4. F. A. Shamoyan, Izv. Akad. Nauk Armen. SSR, Ser. Mat., 8, No. 6, 485-497 (1973).
5. F. A. Shamoyan, J. Sov. Math., 8, No. 1, 152-156 (1977).
6. S. A. Vinogradov and N. A. Shirokov, J. Sov. Math., 2, No. 2, 68-83 (1974).
7. D. A. Stegenga, Am. Math. J., 98, No. 3, 3-10 (1976).

REGULAR AVERAGING OPERATORS

L. B. Shapiro

UDC 513.88

The aim of the present paper is to construct examples of open maps of bicompacta, which do not admit of regular averaging operators.

Definition 1 (see [2]). Let $\varphi: X \to Y$ be a continuous map of bicompactum X onto bicompactum Y. Then we say that φ admits of a regular averaging operator if there exists a linear operator u: $C(X) \to C(Y)$ such that $\|u\| = 1$ and the relation $f \in C(Y)$ is fulfilled for any function $u(f \circ \varphi) = f$.

As Michael [6] showed, any open map of compacta (i.e., of metric bicompacta) admits of a regular averaging operator. In [7] this result was strengthened in the following way: an open map of a Milyutin space onto a compactum admits of a regular averaging operator. We recall that a bicompactum which is the image of the generalized Cantor disctoninuum D^τ under a map admitting of a regular averaging operator is called a Milyutin space. An example of an open map not admitting of a regular averaging operator has been constructed in [5]. In this example the preimage is not a dyadic bicompactum and, all the more, not a Milyutin space. On the other hand, from Shchepin's results [4] it follows that if an open image of Milyutin space has a weight not exceeding $\aleph_1$, then it is a Milyutin space. Therefore, the question naturally arises: does any open map of a Milyutin space of weight not exceeding $\aleph_1$ admit of a regular averaging operator? We construct an example of an open map of $D^{\aleph_1}$ onto itself, which does not admit of a regular averaging operator. In passing we obtain as well an example of an open triple map of dyadic bicompacta of weight $\aleph_1$, which does not admit of a regular averaging operator (it is easy to see that any open double map admits of a regular averaging operator).

We recall that for an arbitrary topological space X, by exp X we denote the space of all its nonempty closed subsets in the Vietoris topology, i.e., the base in exp X is formed by sets of the form

$$\langle U_1, \ldots, U_n \rangle = \left\{ \hat{F} \in \exp X\colon\ F \subset \bigcup_{i=1}^{n} U_i \text{ and } F \cap U_i \neq \phi \text{ for any } i \right\},$$

where $U_1, \ldots, U_n$ are sets open in X and $\hat{F}$ denotes the point from exp X corresponding to the set $F = [F] \subset X$. Also let $\exp_n X = \{\hat{F} \in \exp X\colon |F| \leqslant n\}$ be the n-th symmetric degree of space X and $A_n(X) = \{\hat{F} \in \exp X\colon |F| = n\}$. It is clear that $A_n(X) \subset \exp_n X \subset \exp X$.

Proposition 1. Let $X = D^\tau, \tau \geqslant \aleph_1$. Then a continuous map g: $A_n(X) \to X$ such that $(\hat{F}) \in F$ does not exist.

Proof. Suppose that such a map does exist. We consider the natural map $h_n\colon X^n \to \exp_n X$, "neglecting order," i.e., $h_n(x_1, \ldots, x_n) = \widehat{\{x_1, \ldots, x_n\}}$. It will be continuous. Let $B_n(X) = h_n^{-1}(A_n(X))$ be a dense subset of X^n. The point $\xi = \{x_i\} \in B_n(X)$ if and only if $x_i \neq x_j$ for $i \neq j$. We define the function f: $B_n(X) \to R$ as follows: $f(\xi) = i$ if and only if $gh_n(\xi) = x_i$. Then f is a continuous finite-valued function. Since $X = D^\tau$, $X^n = \beta B_n(X)$ is the Stone–Cech compactification of space $B_n(X)$. Let $\tilde{f}$ be the continuation of function f onto X^n. We take an arbitrary point $\xi = \{x_i\}$, lying on the diagonal, i.e., $x_i = x_j = x$ for any i and j. Let $\tilde{f}(\xi) = k$. Then a neighborhood U of point ξ exists such that $\tilde{f}(U) = k$. Since point ξ lies on the diagonal, we can find a neighborhood V of point $x \in X$, such that $\xi \in V^n \subset U$. We take an arbitrary point $\eta \in V^n \cap B_n(X)$. Then $f(\eta) = k$, i.e., $gh_n(\eta) = x_k$. Let $k_1 \neq k$. We take a point $\eta_1 = \{y_i\}$ such that $y_i = x_i$ for $i \neq k, k_1, y_k = x_{k_1}, y_{k_1} = x_k$. Then $\eta_1 \in V^n \cap B_n(X)$. Since $f(\eta_1) = k$, then

Moscow State University. Translated from Funktsional'nyi Analiz i Ego Prilozheniya, Vol. 13, No. 1, pp. 84-85, January-March, 1979. Original article submitted June 1, 1977.

0016-2663/79/1301-0071$07.50

$gh_n(\eta_1) = y_k = x_k \neq x_k$. But this contradicts the fact that $gh_n(\eta) = gh_n(\eta_1)$, since $h_n(\eta) = h_n(\eta_1)$. The proposition has been proved.

We remark that Proposition 1 with n = 2 was first proved by Shchepin by another method.

As usual, by P(X) we denote the space of probability measures in the weak topology, i.e., $P(X) = \{\mu \in M(X): \|\mu\| = 1$ and $\langle \mu, 1_X \rangle = 1\}$.

Proposition 2. Let $X = D^\tau, \tau \geqslant \aleph_1$. Then a continuous map $g: \exp_3 X \to P(X)$ such that supp $g(\hat{F}) \subset F$ does not exist.

The proof of Proposition 2 uses Proposition 1.

We are now ready to describe the examples.

Example 1. There exists an open map of $D^{\aleph_1}$ onto $D^{\aleph_1}$, not admitting of a regular averaging operator.

Let X and Y be homeomorphic to $D^{\aleph_1}$. Then by virtue of Sirota's theorem [3] there exists a homeomorphism f: X → exp Y. We consider the space $Z = \{(x, y) \in X \times Y: y \in f(x)\}$, viz., the graph of map f. Using the result in [3] we can show that Z is homeomorphic to $D^{\aleph_1}$. Then the map $\pi_X: Z \to X$ is open-closed (see [1]). However, it does not admit of a regular averaging operator. Indeed, if this is not so, then by virtue of the statement on p. 33 in [2] there exists a continuous map $\varphi: X \to P(Z)$ such that $\operatorname{supp} \varphi(x) \subset \pi_X^{-1}(x)$. We consider as well the map $P(\pi_Y): P(Z) \to P(Y)$. Then the composition $P(\pi_y) \circ \varphi \circ f^{-1}$ is a map of space exp Y into space P(Y) such that to every element of $\hat{F}$ there corresponds a measure whose support is contained in set F, which contradicts Proposition 2.

Example 1'. There exists an open triple map of dyadic bicompacta of weight $\aleph_1$, not admitting of a regular averaging operator.

In order to see this it is sufficient to consider the restriction of the map $\pi_X: Z \to X$ to the subspace $\pi_X^{-1} f^{-1}(\exp_3 Y)$. The space $\exp_3 Y$ is, obviously, a dyadic compactum. The proof that the bicompactum $\pi_X^{-1} f^{-1}(\exp_3 Y)$ is dyadic is more cumbersome and we shall not carry it out here.

LITERATURE CITED

1. K. Kuratowski, Topology, Vol. I, Academic Press, New York (1966).
2. A. Pelczyński, Linear Continuations, Linear Averagings, and Their Applications [Russian translation], Mir, Moscow (1970).
3. S. Sirota, Dokl. Akad. Nauk SSSR, 181, No. 5, 1069-1072 (1968).
4. E. V. Shchepin, Usp. Mat. Nauk, 31, No. 5, 191-226 (1976).
5. H. H. Corson and J. Lindenstrauss, Trans. Am. Math. Soc., 121, 492-504 (1966).
6. E. Michael, Proc. Am. Math. Soc., 15, 407-409 (1964).
7. S. Z. Ditor, Trans. Am. Math. Soc., 175, 195-208 (1973).