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Corrigendum to "Metrical and topological properties of a generalized Libera transform" [Arch. Math. 63 (1994) 517–524]

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Abstract. We provide an argument that corrects a flaw in the proof of Theorem 1 of the article.

Theorem 1 of [1] states (among other) that the generalized Libera transform is bounded on the Besov spaces B^p for $1 < p < \infty$. The theorem as stated is correct but its proof as given in [1] is valid only for $2 \leq p < \infty$ and needs to be corrected for the other values $1 < p < 2$. We would like to thank Jie Xiao and S. Stevic for pointing out the gap. The erroneous part of the proof is due to an inequality used in line 5^+ of [1, page 521], which inequality is valid only for $2 \leq p < \infty$. We fill the gap here for the range $1 < p < 2$.

We use the same notation as in [1]. We also denote by $\|f\|_{p,\alpha}$ the norm of f in the weighted Bergman space A^p_α with weight $w(z) = (1 - |z|^2)^\alpha$ as in [2], and observe that for the Besov space seminorm $\rho_{3,p}$ we have $\rho_{3,p}(f) = \|f'\|_{p,p-2}$.

Recall that for a function $f \in B^p$, the generalized Libera transform of f is

$$g(z) = \int_0^1 f(\phi_t(z))\, dt, \quad z \in \mathbb{D},$$

where $\phi_t(z) = tz + (1 - t)z_0$ and $z_0 \in \mathbb{D} \cup \partial\mathbb{D}$. We showed in [1, page 20], for all $1 < p < \infty$, that $\rho_{3,p}$ satisfies

$$\rho_{3,p}(g)^p \leq \int_0^1 \rho_{3,p}(f \circ \phi_t)^p\, dt, \tag{1}$$

and the proof of Theorem 1 was then completed by claiming that $\rho_{3,p}(f \circ \phi_t) \leq \rho_{3,p}(f)$. The proof of this claim, as given in [1], is correct only for $2 \leq p < \infty$. To fill the gap for $1 < p < 2$ we work as follows

$$
\begin{aligned}
\rho_{3,p}(f\circ\phi_t)^p &= \int_{\mathbb{D}} |(f\circ\phi_t)'(z)|^p(1-|z|^2)^{p-2}\,dm(z)\\
&= \int_{\mathbb{D}} |(f'\circ\phi_t)(z)|^p|\phi_t'(z)|^p(1-|z|^2)^{p-2}\,dm(z)\\
&\le t^p\int_{\mathbb{D}} |(f'\circ\phi_t)(z)|^p(1-|z|^2)^{p-2}\,dm(z), \quad (\text{since } |\phi_t'(z)|=t),\\
&= t^p\|f'\circ\phi_t\|^p_{p,p-2}, \quad (\text{norm of } f'\circ\phi_t \text{ in the Bergman space } A^p_{p,p-2}).
\end{aligned}
$$

Now we apply [2, Lemma 1] taking into account that $-1<p-2<0$. It is easy to check that $(\|\phi_t\|_\infty+3|\phi_t(0)|)/(\|\phi_t\|_\infty+|\phi_t(0)|)\le 2$ and that

$$
\frac{\|\phi_t\|_\infty+|\phi_t(0)|}{\|\phi_t\|_\infty-|\phi_t(0)|}=\frac{t+(1-t)|z_0|+(1-t)|z_0|}{t+(1-t)|z_0|-(1-t)|z_0|}=(t+2(1-t)|z_0|)/t\le 2/t
$$

From these and [2, Lemma 1] we have

$$
\|f'\circ\phi_t\|^p_{p,p-2}\le 2^{2-p}(2/t)^p=4t^{-p},
$$

hence $\rho_{3,p}(f\circ\phi_t)^p\le 4\rho_{3,p}(f)^p$. Combining this with (1) we obtain $\rho_{3,p}(g)\le 4^{1/p}\rho_{3,p}(f)$ and the desired conclusion follows.

References

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