}, \quad s_{-1} = \textit{low}, \quad s_0 = \textit{medium},$$
$$s_1 = \textit{high}, \quad s_2 = \textit{very high}, \quad s_3 = \textit{perfect}\}$$

To preserve all the given information, Xu (2004b) extended the discrete linguistic label set S to a continuous linguistic label set $\bar{S} = \{ s_\alpha | \, \alpha \in [-q, q]\}$, where $q\,(q > t)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then s_α is called an original linguistic label, otherwise, s_α is called a virtual linguistic label. In general, the DM uses the original labels to evaluate attributes and alternatives, and the virtual labels can only appear in the course of operation.

Let $s_\alpha, s_\beta \in \bar{S}$, and $\mu \in [0, 1]$, then two operational laws of linguistic variables are given as follows:

(1) $\mu\, s_\alpha = s_{\mu\,\alpha}$;
(2) $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha = s_{\alpha+\beta}$.

Definition 3 (Xu 2004b). Let $LWAA : \bar{S}^n \to \bar{S}$, if

$$LWAA_\omega \left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = \omega_1 s_{\alpha_1} \oplus \omega_2 s_{\alpha_2} \oplus \cdots \oplus \omega_n s_{\alpha_n} = s_{\bar{\alpha}} \tag{4}$$

where $\bar{\alpha} = \sum_{j=1}^{n} \omega_j \alpha_j, \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of the $s_{\alpha_j}\,(j = 1, 2, \ldots, n)$, and $\omega_j \in [0, 1], \sum_{j=1}^{n} \omega_j = 1, s_{\alpha_j} \in \bar{S}$, then *LWAA* is called the linguistic weighted arithmetic averaging (LWAA) operator.

Definition 4 (Xu 2004b). An extended OWA (EOWA) operator of dimension n is a mapping $EOWA : \bar{S}^n \to \bar{S}$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1$. Furthermore

$$EOWA_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = w_1 s_{\beta_1} \oplus w_2 s_{\beta_2} \oplus \cdots \oplus w_n s_{\beta_n} = s_{\bar{\beta}} \tag{5}$$

where $\bar{\beta} = \sum_{j=1}^{n} w_j \beta_j, s_{\beta_j}$ is the jth largest of the $s_{\alpha_i} (i = 1, 2, \ldots, n)$.

From Definitions 3 and 4, we know that the LWAA operator weights the linguistic argument, while the EOWA operator weights the ordered position of the linguistic argument instead of weighting the argument itself. Therefore, weights represent different aspects in both the LWAA and EOWA operators. However, both the operators consider only one of them. To overcome this drawback, in the following, we shall propose a linguistic hybrid arithmetic averaging (LHAA) operator.

Definition 5. A linguistic hybrid arithmetic averaging (LHAA) operator is a mapping $LHAA : \bar{S}^n \to \bar{S}$, which has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ with $w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1$ such that

$$LHAA_{\omega,w}\ \left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = w_1 s_{\beta_1} \oplus w_2 s_{\beta_2} \oplus \cdots \oplus w_n s_{\beta_n}$$

where s_{β_j} is the j th largest of the linguistic weighted arguments $\bar{s}_{\alpha_i}(\bar{s}_{\alpha_i} = n\omega_i s_{\alpha_i}, i = 1, 2, \ldots, n)$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of the $s_{\alpha_i}(i = 1, 2, \ldots, n)$ with $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$, and n is the balancing coefficient, which plays a role of balance (in such a case, if the vector $(\omega_1, \omega_2, \ldots, \omega_n)^T$ goes to $\left(1/n, 1/n, \ldots, 1/n\right)^T$, then the vector $\left(n\omega_1 s_{\alpha_1}, n\omega_2 s_{\alpha_2}, \ldots, n\omega_n s_{\alpha_n}\right)^T$ goes to $\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right)^T$).

Example 1. Assume $\omega = (0.2, 0.3, 0.1, 0.4)^T$, $w = (0.1, 0.4, 0.4, 0.1)^T$, and

$$s_{\alpha_1} = s_2, \quad s_{\alpha_2} = s_3, \quad s_{\alpha_3} = s_1, \quad s_{\alpha_4} = s_0$$

By Definition 5, we have

$$\bar{s}_{\alpha_1} = 4 \times 0.2 \times s_2 = s_{1.6}, \quad \bar{s}_{\alpha_2} = 4 \times 0.3 \times s_3 = s_{3.6}$$
$$\bar{s}_{\alpha_3} = 4 \times 0.1 \times s_1 = s_{0.4}, \quad \bar{s}_{\alpha_4} = 4 \times 0.4 \times s_0 = s_0$$

thus

$$s_{\beta_1} = s_{3.6}, \quad s_{\beta_2} = s_{1.6}, \quad s_{\beta_3} = s_{0.4}, \quad s_{\beta_4} = s_0$$

therefore,

$$LHAA_{\omega,w}\ (s_2, s_3, s_1, s_0) = 0.1 \times s_{3.6} \oplus 0.4 \times s_{1.6} \oplus 0.4 \times s_{0.4} \oplus 0.1 \times s_0 = s_{1.16}$$

Theorem 1. *The LWAA operator is a special case of the LHAA operator.*

Proof: Let $w = (1/n, 1/n, \ldots, 1/n)^T$, then

$$\begin{aligned} LHAA_{\omega,w}\ \left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) &= w_1 s_{\beta_1} \oplus w_2 s_{\beta_2} \oplus \cdots \oplus w_n s_{\beta_n} \\ &= \frac{1}{n}\left(s_{\beta_1} \oplus s_{\beta_2} \oplus \cdots \oplus s_{\beta_n}\right) \\ &= \omega_1 s_{\alpha_1} \oplus \omega_2 s_{\alpha_2} \oplus \cdots \oplus \omega_n s_{\alpha_n} \\ &= s_{\bar{\alpha}} \end{aligned}$$

where $\bar{\alpha} = \sum_{j=1}^{n} \omega_j \alpha_j$. This completes the proof of Theorem 1.

□

Theorem 2. *The EOWA operator is a special case of the LHAA operator.*

Proof: Let $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then

$$\bar{s}_{\alpha_i} = s_{\alpha_i}, \quad i = 1, 2, \ldots, n$$

which completes the proof of Theorem 2. □

From Definition 5 and the above theorems, we know that:

(1) The LHAA operator first weights the given arguments, and then reorders the weighted arguments in descending order and weights these ordered arguments by the LHAA weights, and finally aggregates all the weighted arguments into a collective one.
(1) The LHAA operator generalizes both the LWAA and EOWA operators, and reflects the importance degrees of both the given argument and its ordered position.

3. An Approach to Multiple Attribute Group Decision Making with Linguistic Information

The multiple attribute group decision making problem which is considered in this paper can be represented as follows:

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a discrete set of alternatives, $D = \{d_1, d_2, \ldots, d_l\}$ be the set of DMs, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ be the weight vector of DMs, where $\lambda_k \geq 0, \sum_{k=1}^{l} \lambda_k = 1$. Let $G = \{G_1, G_2, \ldots, G_m\}$ be the set of attributes, and $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the weight vector of attributes, where $\omega_i \geq 0, \sum_{i=1}^{m} \omega_i = 1$. Let $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ be the decision matrix, where $a_{ij}^{(k)} \in S$ is a preference value, which takes the form of linguistic label, given by the DM $d_k \in D$, for the alternative $x_j \in X$ with respect to the attribute $G_i \in G$.

Multiple attribute group decision making problems follow a common resolution scheme (Roubens1997; Herrera and Martínez 2000c) composed by the following two phases:

(1) *Aggregation phase:* It combines the individual preferences to obtain a collective preference value for each alternative.
(2) *Exploitation phase:* It orders the collective preference values to obtain the best alternative(s).

In the following, we shall utilize the LWAA and LHAA operators to develop a practical approach to multiple attribute group decision making with linguistic information.

Step 1: Utilize the linguistic decision information in the matrix $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ provided by the DM $d_k \in D$, and the LWAA operator:

$$z_j^{(k)} = LWAA_\omega\left(a_{1j}^{(k)}, a_{2j}^{(k)} \ldots, a_{mj}^{(k)}\right), \quad j = 1, 2, \ldots, n$$

to derive the individual overall preference values $z_j^{(k)}(j = 1, 2, \ldots, n)$ of the alternatives $x_j(j = 1, 2, \ldots, n)$, where $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ is the weight vector of the attributes $G_i(i = 1, 2, \ldots, m)$, with $\omega_i \geq 0$, $\sum_{i=1}^{m} \omega_i = 1$.

Step 2: Utilize the LHAA operator:

$$z_j = LHAA_{\lambda,w}\left(z_j^{(1)}, z_j^{(2)}, \ldots, z_j^{(l)}\right)$$

to derive the collective overall preference values $z_j(j = 1, 2, \ldots, n)$ of the alternatives $x_j(j = 1, 2, \ldots, n)$, where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ is the weight vector of DMs, with $\lambda_k \geq 0$, $\sum_{k=1}^{l} \lambda_k = 1$, $w = (w_1, w_2, \ldots, w_l)^T$ is the weighting vector of the LHAA operator, with $w_j \in [0, 1]$ $\sum_{j=1}^{l} w_j = 1$, which can be determined by the normal distribution based method, or others (please see Xu (2005b) for more details).

Step 3: Utilize the collective overall preference values $z_j(j = 1, 2, \ldots, n)$ to rank the alternatives $x_j(j = 1, 2, \ldots, n)$, and then to select the best one(s).

Step 4: End.

4. Illustrative Example

In this section, a multiple attribute group decision making problem of evaluating university faculty for tenure and promotion (Bryson and Mobolurin 1995) is used to illustrate the proposed approach.

A practical use of the proposed approach involves the evaluation of university faculty for tenure and promotion. The attributes used at some universities are G_1: teaching, G_2: research, and G_3: service (whose weight vector $\omega = (0.14, 0.26, 0.60)^T$). Five faculty candidates (alternatives) $x_j(j = 1, 2, 3, 4, 5)$ are to be evaluated using the label set

$$S = \{s_{-3} = none,\quad s_{-2} = very\ low,\quad s_{-1} = low,\quad s_0 = medium,\\ s_1 = high,\quad s_2 = very\ high,\quad s_3 = perfect\}$$

by three DMs $d_k(k = 1, 2, 3)$ (whose weight vector $\lambda = (0.2, 0.5, 0.3)^T$) under these three attributes, as listed in Tables 1–3.

To get the best alternative(s), the following steps are involved:

Table 1. Decision matrix $A^{(1)}$.

G_i	x_1	x_2	x_3	x_4	x_5
G_1	s_{-1}	s_{-2}	s_{-1}	s_2	s_1
G_2	s_0	s_1	s_{-2}	s_1	s_1
G_3	s_2	s_2	s_2	s_{-1}	s_0

Table 2. Decision matrix $A^{(2)}$.

G_i	x_1	x_2	x_3	x_4	x_5
G_1	s_0	s_0	s_0	s_2	s_0
G_2	s_{-1}	s_{-1}	s_1	s_0	s_2
G_3	s_1	s_2	s_0	s_0	s_1

Table 3. Decision matrix $A^{(3)}$.

G_i	x_1	x_2	x_3	x_4	x_5
G_1	s_{-1}	s_{-1}	s_{-2}	s_1	s_0
G_2	s_{-2}	s_0	s_0	s_0	s_2
G_3	s_2	s_1	s_2	s_1	s_{-1}

Step 1: Utilize the linguistic decision information in the matrix $A^{(k)}$ provided by the DM d_k and the LWAA operator to derive the individual overall preference values $z_j^{(k)}(j = 1, 2, 3, 4, 5)$ of the alternatives $x_j(j = 1, 2, 3, 4, 5)$:

$$z_1^{(1)} = s_{1.06},\ z_2^{(1)} = s_{1.18},\ z_3^{(1)} = s_{0.54},\ z_4^{(1)} = s_{-0.06}, z_5^{(1)} = s_{0.40}$$
$$z_1^{(2)} = s_{0.34},\ z_2^{(2)} = s_{0.94},\ z_3^{(2)} = s_{0.26},\ z_4^{(2)} = s_{0.28}, z_5^{(2)} = s_{1.12}$$
$$z_1^{(3)} = s_{0.54},\ z_2^{(3)} = s_{0.46},\ z_3^{(3)} = s_{0.92},\ z_4^{(3)} = s_{0.74}, z_5^{(3)} = s_{-0.08}$$

Step 2: Utilize the LHAA operator (let $w = (0.2429, 0.5142, 0.2429)^T$ be its weighting vector (for more details, please see Xu (2005)) to derive the collective overall preference values $z_j(j = 1, 2, 3, 4, 5)$ of the alternatives $x_j(j = 1, 2, 3, 4, 5)$:

$$z_1 = LHAA_{\lambda,w}\left(z_1^{(1)}, z_1^{(2)}, z_1^{(3)}\right) = s_{0.535}$$
$$z_2 = LHAA_{\lambda,w}\left(z_2^{(1)}, z_2^{(2)}, z_2^{(3)}\right) = s_{0.807}$$
$$z_3 = LHAA_{\lambda,w}\left(z_3^{(1)}, z_3^{(2)}, z_3^{(3)}\right) = s_{0.480}$$
$$z_4 = LHAA_{\lambda,w}\left(z_4^{(1)}, z_4^{(2)}, z_4^{(3)}\right) = s_{0.369}$$
$$z_5 = LHAA_{\lambda,w}\left(z_5^{(1)}, z_5^{(2)}, z_5^{(3)}\right) = s_{0.514}$$

and thus

$$z_2 > z_1 > z_5 > z_3 > z_4$$

Step 3: Utilize the collective overall preference values $z_j(j = 1, 2, 3, 4, 5)$ to rank the alternatives $x_j(j = 1, 2, 3, 4, 5)$:

$$x_2 \succ x_1 \succ x_5 \succ x_3 \succ x_4$$

and thus the best alternative is x_2.

In the above example, to reflect the importance degrees of all the preference values, we weight each preference value by using the corresponding attribute weight, and then utilize the LWAA operator to aggregate all the weighted preference values into the individual overall preference values. Consider that some DMs may provide unduly high or unduly low preference values for their preferred or repugnant objects, to relieve the influence of these unfair arguments on the decision results and to reflect the importance degrees of all the DMs, we first weight each individual overall preference value by using the corresponding DM weight, and then utilize the LHAA operator to aggregate the individual weighted overall preference values into the collective overall preference value for each alternative by using the normal distribution based method to assign low weights to those "false" or "biased" ones. Finally, we rank all the alternatives in accordance with the collective overall preference values and then get the best alternative with the largest collective overall preference value.

5. Conclusions

In this paper, we have introduced a linguistic hybrid arithmetic averaging (LHAA) operator. The LHAA operator first weights the given arguments, and then reorders the weighted arguments in descending order and weights these ordered arguments by the LHAA weights which can be determined by the normal distribution based method, and finally aggregates all the weighted arguments into a collective one. Obviously, the LHAA operator generalizes both the linguistic weighted arithmetic averaging (LWAA) operator and the extended OWA (EOWA) operator, and reflects the importance degrees of both the given argument and the ordered position of the argument. Furthermore, the LHAA operator can relieve the influence of unfair arguments on the decision results by using the normal distribution based method to assign low weights to those "false" or "biased" ones. Based on the LHAA operators, we have developed a practical approach to multiple attribute group decision making with linguistic information, and used a multiple attribute group decision making problem of evaluating university faculty for tenure and promotion to illustrate the developed approach. Theoretical analysis and the numerical results show that the approach is straightforward and has no loss of information, and thus can make the decision results more precise and reasonable.

In future research, our work will focus on the situations where both the argument weights and the LHAA weights are expressed linguistically, and on the application of the LHAA operator in other fields such as neural network, artificial intelligence, and pattern recognition, etc.

Acknowledgements

The author is very grateful to the associate editor, Professor C. Carlsson, and the anonymous referees for their insightful and constructive comments and suggestions that led to an improved version of this paper. This work was supported by the National Natural Science Foundation of China under Grant (70571087).

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