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Generalized triangular fuzzy correlated averaging operator and their application to multiple attribute decision making

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ABSTRACT

We investigate the multiple attribute decision making problems with triangular fuzzy information. Motivated by the ideal of Choquet integral [G. Choquet, Theory of capacities, Ann. Instit. Fourier 5 (1953) 131–295] and generalized OWA operator [R.R. Yager, Generalized OWA aggregation operators, Fuzzy Optim. Dec. Making 3 (2004) 93–107], in this paper, we have developed an generalized triangular fuzzy correlated averaging (GTFCA) operator. The prominent characteristic of the operators is that they cannot only consider the importance of the elements or their ordered positions, but also reflect the correlation among the elements or their ordered positions. We have applied the GTFCA operator to multiple attribute decision making problems with triangular fuzzy information. Finally an illustrative example has been given to show the developed method.

1. Introduction

The information aggregation operators are an interesting research topic, which is receiving increasing attention. The fundamental aspect of the OWA operator is a reordering step in which the input arguments are rearranged in descending order [1–5]. Since its appearance, it has been studied and applied in a wide range of problems [6–16]. The ordered weighted geometric (OWG) operator is an aggregation operator that is based on the OWA operator and the geometric mean [17,18]. In some situations, however, the input arguments take the form of fuzzy data rather than numerical ones because of time pressure, lack of knowledge, and the decision maker's limited attention and information processing capabilities. Therefore, Xu [19] and Fan and Wang [20] developed the fuzzy ordered weighted averaging (FOWA) operator. Xu [21] introduced the fuzzy ordered weighted geometric (FOWG) operator. Xu and Wu [22] proposed the fuzzy induced ordered weighted averaging (FIOWA) operator. Xu and Da [23] developed the fuzzy induced ordered weighted geometric (FIOWG) operator. Xu [24] developed some fuzzy harmonic mean operators, such as fuzzy weighted harmonic mean (FWHM) operator, fuzzy ordered weighted harmonic mean (FOWHM) operator, fuzzy hybrid harmonic mean (FHHM) operator. Wei [25] proposed the fuzzy ordered weighted harmonic averaging (FOWHA) operator. Wei [26] developed the fuzzy induced ordered weighted harmonic mean (FIOWHM) operator and applied it to the group decision making.

All of the existed triangular fuzzy information aggregation operators only consider situations where all the elements in the triangular fuzzy variables are independent. However, in many practical situations, the elements in the triangular fuzzy variables are usually correlative. Therefore, we need to find some new ways to deal with these situations in which the decision data in question are correlative. The Choquet integral [27] is a very useful way of measuring the expected utility of an uncertain event, and can be utilized to depict the correlations of the decision data under consideration.

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Motivated by the correlation properties of the Choquet integral [27] and generalized OWA operator [28], in this paper we propose a generalized triangular fuzzy correlated averaging (GTFCA) operator, whose prominent characteristic is that they cannot only consider the importance of the elements or their ordered positions, but also reflect the correlations of the elements or their ordered positions. To do so, the remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to triangular fuzzy variables and some operational laws of triangular fuzzy variables. In Section 3 we have developed the generalized triangular fuzzy ordered weighted averaging (GTFOWA) operator and studied some desirable properties of the GTFOWA operator, such as idempotent, commutative and monotonic. In Section 4 we have developed the generalized triangular fuzzy correlated averaging (GTFCA) operator and studied some desirable properties of the GTFCA operator, such as idempotent, commutative and monotonic. In Section 5, we have developed an approach to multiple attribute decision making based on GTFCA operator with triangular fuzzy information. In Section 6, an illustrative example is pointed out. In Section 7, we conclude the paper and give some remarks.

2. Preliminaries

In this section, we briefly describe some basic concepts and basic operational laws related to triangular fuzzy numbers.

Definition 1 [29]. A triangular fuzzy numbers $\tilde{a}$ can be defined by a triplet (a^L, a^M, a^U). The membership function $\mu_{\tilde{a}}(x)$ is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a^L, \\ \frac{x-a^L}{a^M-a^L}, & a^L \leqslant x \leqslant a^M, \\ \frac{x-a^U}{a^M-a^U}, & a^M \leqslant x \leqslant a^U, \\ 0, & x \geqslant a^U. \end{cases} \quad (1)$$

where $0 < a^L \leqslant a^M \leqslant a^U$, a^L and a^U stand for the lower and upper values of the support of $\tilde{a}$, respectively, and a^M for the modal value.

Definition 2 [29]. Basic operational laws related to triangular fuzzy numbers:

$$\tilde{a} \oplus \tilde{b} = [a^L, a^M, a^U] \oplus [b^L, b^M, b^U] = [a^L + b^L, a^M + b^M, a^U + b^U],$$

$$\tilde{a} \otimes \tilde{b} = [a^L, a^M, a^U] \otimes [b^L, b^M, b^U] = [a^L b^L, a^M b^M, a^U b^U],$$

$$\lambda \otimes \tilde{a} = \lambda \otimes [a^L, a^M, a^U] = [\lambda a^L, \lambda a^M, \lambda a^U], \quad \lambda > 0,$$

$$\frac{1}{\tilde{a}} = [1/a^U, 1/a^M, 1/a^L].$$

Definition 3 [21]. Let $\tilde{b} = [b^L, b^M, b^U]$ and $\tilde{a} = [a^L, a^M, a^U]$ be two triangular fuzzy numbers, then the degree of possibility of $a \geqslant b$ is defined as

$$p(a \geqslant b) = \lambda \max\left\{1 - \max\left[\frac{b^M - a^L}{a^M - a^L + b^M - b^L}, 0\right], 0\right\} + (1-\lambda)\max\left\{1 - \max\left[\frac{b^U - a^M}{a^U - a^M + b^U - b^M}, 0\right], 0\right\}, \quad (2)$$

where the value λ is an index of rating attitude. It reflects the decision maker's risk-bearing attitude. If $\lambda > 0.5$, the decision maker is risk lover. If $\lambda = 0.5$, the decision maker is neutral to risk. If $\lambda < 0.5$, the decision maker is risk a vector.

From Definition 3, we can easily get the following results easily:

(1) $0 \leqslant p(a \geqslant b) \leqslant 1$, $0 \leqslant p(b \geqslant a) \leqslant 1$;
(2) $p(a \geqslant b) + p(b \geqslant a) = 1$. Especially, $p(a \geqslant a) = p(b \geqslant b) = 0.5$.

3. Generalized triangular fuzzy aggregating operators

The GOWA operator [28] is a generalization of the OWA operator [1–3] by using generalized means. It can be defined as follows.

Definition 4 [28]. A GOWA operator of dimension n is a mapping GOWA: $R^n \to R$ that has an associated weight vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^n w_j = 1$. Furthermore,

$$\mathrm{GOWA}(a_1,a_2,\ldots,a_n)=\left(\sum_{j=1}^{n} w_j a_{\sigma(j)}^{\lambda}\right)^{1/\lambda}, \quad (3)$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\alpha_{\sigma(j-1)} \geqslant \alpha_{\sigma(j)}$ for all $j = 2,\ldots,n$.

In the following, we shall extend the GOWA operators to accommodate the situations where the input arguments are triangular fuzzy information.

Definition 5. Let $\tilde{a}_j = \left[a_j^L, a_j^M, a_j^U\right]$ $(j = 1,2,\ldots,\boldsymbol{n})$ be a collection of triangular fuzzy numbers, A generalized triangular fuzzy ordered weighted averaging (GTFOWA) operator of dimension n is a mapping GTFOWA: $Q^n \to Q$, that has an associated weight vector $w = (w_1,w_2,\ldots,w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\left(\bigoplus_{j=1}^{n} w_j\left(\tilde{a}_{\sigma(j)}\right)^{\lambda}\right)^{1/\lambda}=\left[\left(\sum_{j=1}^{n} w_j\left(a_{\sigma(j)}^{L}\right)^{\lambda}\right)^{1/\lambda},\left(\sum_{j=1}^{n} w_j\left(a_{\sigma(j)}^{M}\right)^{\lambda}\right)^{1/\lambda},\left(\sum_{j=1}^{n} w_j\left(a_{\sigma(j)}^{U}\right)^{\lambda}\right)^{1/\lambda}\right], \quad (4)$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\tilde{\alpha}_{\sigma(j-1)} \geqslant \tilde{\alpha}_{\sigma(j)}$ for all $j = 2,\ldots,n$.

Now we consider some special cases of the GTFOWA operator:

(1) If $\lambda = 1$, then the GTFOWA operator reduces to the triangular fuzzy ordered weighted averaging (TFOWA) [19,20]:

$$\mathrm{TFOWA}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\bigoplus_{j=1}^{n} w_j\tilde{a}_{\sigma(j)}=\left[\sum_{j=1}^{n} w_j a_{\sigma(j)}^{L},\sum_{j=1}^{n} w_j a_{\sigma(j)}^{M},\sum_{j=1}^{n} w_j a_{\sigma(j)}^{U}\right].$$

(2) If $\lambda = 0$, then the GTFOWA operator reduces to the triangular fuzzy ordered weighted geometric (TFOWG) [21]:

$$\mathrm{TFOWG}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\bigotimes_{j=1}^{n}\left(\tilde{a}_{\sigma(j)}\right)^{w_j}=\left[\prod_{j=1}^{n}\left(a_{\sigma(j)}^{L}\right)^{w_j},\prod_{j=1}^{n}\left(a_{\sigma(j)}^{M}\right)^{w_j},\prod_{j=1}^{n}\left(a_{\sigma(j)}^{U}\right)^{w_j}\right].$$

(3) If $\lambda = -1$, then the GTFOWA operator reduces to the triangular fuzzy ordered weighted harmonic mean (TFOWHM) [26]:

$$\mathrm{TFOWHM}_w(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\frac{1}{\bigoplus_{j=1}^{n}\frac{w_j}{\tilde{a}_{\sigma(j)}}}=\left[\frac{1}{\sum_{j=1}^{n}\frac{w_j}{a_{\sigma(j)}^{L}}},\frac{1}{\sum_{j=1}^{n}\frac{w_j}{a_{\sigma(j)}^{M}}},\frac{1}{\sum_{j=1}^{n}\frac{w_j}{a_{\sigma(j)}^{U}}}\right].$$

The GTFOWA operator has the following properties similar to those of the GOWA operator [28].

Theorem 1 (Commutativity).

$$\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n),$$

where $(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)$ *is any permutation of* $(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$.

Theorem 2 (Idempotency). *If* $\tilde{a}_j\left(\tilde{a}_j=\left[a_j^L,a_j^M,a_j^U\right]\right)=\tilde{a}\left(\tilde{a}=\left[a^L,a^M,a^U\right]\right)$ *for all j, then*

$$\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\tilde{a}.$$

Theorem 3 (Monotonicity). *If* $\tilde{a}_j \leqslant \tilde{a}'_j$ *for all j, then*

$$\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leqslant \mathrm{GTFOWA}_{w,\lambda}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n).$$

4. Generalized triangular fuzzy correlated aggregating operators

However, the above aggregation operators with triangular fuzzy information are based on the assumption that the attribute of decision makers are independent, which is characterized by an independence axiom [30,31], that is, these operators are based on the implicit assumption that attributes of decision makers are independent; their effects are viewed as additive. For real decision making problems, there is always some degree of inter-dependent characteristics between attributes. Usually, there is interaction among attributes of decision makers. However, this assumption is too strong to match decision behaviors in the real world. The independence axiom generally cannot be satisfied. Thus, it is necessary to consider this issue.

Let $m(x_j)$ $(j = 1,2,\ldots,n)$ be the weight of the elements $x_j \in X$ $(j = 1,2,\ldots,n)$, where m is a fuzzy measure, defined as follows:

Definition 6 [32]. A fuzzy measure m on the set X is a set function $m: \theta(x) \to [0,1]$ satisfying the following axioms:

(1) $m(\phi) = 0$, $m(X) = 1$;
(2) $A \subseteq B$ implies $m(A) \leqslant m(B)$, for all $A, B \subseteq X$;
(3) $m(A \cup B) = m(A) + m(B) + \rho m(A)m(B)$, for all $A, B \subseteq X$ and $A \cap B = \phi$, where $\rho \in (-1, \infty)$.

Especially, if $\rho = 0$, then the condition (3) reduces to the axiom of additive measure:

$$m(A \cup B) = m(A) + m(B), \quad \text{for all } A, B \subseteq X \quad \text{and} \quad A \cap B = \phi.$$

If all the elements in X are independent, and we have

$$m(A) = \sum_{x_j \in A} m(\{x_j\}), \quad \text{for all } A \subseteq X.$$

Definition 7 ([27,33]). Let f be a positive real-valued function $f: X \to R^+$ and m be a fuzzy measure on X. The discrete Choquet integral of f with respective to m is defined by

$$C_m(f) = \sum_{j=1}^{n} f_{\sigma(j)}[m(A_{\sigma(j)}) - m(A_{\sigma(j-1)})], \tag{5}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $f_{\sigma(j-1)} \geqslant f_{\sigma(j)}$ for all $j = 2,\ldots,n$, $A_{\sigma(k)} = \{x_{\sigma(j)} - j \leqslant k\}$, for $k \geqslant 1$, and $A_{\sigma(0)} = \phi$.

In the following, we shall develop generalized triangular fuzzy correlated averaging (GTFCA) operator based on the Definition 7 and GOWA operator.

Definition 8. Let $X(x_1, x_2, \ldots, x_n)$ be a finite set, m be a fuzzy measure on X, and $\tilde{a}_j = \left[a_j^L, a_j^M, a_j^U\right]$ $(j = 1, 2, \ldots, \boldsymbol{n})$ be a collection of triangular fuzzy numbers on X. A generalized triangular fuzzy correlated averaging (GTFCA) operator of dimension n is a function GTFCA: $H^n \to H$, which is defined to aggregate the set of second arguments of a collection of triangular fuzzy numbers $\tilde{a}_j = \left[a_j^L, a_j^M, a_j^U\right]$ $(j = 1, 2, \ldots, \boldsymbol{n})$ according to the following expression:

$$\begin{aligned}
\text{GTFCA}_\lambda(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) &= \left(\bigoplus_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))\left(\tilde{a}_{\sigma(j)}\right)^\lambda\right)^{1/\lambda} \\
&= \left[\left(\sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))\left(a_{\sigma(j)}^L\right)^\lambda\right)^{1/\lambda}, \left(\sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))\left(a_{\sigma(j)}^M\right)^\lambda\right)^{1/\lambda},\right. \\
&\quad \left.\left(\sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))\left(a_{\sigma(j)}^U\right)^\lambda\right)^{1/\lambda}\right],
\end{aligned} \tag{6}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\tilde{\alpha}_{\sigma(j-1)} \geqslant \tilde{\alpha}_{\sigma(j)}$ for all $j = 2,\ldots,n$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \ldots, x_{(i)}\}$ when $i \geqslant 1$ and $A_{\sigma(0)} = \phi$.

Now we consider some special cases of the GTFCA operator:

(1) If $\lambda = 1$, then the GTFCA operator reduces to the triangular fuzzy correlated averaging (TFCA):

$$\begin{aligned}
\text{TFCA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) &= \bigoplus_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))\tilde{a}_{\sigma(j)} \\
&= \left[\sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))a_{\sigma(j)}^L, \sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))a_{\sigma(j)}^M, \sum_{j=1}^{n} (m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))a_{\sigma(j)}^U\right].
\end{aligned}$$

(2) If $\lambda = 0$, then the GTFOWA operator reduces to the triangular fuzzy correlated geometric (GTFCG):

$$\begin{aligned}
\text{TFCG}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) &= \bigotimes_{j=1}^{n} \left(\tilde{a}_{\sigma(j)}\right)^{(m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))} \\
&= \left[\prod_{j=1}^{n} \left(a_{\sigma(j)}^L\right)^{(m(A_{\sigma(j)}) - m(A_{\sigma(j-1)}))}, \prod_{j=1}^{n} \left(a_{\sigma(j)}^M\right)^{\left(m(A_{\sigma(j)}) - m(A_{\sigma(j-1)})\right)}, \prod_{j=1}^{n} \left(a_{\sigma(j)}^U\right)^{\left(m(A_{\sigma(j)}) - m(A_{\sigma(j-1)})\right)}\right].
\end{aligned}$$

If $\lambda = -1$, then the GTFOWA operator reduces to the triangular fuzzy correlated harmonic (TFCH):

$$\mathrm{TFCH}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\frac{1}{\bigoplus_{j=1}^{n}\frac{(m(A_{\sigma(j)})-m(A_{\sigma(j-1)}))}{\tilde{a}_{\sigma(j)}}}=\left[\frac{1}{\sum_{j=1}^{n}\frac{(m(A_{\sigma(j)})-m(A_{\sigma(j-1)}))}{a^L_{\sigma(j)}}},\frac{1}{\sum_{j=1}^{n}\frac{\left(m(A_{\sigma(j)})-m(A_{\sigma(j-1)})\right)}{a^M_{\sigma(j)}}},\frac{1}{\sum_{j=1}^{n}\frac{(m(A_{\sigma(j)})-m(A_{\sigma(j-1)}))}{a^U_{\sigma(j)}}}\right].$$

It is easy to prove that the GTFCA operator has the following properties.

Theorem 4 (Commutativity).

$$\mathrm{GTFCA}_\lambda(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\mathrm{GTFCA}_{w,\lambda}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n),$$

where $(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n)$ *is any permutation of* $(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$.

Theorem 5 (Idempotency). *If* $\tilde{a}_j\left(\tilde{a}_j=\left[a^L_j,a^M_j,a^U_j\right]\right)=\tilde{a}(\tilde{a}=[a^L,a^M,a^U])$ *for all j, then*

$$\mathrm{GTFCA}_\lambda(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\tilde{a}.$$

Theorem 6 (Monotonicity). *If* $\tilde{a}_j\leqslant\tilde{a}'_j$ *for all j, then*

$$\mathrm{GTFCA}_\lambda(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leqslant\mathrm{GTFOWA}_{w,\lambda}(\tilde{a}'_1,\tilde{a}'_2,\ldots,\tilde{a}'_n).$$

5. An approach to multiple attribute decision making with triangular fuzzy information

Let $A=\{A_1,A_2,\ldots,A_m\}$ be a discrete set of alternatives, and $G=\{G_1,G_2,\ldots,G_n\}$ be the set of attributes, $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$ is the weighting vector of the attribute G_j $(j=1,2,\ldots,n)$, where $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Suppose that $A=(\tilde{a}_{ij})_{m\times n}=\left[a^L_{ij},a^M_{ij},a^U_{ij}\right]_{m\times n}$ is the decision making matrix, where $\tilde{a}_{ij}$ is the preference values, which take the form of triangular fuzzy numbers, given by the decision maker, for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$.

In what follows, we shall apply the GTFCA operator to solve the MADM problems in which both the attribute weights are usually correlative and attribute values take the form of triangular fuzzy numbers.

Step 1. Normalize each attribute value $\tilde{a}_{ij}$ in the matrix A into a corresponding element in the matrix $R=(\tilde{r}_{ij})_{m\times n}=\left[r^L_{ij},r^M_{ij},r^U_{ij}\right]_{m\times n}$ using the following formulas:

$$\begin{cases} r^L_{ij}=a^L_{ij}\Big/\sum_{i=1}^{m}a^U_{ij},\\ r^M_{ij}=a^M_{ij}\Big/\sum_{i=1}^{m}a^M_{ij}, \quad \text{for benefit attribute } G_j,\quad i=1,2,\ldots,m,\ j=1,2,\ldots,n,\ k=1,2,\ldots,t.\\ r^U_{ij}=a^U_{ij}\Big/\sum_{i=1}^{m}a^L_{ij},\end{cases} \tag{7}$$

$$\begin{cases} r^L_{ij}=\left(1/a^U_{ij}\right)\Big/\sum_{i=1}^{m}\left(1/a^L_{ij}\right),\\ r^M_{ij}=\left(1/a^M_{ij}\right)\Big/\sum_{i=1}^{m}\left(1/a^M_{ij}\right), \quad \text{for cost attribute } G_j,\quad i=1,2,\ldots,m,\ j=1,2,\ldots,n,\ k=1,2,\ldots,t.\\ r^U_{ij}=\left(\left(1/a^L_{ij}\right)\right)\Big/\sum_{i=1}^{m}\left(1/a^U_{ij}\right),\end{cases} \tag{8}$$

Step 2. Calculate the fuzzy measure of attribute of G_j $(j=1,2,\ldots,n)$ and attribute sets of G.

Step 3. We utilize the decision information given in matrix R, and the GTFCA operator

$$\begin{aligned}\tilde{r}_i&=(r^L_i,r^M_i,r^U_i)=\mathrm{TFCA}(\tilde{r}_{i1},\tilde{r}_{i2},\ldots,\tilde{r}_{in})=\bigoplus_{j=1}^{n}\left(m(A_{\sigma(j)})-m(A_{\sigma(j-1)})\right)\tilde{r}_{\sigma(ij)}\\ &=\left[\sum_{j=1}^{n}\left(m\left(A_{\sigma(j)}\right)-m\left(A_{\sigma(j-1)}\right)\right)a^L_{\sigma(ij)},\sum_{j=1}^{n}(m(A_{\sigma(j)})-m(A_{\sigma(j-1)}))a^M_{\sigma(ij)},\sum_{j=1}^{n}(m(A_{\sigma(j)})-m(A_{\sigma(j-1)}))a^U_{\sigma(ij)}\right],\end{aligned} \tag{9}$$

to derive the overall preference values $\tilde{r}_i$ $(i=1,2,\ldots,m)$ of the alternative A_i.

Step 4. To rank these collective overall preference values $\tilde{r}_i$ $(i = 1, 2, \ldots, m)$, we first compare each $\tilde{r}_i$ with all the $\tilde{r}_j$ $(j = 1, 2, \ldots, m)$ by using Eq. (2). For simplicity, we let $p_{ij} = p(\tilde{r}_i \geqslant \tilde{r}_j)$, then we develop a complementary matrix as $P = (p_{ij})_{m\times m}$, where $p_{ij} \geqslant 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $i, j = 1,2,\ldots,n$.
Summing all the elements in each line of matrix P, we have

$$p_i = \sum_{j=1}^{m} p_{ij}, \quad i = 1, 2, \ldots, m. \tag{10}$$

Then we rank the collective overall preference values $\tilde{r}_i$ $(i = 1, 2, \ldots, m)$ in descending order in accordance with the values of p_i $(i = 1,2,\ldots,m)$.

Step 5. Rank all the alternatives A_i $(i = 1,2,\ldots,m)$ and select the best one(s) in accordance with the collective overall preference values $\tilde{r}_i$ $(i = 1, 2, \ldots, m)$.

Step 6. End.

6. Illustrative example

In this section, we utilize a practical multiple attribute decision making problems to illustrate the application of the developed approaches.

Suppose an organization plans to implement ERP system (adapted from [34]). The first step is to form a project team that consists of CIO and two senior representatives from user departments. By collecting all possible information about ERP vendors and systems, project term choose five potential ERP systems A_i $(i = 1,2,\ldots,5)$ as candidates. The company employs some external professional organizations (or experts) to aid this decision-making. The Project team selects four attributes to evaluate the alternatives: (1) function and technology G_1, (2) strategic fitness G_2, (3) vendor's ability G_3; (4) vendor's reputation G_4. The five possible alternatives A_i $(i = 1,2,\ldots,5)$ are to be evaluated using the triangular fuzzy numbers by the decision makers under the above four attributes, and construct the following matrix $A = (\tilde{a}_{ij})_{5\times 4}$ is shown in Table 1.

Then, we utilize the approach developed to get the most desirable alternative(s).

Step 1. Calculate the normalized decision matrix $\widetilde{R}$. The result is shown in Table 2.

Step 2. Suppose the fuzzy measure of attribute of G_j $(j = 1,2,\ldots,n)$ and attribute sets of G are as follows:

$$\mu(G_1) = 0.20, \quad \mu(G_2) = 0.35, \quad \mu(G_3) = 0.25, \quad \mu(G_4) = 0.20,$$

$$\mu(G_1, G_2) = 0.65, \quad \mu(G_1, G_3) = 0.70, \quad \mu(G_1, G_4) = 0.60,$$

$$\mu(G_2, G_3) = 0.55, \quad \mu(G_2, G_4) = 0.65, \quad \mu(G_3, G_4) = 0.50,$$

$$\mu(G_1, G_2, G_3) = 0.80, \quad \mu(G_1, G_2, G_4) = 0.85, \quad \mu(G_1, G_3, G_4) = 0.85,$$

$$\mu(G_2, G_3, G_4) = 0.70, \quad \mu(G_1, G_2, G_3, G_4) = 1.00.$$

Step 3. We utilize the decision information given in matrix R, and the GTFCA operator to obtain the overall preference values $\tilde{r}_i$ of the alternatives A_i $(i = 1,2,3,4,5)$. The results are shown in Table 3.

Table 1
Decision matrix A.

	G_1	G_2	G_3	G_4
A_1	(0.64,0.66,0.69)	(0.35,0.38,0.41)	(0.53,0.58,0.63)	(0.45,0.49,0.53)
A_2	(0.33,0.40,0.43)	(0.42,0.45,0.48)	(0.61,0.63,0.66)	(0.40,0.50,0.56)
A_3	(0.50,0.52,0.55)	(0.66,0.68,0.69)	(0.68,0.70,0.71)	(0.63,0.66,0.68)
A_4	(0.71,0.72,0.73)	(0.70,0.72,0.73)	(0.55,0.58,0.61)	(0.70,0.71,0.73)
A_5	(0.56,0.58,0.61)	(0.67,0.70,0.72)	(0.51,0.52,0.54)	(0.51,0.52,0.54)

Table 2
Decision matrix $\widetilde{R}$.

	G_1	G_2	G_3	G_4
A_1	(0.148,0.167,0.182)	(0.116,0.130,0.146)	(0.168,0.193,0.219)	(0.148,0.170,0.197)
A_2	(0.238,0.276,0.352)	(0.139,0.154,0.171)	(0.194,0.209,0.229)	(0.132,0.174,0.208)
A_3	(0.186,0.212,0.232)	(0.218,0.232,0.246)	(0.216,0.233,0.247)	(0.207,0.229,0.253)
A_4	(0.140,0.153,0.164)	(0.231,0.246,0.261)	(0.175,0.193,0.212)	(0.230,0.247,0.271)
A_5	(0.168,0.190,0.208)	(0.221,0.239,0.257)	(0.162,0.173,0.188)	(0.168,0.181,0.201)

Table 3
The overall preference values of the alternatives.

	TFCA	TFCG	TFCH
A_1	(0.148, 0.169, 0.189)	(0.147, 0.168, 0.188)	(0.146, 0.166, 0.186)
A_2	(0.185, 0.209, 0.242)	(0.181, 0.205, 0.236)	(0.177, 0.202, 0.230)
A_3	(0.206, 0.226, 0.243)	(0.206, 0.226, 0.243)	(0.205, 0.225, 0.243)
A_4	(0.201, 0.216, 0.231)	(0.196, 0.211, 0.226)	(0.191, 0.206, 0.221)
A_5	(0.185, 0.203, 0.221)	(0.184, 0.201, 0.219)	(0.182, 0.199, 0.217)

Table 4
Ordering of the alternative by utilizing the GTFCA operator.

	Ordering
TFCA	$A_3 > A_4 > A_2 > A_5 > A_1$
TFCG	$A_3 > A_2 > A_4 > A_5 > A_1$
TFCH	$A_3 > A_2 > A_4 > A_5 > A_1$

Step 4. According to the aggregating results shown in Table 3 and the formula of degree of possibility (2), the ordering of the alternatives are shown in Table 4. Note that?means "preferred to". As we can see from Table 4, depending on the aggregation operators used, the ordering of the alternatives may be different, and thus, the organization can properly select the desirable alternative according to his interest and the actual needs.

7. Conclusion

The traditional Choquet aggregation operators and generalized OWA operator are generally suitable for aggregating the information taking the form of numerical values, and yet they will fail in dealing with triangular fuzzy information. Motivated by the ideal of Choquet integral [27] and generalized OWA operator [28], in this paper, we propose a generalized triangular fuzzy correlated averaging (GTFCA) operator, whose prominent characteristic is that they cannot only consider the importance of the elements or their ordered positions, but also reflect the correlations of the elements or their ordered positions and study some desirable properties of the GTFCA operator, such as commutativity, idempotency and monotonicity. We have applied the GTFCA operator to multiple attribute decision making problems with triangular fuzzy information. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, we shall continue working in the extension and application of the developed operators to other domains.

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