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Common Fixed Point Theorems for Multi-valued Contractions

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Abstract. In this paper, a common fixed point theorem for weakly compatible maps in fuzzy metric spaces is proved.

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1. Introduction and Preliminaries

The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and application. George and Veeramani [7] and Kramosil and Michalek [11] have introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics particularly in connections with both string and E-infinity theory which were given and studied by El Naschie [1, 2, 3, 4, 16]. Many authors [8, 13, 14] have proved fixed point theorem in fuzzy (probabilistic) metric spaces.

Definition 1.1. A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm if it satisfies the following conditions

1. $*$ is associative and commutative,

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2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0, 1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 1.2. A 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
5. $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Let $(X, M, *)$ be a fuzzy metric space . For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}.$$

Let $(X, M, *)$ be a fuzzy metric space. Let τ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then τ is a topology on X (induced by the fuzzy metric M). This topology is Hausdorff and first countable. A sequence $\{x_n\}$ in X converges to x if and only if $M(x_n, x, t) \to 1$ as $n \to \infty$, for each $t > 0$. It is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exits $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$. The fuzzy metric space $(X, M, *)$ is said to be complete if every Cauchy sequence is convergent. A subset A of X is said to be F-bounded if there exists $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Lemma 1.3. [7] *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.*

Example 1.4. Let $X = \mathbb{R}$. Denote $a * b = a.b$ for all $a, b \in [0, 1]$. For each $t \in]0, \infty[$, define

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$.

Definition 1.5. Let $(X, M, *)$ be a fuzzy metric space. M is said to be continuous on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t).$$

Whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$ i.e.

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \text{ and } \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t)$$

Lemma 1.6. *Let $(X, M, *)$ be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.*

Proof. See Proposition 1 of [12]. □

Lemma 1.7. *Let $(X, M, *)$ be a fuzzy metric space. If we define $E_{\lambda,M} : X^2 \to \mathbb{R}^+ \cup \{0\}$ by*

$$E_{\lambda,M}(x, y) = \inf\{t > 0 \ : \ M(x, y, t) > 1 - \lambda\}$$

for each $\mu \in (0, 1)$ there exists $\lambda \in (0, 1)$ such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)$$

for any $x_1, x_2, ..., x_n \in X$
(ii) *The sequence $\{x_n\}_{n\in\mathbb{N}}$ is convergent in fuzzy metric space $(X, M, *)$ if and only if $E_{\lambda,M}(x_n, x) \to 0$. Also the sequence $\{x_n\}_{n\in\mathbb{N}}$ is Cauchy sequence if and only if it is Cauchy with $E_{\lambda,M}$.*

Proof. (i). For every $\mu \in (0, 1)$, we can find a $\lambda \in (0, 1)$ such that

$$\overbrace{(1 - \lambda) * (1 - \lambda) * \cdots * (1 - \lambda)}^{n} \geq 1 - \mu$$

by triangular inequality we have

$$\begin{aligned} &M(x_1, x_n, E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)) + n\delta) \\ &\geq M(x_1, x_2, E_{\lambda,M}(x_1, x_2) + \delta) * \cdots * M(x_{n-1}, x_n, E_{\lambda,M}(x_{n-1}, x_n) + \delta) \\ &\geq \overbrace{(1 - \lambda) * (1 - \lambda) * \cdots * (1 - \lambda)}^{n} \geq 1 - \mu \end{aligned}$$

for very $\delta > 0$, which implies that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n) + n\delta.$$

Since $\delta > 0$ is arbitrary, we have

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)$$

(ii). Note that since M is continuous in its third place and

$$E_{\lambda,M}(x, y) = \inf\{t > 0 \ : \ M(x, y, t) > 1 - \lambda\}.$$

Hence, we have

$$M(x_n, x, \eta) > 1 - \lambda \Longleftrightarrow E_{\lambda,M}(x_n, x) < \eta$$

for every $\eta > 0$. □

Lemma 1.8. *Let (X,M,*) be a fuzzy metric space. If*

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, k^n t)$$

for some $k > 1$ and for every $n \in \mathbb{N}$. Then sequence $\{x_n\}$ is a Cauchy sequence.

Proof. For every $\lambda \in (0,1)$ and $x_n, x_{n+1} \in X$, we have

$$\begin{aligned} E_{\lambda,M}(x_{n+1},x_n) &= \inf\{t>0 \;:\; M(x_{n+1},x_n,t) > 1-\lambda\} \\ &\leq \inf\{t>0 \;:\; M(x_0,x_1,k^n t) > 1-\lambda\} \\ &= \inf\{\frac{t}{k^n} \;:\; M(x_0,x_1,t) > 1-\lambda\} \\ &= \frac{1}{k^n}\inf\{t>0 \;:\; M(x_0,x_1,t) > 1-\lambda\} \\ &= \frac{1}{k^n}E_{\lambda,M}(x_0,x_1). \end{aligned}$$

By Lemma 1.7, for every $\mu \in (0,1)$ there exists $\lambda \in (0,1)$ such that

$$\begin{aligned} E_{\mu,M}(x_n,x_m) &\leq E_{\lambda,M}(x_n,x_{n+1}) + E_{\lambda,M}(x_{n+1},x_{n+2}) + \cdots + E_{\lambda,M}(x_{m-1},x_m) \\ &\leq \frac{1}{k^n}E_{\lambda,M}(x_0,x_1) + \frac{1}{k^{n+1}}E_{\lambda,M}(x_0,x_1) + \cdots + \frac{1}{k^{m-1}}E_{\lambda,M}(x_0,x_1) \\ &= E_{\lambda,M}(x_0,x_1)\sum_{j=n}^{m-1}\frac{1}{k^j} \longrightarrow 0. \end{aligned}$$

Hence sequence $\{x_n\}$ is Cauchy sequence. □

In 1998, Jungck and Rhoades [10] introduced the following concept of weak compatibility.

Definition 1.9. Let A and S be mappings from a fuzzy metric space $(X, M, *)$ into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is, $Ax = Sx$ implies that $ASx = SAx$.

Example 1.10. Let $(X, M, *)$ be a fuzzy metric space, in which X=[0,2], $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ and $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $t > 0$. Define self-maps A and S on X as follows:

$$Ax = \begin{cases} 2 & \text{if } 0 \leq x \leq 1, \\ \frac{x}{2} & \text{if } 1 < x \leq 2, \end{cases} \qquad Sx = \begin{cases} 2 & \text{if } x = 1, \\ \frac{x+3}{5} & \text{otherwise,} \end{cases}$$

Then we have $S1 = A1$=2 and $S2 = A2 = 1$. Also $SA1 = AS1 = 1$ and $SA2 = AS2 = 2$. Thus (A, S) is weak compatible.

Recently, Jungck and Rhoades [9, 10] defined the concepts of $\delta-$ compatible which extend the concept of compatible mappings in the single-valued setting to set-valued mappings. The following definition is given by Jungck and Rhoades [10].

Definition 1.11. The mappings $I : X \longrightarrow X$ and $F : X \longrightarrow B(X)$ are weakly compatible if they commute at coincidence points, i.e., for each point u in X such that $Fu = \{Iu\}$, we have $FIu = IFu$. (Note that the equation $Fu = \{Iu\}$ implies that Fu is a singleton).

Throughout this paper, $B(X)$ is the set of all nonempty bounded subsets of X. For every $t > 0$ let $\delta(A, B, t)$ be the function defined by

$$\delta(A, B, t) = inf\{M(a, b, t); a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta(A, B, t) = \delta(a, B, t)$. If B also consists of a single point b, we write $\delta(A, B, t) = M(a, b, t)$.

It follows immediately from the definition that

$$\begin{aligned} \delta(A, B, t) &= \delta(B, A, t) \geq 0, \\ \delta(A, B, t + s) &\geq \delta(A, C, t) * \delta(C, B, t), \\ \delta(A, B, t) &= 1 \Longleftrightarrow A = B = \{a\}, \end{aligned}$$

$for\ all\ A, B, C\ in\ B(X).$

Lemma 1.12. *Let $(X, M, *)$ be a fuzzy metric space. Then $\delta(A, B, t) \leq \delta(A, B, kt)$, for all A, B in $B(X)$ and some $k > 1$.*

Proof. If $\delta(A, B, t) > \delta(A, B, kt)$ for some A, B in $B(X)$, then for $a \in A$ and $b \in B$ we set $\delta(A, B, kt) = M(a, b, kt)$ for some $k > 1$. On the other hand we have

$$M(a, b, t) \geq \delta(A, B, t) > \delta(A, B, kt) = M(a, b, kt),$$

which is contradiction. □

Definition 1.13. [5]. A sequence $\{A_n\}$ of subsets of X is said to be convergent to a subset A of X if

(i) given $a \in A$, there is a sequence $\{a_n\}$ in X such that $a_n \in A_n$ for $n = 1, 2, \cdots$, and $\{a_n\}$ converges to a.

(ii) given $\epsilon > 0$, there exists a positive integer N such that $A_n \subseteq A_\epsilon$ for $n > N$ where A_ϵ is the union of all open spheres with centers in A and radius ϵ.

Lemma 1.14. [6, 15]. *Let $\{A_n\}$ be a sequence in $B(X)$ and y a point in X such that $\delta(A_n, y, t) \longrightarrow 1$. Then the sequence $\{A_n\}$ converges to the set $\{y\}$ in $B(X)$.*

2. THE MAIN RESULTS

We begin by recalling some basic concepts of the main theory of this paper.

Theorem 2.1. *Let F, G be mappings of a complete fuzzy metric space $(X, M, *)$ into $B(X)$ and I, J be mappings of X into itself satisfying:*

(i) $F(x) \subseteq J(X)$, $G(x) \subseteq I(X)$ *for every* $x \in X$,

(ii) *The pair (F, I) and (G, J) are weakly compatible,*

(iii)*let $\phi : [0, 1]^3 \longrightarrow [0, 1]$, is a continuous function and increasing in any co-ordinate and $\phi(t, t, t) > t$ for every $t \in [0, 1)$.*

(iv) $\delta(Fx, Gy, t) \geq \phi(M(Ix, Jy, kt), \delta(Ix, Fx, kt), \delta(Jy, Gy, kt))$
for every x, y *in* X *and some* $k > 1$. *Suppose that one of* $J(X)$ *or* $S(X)$ *is a closed subset of* X, *then there exists a unique* $z \in X$ *such that* $\{z\} = \{Iz\} = \{Jz\} = Fz = Gz$.

Proof. Let x_0 be an arbitrary point in X. By (i), we choose a point x_1 in X such that $Jx_1 \in Fx_0 = Z_0$. For this point x_1 there exists a point x_2 in X such that $Ix_2 \in Gx_1 = Z_1$, and so on. Continuing in this manner we can define a sequence $\{x_n\}$ as follows

$$Jx_{2n+1} \in Fx_{2n} = Z_{2n}, \ IX_{2n+2} \in Gx_{2n+1} = Z_{2n+1},$$

for $n = 0, 1, 2, \cdots$. For simplicity, we put $V_n(t) = \delta(Z_n, Z_{n+1}, t)$, for $n = 0, 1, 2, \cdots$. We prove that sequence $\{V_n(t)\}$ is an increasing and convergent to 1. Since

$$\begin{aligned} V_{2n}(t) &= \delta(Z_{2n}, Z_{2n+1}, t) = \delta(Fx_{2n}, Gx_{2n+1}, t) \\ &\geq \phi(M(Ix_{2n}, Jx_{2n+1}, kt), \delta(Ix_{2n}, Fx_{2n}, kt), \delta(Jx_{2n+1}, Gx_{2n+1}, kt)) \\ &\geq \phi(\delta(Gx_{2n-1}, Fx_{2n}, kt), \delta(Gx_{2n-1}, Fx_{2n}, kt), \delta(Fx_{2n}, Gx_{2n+1}, kt)) \\ &= \phi(\delta(Z_{2n-1}, Z_{2n}, kt), \delta(Z_{2n-1}, Z_{2n}, kt), \delta(Z_{2n}, Z_{2n+1}, kt)) \\ &= \phi(V_{2n-1}(kt), V_{2n-1}(kt), V_{2n}(kt)). \end{aligned}$$

We prove that $V_{2n}(kt) \geq V_{2n-1}(kt)$. Now, if $V_{2n}(kt) < V_{2n-1}(kt)$ for some $n \in \mathbb{N}$, since ϕ is an increasing function , then the last inequality above we get

$$V_{2n}(t) \geq \phi(V_{2n}(kt), V_{2n}(kt), V_{2n}(kt)) > V_{2n}(kt).$$

That is $V_{2n}(t) > V_{2n}(kt)$, by Lemma1.12 this is a contradiction. Hence $V_{2n}(kt) \geq V_{2n-1}(kt)$. Similarly, one can show that

$$V_{2n+1}(kt) \geq V_{2n}(kt).$$

Then we deduce that

$$V_0(t) \leq V_1(t) \leq V_2(t) \leq \cdots$$

Thus $\{V_n(t)\}$ is increasing sequence in $[0, 1]$. Therefore , tends to a limit $a \leq 1$. We claim that $a = 1$. For if $a < 1$ on making $n \to \infty$ the following inequality,

$$\delta(Z_n, Z_{n+1}, t) = V_{2n}(t) \geq \phi(V_{2n-1}(kt), V_{2n-1}(kt), V_{2n-1}(kt)),$$

we get

$$a \geq \phi(a, a, a) > a,$$

is a contradiction. Hence $a = 1$, i.e.,

$$V_n(t) = \delta(Z_n, Z_{n+1}, t) \longrightarrow 1.$$

It is easily seen that

$$V_n(t) \geq V_{n-1}(kt) \geq \cdots \geq V_0(k^n t) = \delta(Z_0, Z_1, k^n t).$$

By the above inequality , then if z_n is an arbitrary point in the set Z_n , for $n = 0, 1, 2, \cdots$, it follows that

$$M(z_n, z_{n+1}, t) \geq M(z'_0, z'_1, k^n t),$$

for $z'_0 \in Z_0$ and $z'_1 \in Z_1$. Thus for some $a \in A$ and $b \in B$ if we set

$$M(a, b, t) = \inf\{M(x, y, t); x, y \in X\}.$$

Then $M(z_n, z_{n+1}, t) \geq M(a, b, k^n t)$.

By Lemma 1.8, the sequence $\{z_n\}$, and hence any sequence of Z_n, is a Cauchy sequence in X.

Suppose that $J(X)$ is complete. But $Jx_{2n+1} \in Fx_{2n} = Z_{2n}$, for $n = 0, 1, 2, \cdots$. Therefore by the above, the sequence $\{Jx_{2n+1}\}$ is Cauchy and hence $Jx_{2n+1} \longrightarrow p = Jv \in J(X)$, for some $v \in X$. But $Ix_{2n} \in Gx_{2n-1} = Z_{2n-1}$ that is ,

$$M(Ix_{2n}, Jx_{2n+1}, t) \geq \delta(Z_{2n-1}, Z_{2n}, t) = V_{2n-1}(t) \longrightarrow 1.$$

Consequently, $Ix_{2n} \longrightarrow p$. Moreover, we have for $n = 1, 2, 3, \cdots$

$$\begin{aligned} \delta(Fx_{2n}, p, t) &\geq \delta(Fx_{2n}, Ix_{2n}, \frac{t}{2}) * \delta(Ix_{2n}, p, \frac{t}{2} \\ &\geq \delta(Z_{2n}, Z_{2n-1}, \frac{t}{2}) * \delta(Ix_{2n}, p, \frac{t}{2}). \end{aligned}$$

Therefore, $\lim_{n\to\infty} \delta(Fx_{2n}, p, t) = 1 * 1 = 1$. Hence from Lemma 1.14 it follows that $\lim_{n\to\infty} Fx_{2n} = \{p\}$. In like manner it follows that $\delta(Gx_{2n-1}, p, t) \longrightarrow 1$.

Since , for $n = 1, 2, 3, \cdots$,

$$\delta(Fx_{2n}, Gv, t) \geq \phi(M(Ix_{2n}, Jv, kt), \delta(Ix_{2n}, Fx_{2n}, kt), \delta(Jv, Gv, kt)).$$

On making $n \longrightarrow \infty$ if $Gv \neq p$, then

$$\begin{aligned} \delta(p, Gv, t) &> \phi(M(p, Jv, kt), \delta(p, p, kt), \delta(Jv, Gv, kt)) \\ &> \delta(p, Gv, kt). \end{aligned}$$

Hence $\delta(p, Gv, t) > \delta(p, Gv, kt)$, by Lemma1.12 this is a contradiction. Thus $Gv = \{p\} = \{Jv\}$, since $Gv \subseteq IX$, so $u \in X$ exists such that $\{Iu\} = Gv = \{Jv\}$. Now if $Fu \neq Gv$, then

$$\begin{aligned} \delta(Fu, Gv, t) &\geq \phi(M(Iu, Jv, kt), \delta(Iu, Fu, kt), \delta(Jv, Gv, kt)) \\ &> \delta(Fu, Gv, kt), \end{aligned}$$

by Lemma1.12 this is a contradiction. So we have $Fu = Gv$. Hence $Fu = \{p\} = Gv = \{Iu\} = \{Jv\}$. Since $Fu = \{Iu\}$ and the pair $\{F, I\}$ is weakly compatible, we obtain $Fp = FIu = IFu = \{Ip\}$. If $Fp \neq \{p\}$, we have

$$\begin{aligned} \delta(Fp, p, t) &= \delta(Fp, Gv, t) \\ &\geq \phi(M(Ip, Jv, kt), \delta(Ip, Fp, kt), \delta(Jv, Gv, kt)) \\ &> \delta(Fp, p, kt), \end{aligned}$$

by Lemma1.12 this is a contradiction. It follows that $Fp = \{Ip\} = \{p\}$. Similarly, $Gp = \{Jp\} = \{p\}$. Therefore, we obtain $Fp = Gp = \{Jp\} = \{Ip\} = \{p\}$. To see the p is unique, suppose that $\{q\} = \{Iq\} = \{Jq\} = Fq = Gq$. If $p \neq q$, then

$$\begin{aligned} M(p,q,t) &\geq \delta(Fp,Gq,t) \\ &\geq \phi(M(Ip,Jq,kt),\delta(Ip,Fp,kt),\delta(Jq,Gq,kt)) \\ &> M(p,q,kt), \end{aligned}$$

is a contradiction. It follows that $p = q$. □

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