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A continuous passage from trigonometric to Hausdorff moments : an attempt with consequences

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The objective of this talk is a discussion of the the families T_γ with $0 \le \gamma \le 1$, of functions F which are analytic in the domains

$$\Omega_\gamma^* := \begin{cases} \left\{ \frac{(\gamma^2-1)z^2}{(1+\gamma z)^2} \ : \ |z|<1 \right\}, & 0 \le \gamma < 1, \\ \mathbb{C} \setminus [-1,\infty], & \gamma = 1, \end{cases}$$

and satisfy the conditions $F(0) = 1$,

$$\text{Re } \frac{1}{1+\gamma z} F\left(\frac{(1+\gamma x)z}{(1+\gamma z)x} \right) > \frac{1}{2}, \quad |z|<1, |x|=1.$$

It turns out that these functions, for $\gamma = 0$, are the generating functions for trigonometric moment sequences, and for $\gamma = 1$ the generating functions for Hausdorff moment sequences. The cases corresponding to $0 < \gamma < 1$ are not yet completely understood, but many interesting properties of the members in T_γ are already known and will be discussed here. For instance, it turns out that the classes T_γ are closed under the Hadamard product, and more results of the type of the Pólya–Schoenberg conjecture will be derived.