International Mathematical Forum, 2, 2007, no. 32, 1577 - 1586

On Fuzzy KS-Semigroups

D.R Prince Williams

Department of Information Technology
Salalah College of Technology
Post Box:608, Salalah-211
Sultanate of Oman
princeshree1@gmail.com

Shamshad Husain

Department of Applied Mathematics
Z.H.College of Engineering and Technology
Aligarh Muslim University
Alighar-202002,India
s_husain123@yahoo.co.in

Abstract

In this paper ,we fuzzify the new class of algebraic structure introduced by Kim[5].In this fuzzification(called fuzzy KS-semigroups),we introduce the notions of fuzzy sub KS-semigroups,fuzzy KS-ideal,fuzzy KS-p-ideal and investigate some of their related properties.The purpose of this study is to implement the fuzzy set theory and ideal theory in the KS-semigroups.This fuzzification leads to development of new notions over fuzzy KS-semigroups.

Mathematics Subject Classification: 06F35, 03G25, 03E72

Keywords: fuzzy sub KS-semigroup, fuzzy KS-ideal, fuzzy KS-p-ideal

1 Introduction

The notion of BCK-semigroups was proposed by Imai and Isèki [2]as a generalization of the concept of set-theoretic difference and propositional calculus.
Since then a great deal of literature has been produced on the theory of BCK-algebras. Isèki[3] introduced the notion of BCI-algebras, which is a generalization of BCK-algebras.For the general development of BCK/BCI-algebras,the ideal theory plays an important role. Zadeh[8] introduced the notion of fuzzy sets.After the introduction of the concept of fuzzy sets by Zadeh several researches were conducted on the generalization of the the notion of fuzzy sets, and Rosenfeld[6] introduced

the notion of fuzzy group.Following the idea of fuzzy groups,Xi[7] introduced the notion of fuzzy BCK-algebras.Jun and Meng[6] studied fuzzy BCK-algebras.After that Ahmed[1] fuzzified BCI-algebras.

Recently,the new class of algebraic structure introduced by Kim[5],called KS-semigroups,which is the combination of BCK-algebras and semigroups. In this paper ,we fuzzify the new class of algebraic structure introduced by Kim[5].In this fuzzification(called fuzzy KS-semigroups),we introduce the notions of fuzzy sub KS-semigroups,fuzzy KS-ideal,fuzzy KS-p-ideal and investigate some of their related properties.Using these new notions,we claim that some of the results of fuzzy KS-semigroups are very closely related to the results of fuzzy BCK-algebras.

For convenience of readers,in section 2,we list the basic definitions of BCK-algebras,fuzzy set theory,fuzzy BCK-algebras,ideal theory and new structure KS-semigroups.Also in this section we introduce the new notion called sub KS-subgroup with example.In section 3,we introduce the notion fuzzy sub KS- semigroup with suitable example.In section 4,we introduce the notion of fuzzy KS-ideal and fuzzy KS-p-ideal with examples.The constructions of these notions are based on ideal theory.In this section, we have proved some interesting results which are very closer to the results of fuzzy ideal in BCK-algebras.In section 5,we have proved some results on fuzzy KS-ideals in KS-semigroup homomorphism.

2 Preliminary

In this section, we cite the fundamental definitions that will be used in the sequel:

Definition 2.1 *An algebraic system* $(X, *, 0)$ *of type* $(2, 0)$ *is called a BCK-algebra if it satisfies the following conditions:*

BCK1. $((x * y) * (x * z)) * (z * y) = 0$,

BCK2. $((x * (x * y)) * y = 0$,

BCK3. $x * x = 0$,

BCK4. $0 * x = 0$

BCK5. $x * y = 0$ *and* $y * x = 0 \Rightarrow x = y$*, for all* $x, y, z \in X$

Remarks

- A partial ordering " $\leq$ " on X can be defined by $x \leq y$ if and only if $x * y = 0$.
- A BCK-algebra X has the following properties:
  1. $x * 0 = 0$,
  2. $(x * y) * z = (x * z) * y$,
  3. $x \leq y$ implies that $x * z \leq y * z$ and $z * y \leq z * x$,
  4. $(x * z) * (y * z) \leq x * y$,for all $x, y, z \in X$.

Definition 2.2 *A KS-semigroup is a non-empty set X with two binary operation $*$ and . ,and a constant 0 satisfies the following axioms:*

KS1. $(X, *, 0)$ *is a BCK-algebra,*

KS2. $(X, .)$*is a semigroup,*

KS3. $x.(y * z) = (x.y) * (x.z)$ *and* $(x * y).z = (x.z) * (y.z)$*,for all* $x, y, z \in X$.

Remarks

- Throughout this paper X denotes the KS-semigroup unless otherwise specified.
- For the sake of convenience ,we shall write the multiplication $x.y$ by xy.

Example 2.3 *Let* $X = \{0, 1, 2, 3, 4\}$*be defined by the following Cayley tables:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	0	0

.	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	2
3	0	0	0	1	2
4	0	1	2	3	4

Then ,by usual calculations, we can prove that X is a KS-semigroup.

Definition 2.4 *A non-empty subset A of a semigroup $(X, .)$ is said to be left(resp. right) stable if* $xa \in A$ *(resp.*$ax \in A$*) whenever* $x \in X$ *and* $a \in A$.

Both left and right stable is called *two-sided stable* or simply *stable.*

Definition 2.5 *A non-empty subset A of a KS-semigroup X is said to be left(resp. right) ideal of X if*

KSI1. *A is left(resp.right) stable subset of* $(X, .)$ *and*

KSI2. $x * y \in A$ *and* $y \in A$ *imply that* $x \in A$ *, for all* $x, y \in X$.

If A is both left and right ideal then A is called *two-sided ideal*or simply *an ideal* .

Example 2.6 *Let* $X = \{0, 1, 2, 3\}$*be defined by the following Cayley tables:*

*	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	0
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Then by usual calculations, we can prove that $X$is a KS-semigroup. If $A = \{0, 1\}$,then A is an ideal of a KS-semigroup of X.

Definition 2.7 *A non-empty subset A of a KS-semigroup X is said to be left(resp. right)p-ideal of X if*

KSPI1. *A is a left(resp. right)stable subset of $(X, .)$ and,*

KSPI2. *$(x * y) * z \in A$ and $y * z \in A$ imply that $x * z \in A$, for all $x, y, z \in X$.*

Example 2.8 *Let $X = \{0, 1, 2, 3\}$ be defined by the following Cayley tables:*

*	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	2	2	0	2
3	1	1	1	0

.	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	3	0	0

Then by usual calculation,we can prove that X is a KS-semigroup.If $A = \{0, 1\}$,then A is a p-ideal of a KS-semigroup of X.

Definition 2.9 *Let X be a non-empty set.A fuzzy subset of X is a function $\mu : X \to [0, 1]$.Let μ be the fuzzy subset of X.For a fixed $0 \le t \le 1$,the set $\mu_t = \{x \in X | \mu(x) \ge t\}$ is called an upper level set of μ.*

Definition 2.10 *A non-empty subset S of X with binary operations $*$ and .,is called sub KS-semigroup of X if it satisfies the following conditions:*

KSS1. *$x * y \in S$ and,*

KSS2. *$xy \in S$,for all $x, y \in X$.*

Example 2.11 *Let $X = \{0, 1, 2\}$ be a KS-semigroup with the following Cayley tables:*

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

.	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

If $S = \{0, 1\}$,then S is a sub KS-semigroup of X.

3 Fuzzy sub KS-semigroup

In this section,we introduce the notion of *fuzzy sub KS-semigroups* and study their related properties

Definition 3.1 *A fuzzy subset μ in X is called a fuzzy sub KS-semigroup of X if it satisfies the following conditions:*

FSKS1. $\mu(x * y) \geq min\{\mu(x), \mu(y)\}$,

FSKS2. $\mu(xy) \geq min\{\mu(x), \mu(y)\}$, *for all* $x, y, \in X$.

Example 3.2 *Let* $X = \{0, 1, 2\}$ *be a KS-semigroup with the following Cayley tables:*

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

.	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Define a fuzzy subset $\mu : X \longrightarrow [0, 1]$ *by* $\mu(0) = 0.7, \mu(x) = 0.4$,*for all* $x \neq 0$.*Then by usual calculations,we can prove that* μ *is a fuzzy sub KS-semigroup of* X.

Theorem 3.3 *A fuzzy set* μ *of* X *is a fuzzy sub KS-semigroup if and only if for every* $0 \leq t \leq 1$,*the upper level set* μ_t *is either empty or a sub KS-semigroup of* X.

Proof: Suppose that μ is a fuzzy sub KS-semigroup of X. and $\mu_t \neq 0$,then for any $x, y \in \mu_t$,we have $\mu(x * y) \geq min\{\mu(x), \mu(y)\} = t$.Thus $x * y \in \mu_t$.Also $\mu(xy) \geq min\{\mu(x), \mu(y)\} = t$.Thus $xy \in \mu_t$. Hence μ_t is a sub KS-semigroup of X.

Conversely,take $t = min\{\mu(x), \mu(y)\}$, for all $x, y \in X$. Since $\mu_t(\neq 0)$ is a sub KS-semigroup of X, we have $\mu(x * y) \geq t = min\{\mu(x), \mu(y)\}$ and $\mu(xy) \geq t = min\{\mu(x), \mu(y)\}$.Hence μ is a fuzzy sub KS-semigroup.

Theorem 3.4 *Any sub KS-semigroup of* X *can be realized as a level sub KS-semigroup of some fuzzy sub KS-semigroup.*

Proof: Let A be a sub KS-semigroup of X and μ be a fuzzy subset in X defined by

$$\mu(x) = \begin{cases} t & if \quad x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

where $0 < t < 1$ .It is clear that $\mu_t = A$.

If $x, y \in A$,then $\mu(x) = \mu(y) = t$.Thus $\mu(x * y) \geq min\{\mu(x), \mu(y)\} = t$ and $\mu(xy) \geq min\{\mu(x), \mu(y)\} = t$.Hence $x * y \in \mu_t$ and $xy \in \mu_t$.

If $x, y \notin A$,then $\mu(x) = \mu(y) = 0$.Thus$\mu(x * y) \geq min\{\mu(x), \mu(y)\} = 0$ and $\mu(xy) \geq min\{\mu(x), \mu(y)\} = 0$.Hence $x * y \in \mu_t$ and $xy \in \mu_t$.

If at most one of $x, y \in A$,then at least one of $\mu(x)$ and $\mu(y)$ is equal to 0.Thus $\mu(x * y) \geq min\{\mu(x), \mu(y)\} = 0$ and $\mu(xy) \geq min\{\mu(x), \mu(y)\} = 0$.Hence $x * y \in \mu_t$ and $xy \in \mu_t$.

Theorem 3.5 *Two level sub KS-semigroups* μ_s *and* μ_t *of a fuzzy sub KS-semigroup are equal if and only if there is no* $x \in X$ *such that* $s \leq \mu(x) \leq t$.

Proof: The proof is simple and straight forward.

4 Fuzzy KS-ideal

In this section,we introduce the notions of *fuzzy KS-ideal* and *fuzzy KS-p-ideal*, and investigate related properties.

Definition 4.1 *A fuzzy subset μ of X is called a left fuzzy KS-ideal if*

KSI1. $\mu(0) \geq \mu(x)$,

KSI2. $\mu(x) \geq min\{\mu(x * y), \mu(y)\}$,

KSI3. $\mu(xa) \geq min\{\mu(x), \mu(a)\}$, *for all* $x, y, a \in X$.

A fuzzy subset μ is called a *right fuzzy KS-ideal* if it satisfies **KSI1,KSI2** and **KSI4:** $\mu(ax) \geq min\{\mu(x), \mu(a)\}$, for all $x, y, a \in X$.

A fuzzy subset μ of X is called a *fuzzy KS-ideal* if it is both left and right fuzzy KS-ideal of X.

Example 4.2 *Let $X = \{0, 1, 2, 3, 4\}$ be a KS-semigroup defined by the following Cayley tables:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	4	0

*	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	2
3	0	0	0	2	3
4	0	1	2	3	4

Define a fuzzy subset $\mu : X \longrightarrow [0, 1]$ by $\mu(0) = 0.7$ and $\mu(x) = 0.4$,for all $x \neq 0$.Then by usual calculations,we can prove that μ is a left fuzzy KS-ideal of X.

Definition 4.3 *A fuzzy subset μ of X is called a left fuzzy KS-p-ideal if*

KSP1. $\mu(0) \geq \mu(x)$,

KSP2. $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y * z)\}$,

KSP3. $\mu(xa) \geq min\{\mu(x), \mu(a)\}$, *for all* $x, y, z, a \in X$.

A fuzzy subset μ is called a *right fuzzy KS-p-ideal* if it satisfies **KSP1,KSP2** and **KSP4:** $\mu(ax) \geq min\{\mu(x), \mu(a)\}$, for all $x, y, a \in X$.

A fuzzy subset μ of X is called a *fuzzy KS-p-ideal* if it is both left and right fuzzy KS-p-ideal of X.

Example 4.4 *Let $X = \{0, 1, 2\}$ be a KS-semigroup with the following Cayley tables:*

*	0	1	2
0	0	0	0
1	2	0	1
2	2	1	0

.	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Define a fuzzy subset $\mu : X \longrightarrow [0,1]$ by $\mu(0) = 0.7$ and $\mu(x) = 0.4$,for all $x \neq 0$.Then by usual calculations,we can prove that μ is a left fuzzy KS-p-ideal of X.

Theorem 4.5 *Every left(resp. right) fuzzy KS-p-ideal of X is a left(resp. right) fuzzy KS-ideal of X.*

Proof: Let μ be a left fuzzy KS-p-ideal of X.Then μ satisfies the conditions of $KSI1$ and $KSI3$.We have $\mu(x * z) \geq min\{\mu((x * y) * z), \mu(y * z)\}$,put $z = 0$,we get $\mu(x * 0) \geq min\{\mu((x * y) * 0), \mu(y * 0)\}$.Thus $\mu(x) \geq min\{\mu(x * y), \mu(y)\}$.Hence μ is a left fuzzy KS-ideal of X.

However the converse of the theorem is not true.

Example 4.6 *Let $X = \{0, 1, 2\}$ be a KS-semigroup with the following Cayley tables:*

*	0	1	2
0	0	0	0
1	2	0	0
2	2	1	0

.	0	1	2
0	0	0	0
1	0	1	0
2	0	0	2

Define a fuzzy subset $\mu : X \to [0,1]$ by $\mu(0) = 0.7, \mu(x) = 0.4$, for all $x \neq 0$.Then by usual calculations,we can prove that μ is a left fuzzy KS-ideal of X. But it is not left fuzzy KS-p-ideal of X.Since $\mu(2 * 1) < min\{\mu((2 * 1) * 1), \mu(1 * 1)\}$.

Theorem 4.7 *Let μ be a left (resp. right) fuzzy KS-ideal of X.Then the non-empty level set μ_t is also a left(resp. right) fuzzy KS-ideal of X.*

Proof: We have μ is a left fuzzy KS-ideal of X.If $x, y, a \in \mu_t$ then $\mu(x) \geq t$,$\mu(y) \geq t$ and $\mu(a) \geq t$.(i)We have $\mu(0) \geq \mu(x) \geq t$, thus $\mu_t(0) \geq \mu_t(x)$(ii) Define $t = min\{\mu(x*y), \mu(y)\}$.We have $\mu(x) \geq t$,then $\mu(x) \geq t = min\{\mu(x*y), \mu(y)\}$.(iii)Define $t = min\{\mu(x), \mu(a)\}$.We have $\mu(xa) \geq t$,then $\mu(xa) \geq t = min\{\mu(x), \mu(a)\}$. Hence μ_t is a left fuzzy KS-ideal of X.

Definition 4.8 *Let λ and μ be the fuzzy subsets in a set X.The cartesian product $\lambda \times \mu : X \times X \longrightarrow [0,1]$ is defined by $(\lambda \times \mu)(x, y) = min\{\lambda(x), \mu(y)\}$, for all $x, y \in X$.*

Theorem 4.9 *Let λ and μ be a left(resp.right)fuzzy KS-ideal of X.Then $\lambda \times \mu$ is also a left (resp.right)fuzzy KS-ideal of X.*

Proof: For any $(x, y) \in X \times X$,we have

(i) $(\lambda \times \mu)(0, 0) = min\{\lambda(0), \mu(0)\} \geq min\{\lambda(x), \mu(x)\} = (\lambda \times \mu)(x, y)$.

(ii) For any $(x_1, x_2), (y_1, y_2) \in X \times X$,we have

$(\lambda \times \mu)(x_1, x_2) = min\{\lambda(x_1), \mu(x_2)\}$

$\geq min\{min\{\lambda(x_1 * y_1), \lambda(y_1)\}, min\{\mu(x_2 * y_2), \mu(y_2)\}\}$

$= min\{min\{\lambda((x_1 * y_1), \mu(x_2 * y_2))\}, min\{\lambda(y_1), \mu(y_2)\}\}$

$= min\{(\lambda \times \mu)((x_1, x_2) * (y_1, y_2)), (\lambda \times \mu)(y_1, y_2)\}$

(iii) For any $x, a \in X$ then $xa \in X$ and $(x, y), (a_1, a_2) \in X \times X$,we have
$(\lambda \times \mu)(x, y)(a_1, a_2) = (\lambda \times \mu)(xa_1, ya_2) = min\{\lambda(xa_1), \mu(ya_2)\}$
$\geq min\{min\{\lambda(x), \lambda(a_1)\}, min\{\mu(y), \mu(a_2)\}\}$
$= min\{min\{\lambda(x), \mu(y)\}, min\{\lambda(a_1, \mu(a_2)\}\}$
$= min\{(\lambda \times \mu)(x, y), (\lambda \times \mu)(a_1, a_2)\}$.

Theorem 4.10 *Let λ and μ be fuzzy subsets of X such that $\lambda \times \mu$ is a left (resp.right)fuzzy KS-ideal of $X \times X$.Then*
(i) either$\lambda(0) \geq \lambda(x)or\mu(0) \geq \mu(x)$, for all $x \in X$.
(ii) if $\lambda(0) \geq \lambda(x)$,then either $\mu(0) \geq \lambda(x)$or$\mu(0) \geq \mu(x)$, for all $x \in X$.
(iii) if $\mu(0) \geq \mu(x)$,then either $\lambda(0) \geq \lambda(x)$ or $\lambda(0) \geq \mu(x)$, for all $x \in X$.

Proof: By using reduction and absurdity , we can prove this theorem easily.

Theorem 4.11 *Let λ and μ be fuzzy subsets of X.If $\lambda \times \mu$ is a left(resp.right)fuzzy KS-ideal of $X \times X$ then either λ or μ is a left (resp.right)fuzzy KS-ideal of X.*

Proof: By Theorem 4.10 of (i),without lose of generality,we assume that $\mu(0) \geq \mu(x)$,for all $x \in X$.By Theorem 4.10 of (iii) that either $\lambda(0) \geq \lambda(x)$ or $\lambda(0) \geq \mu(x)$, for all $x \in X$.

If $\lambda(0) \geq \mu(x)$,then$(\lambda \times \mu)(0, x) = min\{\lambda(0), \mu(x)\} = \mu(x) - - - - - (A)$

Since $\lambda \times \mu$ is a left fuzzy KS-ideal of $X \times X$,therefore for all $(x_1, x_2), (y_1, y_2)$and $(a_1, a_2) \in X \times X$,then
$(\lambda \times \mu)((x_1, x_2)) \geq min\{(\lambda \times \mu)((x_1, x_2) * (y_1, y_2)), (\lambda \times \mu)((y_1, y_2))\}$ and
$((\lambda \times \mu))((x_1, x_2)(a_1, a_2)) \geq min\{(\lambda \times \mu)((x_1, x_2)), (\lambda \times \mu)(a_1, a_2)\}$.
$(\lambda \times \mu)((x_1, x_2)) \geq min\{(\lambda \times \mu)((x_1 * y_1), (x_2 * y_2)), (\lambda \times \mu)((y_1, y_2))\}$ and
$(\lambda \times \mu)((x_1, x_2)(a_1, a_2)) \geq min\{(\lambda \times \mu)((x_1, x_2)), (\lambda \times \mu)((a_1, a_2))\}$.
If $x_1 = y_1 = a_1 = 0$
$(\lambda \times \mu)((0, x_2)) \geq min\{(\lambda \times \mu)((0 * 0), (x_2 * y_2)), (\lambda \times \mu)(0, y_2)\}$ and
$((\lambda \times \mu)((0, x_2)(0, a_2)) \geq min\{(\lambda \times \mu)((0, x_2)), (\lambda \times \mu)(0, a_2)\}$.
Using(A) we get $\mu(x_2) \geq min\{\mu(x_2 * y_2), \mu(y_2)\}$ and
$\mu(x_2a_2) \geq min\{\mu(x_2 * a_2), \mu(a_2)\}$. This complete the proof.

Definition 4.12 *Let A be a fuzzy subset in S,the strongest fuzzy relation on S,that is fuzzy relation on A is μ_A given by $\mu_A(x, y) = min\{A(x), A(y)\}$.*

Theorem 4.13 *Let A be a fuzzy subset in X and μ_A be a strongest fuzzy relation on X and $xx = x$ for all $x \in X$.Then A is a left(resp. right) fuzzy KS-ideal of X if and only if μ_A is a left(resp. right)fuzzy KS-ideal of $X \times X$.*

Proof: Suppose that A is a left fuzzy KS-ideal of X.
(i)For all $x, y \in X$,we have
$\mu_A(0, 0) = min\{A(0), A(0)\} \geq min\{A(x), A(y)\} = \mu_A(x, y)$
(ii) For all (x_1, x_2) and (y_1, y_2) in $X \times X$,then we have
$\mu_A(x_1, x_2) = min\{A(x_1), A(x_2)\}$
$\geq min\{min\{A(x_1 * y_1), A(y_1)\}, min\{A((x_2 * y_2)), A(y_2)\}$

$= min\{min\{A(x_1 * y_1), A(x_2 * y_2))\}, min\{A(y_1), A(y_2)\}$
$= min\{\mu_A((x_1 * y_1), (x_2 * y_2)), \mu_A(y_1, y_2)\}$
$= min\{\mu_A((x_1, x_2) * (y_1, y_2)), \mu_A(y_1, y_2)\}$
(iii) For all (x_1, x_2) and (a_1, a_2) in $X \times X$,then we have
$\mu_A((x_1, x_2)(a_1, a_2)) = \mu_A((x_1a_1), (x_2a_2)) = min\{A(x_1a_1), A(x_2a_2)\}$
$\geq min\{min\{A(x_1), A(a_1)\}, min\{A(x_2)), A(a_2)\}$
$= min\{min\{A(x_1), A(x_2)\}, min\{A(a_1), A(a_2)\}\}$
$= min\{\mu_A(x_1, x_2), \mu_A(a_1, a_2)\}\}$
Hence μ_A is a left fuzzy KS-ideal of $X \times X$.
Conversely, μ_A is a left fuzzy KS-ideal of $X \times X$.Then
for all $(x, y) \in X$,we have
$min\{A(0), A(0)\} = \mu_A(0, 0) \geq \mu_A(x, y) = min\{A(x), A(y)\}$.
It follows that $A(0) \geq A(x)$.
(ii)For all (x_1, x_2) and (y_1, y_2) in $X \times X$,then we have
$min\{A(x_1), A(x_2)\} = \mu_A(x_1, x_2)$
$\geq min\{\mu_A((x_1, x_2) * (y_1, y_2)), \mu_A(y_1, y_2)\}$
$= min\{\mu_A((x_1 * y_1), (x_2 * y_2)), \mu_A(y_1, y_2)\}$
$= min\{min\{A(x_1 * y_1), A(x_2 * y_2)\}, min\{A(y_1), A(y_2)\}\}$
$= min\{min\{A(x_1 * y_1), A(y_1)\}, min\{A(x_2 * y_2), A(y_2)\}\}$
Put $x_2 = y_2 = 0$,then
$A(x_1) = min\{A(x_1 * y_1), A(y_1)\}$.
(iii)For all (x_1, x_2) and (a_1, a_2) in $X \times X$,then we have
$min\{A(x_1a_1), A(x_2a_2)\} = \mu_A((x_1a_1), (x_2a_2)) = \mu_A((x_1, x_2)(a_1, a_2))$
$\geq min\{min\{A((x_1), A(x_2)\}, min\{A(a_1), A(a_2)\}\}$
$= min\{min\{A(x_1), A(a_1)\}, min\{A(x_2), A(a_2)\}\}$
Put $x_2 = a_2 = 0$,then
$A(x_1a_1)\} \geq min\{A(x_1), A(a_1)\}$
Hence A is a left fuzzy KS-ideal of X.

5 Some results on homomorphism of KS - semigroup

In this section,we investigate some results on homomorphism of KS-semigroup.

Definition 5.1 *Let X and Y be a KS-semigroup.A mapping $f : X \longrightarrow Y$ of KS-semigroup is called a homomorphism if $f(x*y) = f(x)*f(y)$ and $f(xy) = f(x)f(y)$, for all $x, y \in X$.*

Note that if $f : X \longrightarrow Y$ is a homomorphism of KS-semigroup then $f(0) = 0$.

Definition 5.2 *Let $f : X \longrightarrow Y$ be a mapping of KS-semigroup and μ be a fuzzy subset of Y.The map μ^f is the pre-image of μ under f if $\mu^f = \mu(f(x))$,for all $x \in X$.*

Theorem 5.3 *Let $f : X \longrightarrow Y$ be a homomorphism . If μ is a left (resp. right) fuzzy KS-ideal of Y then μ^f is a left (resp. right) fuzzy KS-ideal of X.*

Proof: (i) For any $x \in X$,then $\mu^f(x) = \mu(f(x)) \leq \mu(0) = \mu(f(0)) = \mu^f(0)$. (ii) For any $x, y \in X$,then $\mu^f(x) = \mu(f(x)) \geq min\{\mu(f(x) * f(y)), \mu(f(y))\} = min\{\mu(f(x * y))\mu(f(y))\} = min\{\mu^f(x*y)\mu^f(y)\}$. (iii)For any $x, a \in X$,then $\mu^f(xa) = \mu(f(xa)) = \mu(f(x)(f(a)) \geq min\{\mu(f(x))\mu(f(a))\} = min\{\mu^f(x)\mu^f(a)\}$. Hence μ^f is a left fuzzy KS-ideal of X.

Theorem 5.4 *Let $f : X \longrightarrow Y$ be an epimorphism. If μ^f is a left(resp.right) fuzzy KS-ideal of X then μ is a left(resp. right)fuzzy KS-ideal of Y.*

Proof: (i) Let $y \in Y$, then there exists $x \in X$ such that $f(x) = y$.Then $\mu(y) = \mu(f(x)) = \mu^f(x) \leq \mu^f(0) = \mu(f(0)) = \mu(0)$. (ii) Let $x, y \in Y$,then there exists $a, b \in X$ such that $f(a) = x, f(b) = y$.Then $\mu(x) = \mu(f(a)) = \mu^f(a) \geq min\{\mu^f(a * b), \mu^f(b))\} = min\{\mu(f(a * b)), \mu(f(b))\} = min\{\mu(f(a) * f(b)), \mu(f(b))\} = min\{\mu(x * y), \mu(y)\}$$(iii)$ Let $x, a \in Y$,then there exists $l, m \in X$ such that $f(l) = x, f(m) = a$.Then $\mu(xa) = \mu(f(l)f(m)) = \mu^f(lm) \geq min\{\mu^f(l)\mu^f(m))\} = min\{\mu(f(l)), \mu(f(m))\} = min\{\mu(x), \mu(a)\}$.Hence μ is a left fuzzy KS-ideal of Y.

References

[1] D.H. Ahmad, BCI-algrbra, *J.Fuzzy Math.,* **1** (1993),445-452.

[2] Y.Imai and K.Iséki, On axiom systems of propositional calculi*XIV,Proc.Japan Acad.,* **42** (1966),19-22.

[3] K.Iséki,An algebra related with a propositional calculus, *XIV,Proc.Japan Acad.,* **1** (1993),26-29.

[4] Y.B Jun and J.Meng,Characterization of fuzzy subalgebras by their level subalgebras, Selected papers on BCK and BCI-algebras , *Shanxi Sci. Tech. Press.,* **1** (1992),60-65.

[5] K.H Kim, On structure of KS-semigroups, *Int. Math. Forum,* **1** (2006),67-76.

[6] A. Rosenfeld , Fuzzy groups, *J. Math. Anal. and Appl.,* **35** (1971),512-517.

[7] O.G Xi , Fuzzy BCK-algebras, *Math.Japon,* **36** (1991),935-942.

[8] L.A Zadeh , Fuzzy sets, *Information Control,* **8** (1965),338-353.

Received: October 5, 2006