5.1. Let $(X_i)_{i\in I}$ be a family of topological spaces. Show that $\prod_{i\in I} X_i$ is locally compact if and only if each X_i is locally compact and only finitely many of the X_i's are noncompact.

5.2. Show that the counting measure on a nondiscrete Hausdorff topological space is not a Radon measure.

5.3. Let X be a locally compact, σ-compact Hausdorff topological space. Show that each outer Radon measure on X is inner regular on all Borel sets.

5.4. Let X be a locally compact, second countable Hausdorff topological space. Show that each Borel measure on X that is finite on compact sets is a Radon measure.

5.5. Let X be a locally compact topological space, and let $C_c(X)$ denote the space of all continuous compactly supported functions on X. Given a compact set $K \subset X$, let $C_K(X) = \{f \in C_c(X) : \operatorname{supp} f \subset K\}$. We endow $C_K(X)$ with the topology generated by the sup-norm $\|f\| = \sup_{x\in K} |f(x)|$. Show that every positive linear functional $I\colon C_c(X) \to \mathbb{C}$ is continuous on $C_K(X)$, for each compact set $K \subset X$. (Equivalently, this means that I is continuous w.r.t. the inductive limit topology on $C_c(X) = \varinjlim_K C_K(X)$.) Of course, you are not allowed to use the Riesz-Markov-Kakutani theorem, otherwise the exercise would be trivial...

5.6. Give an example of a locally compact group G and a left uniformly continuous function $f\colon G \to \mathbb{C}$ that is not right uniformly continuous.

5.7. Find explicity the left and the right Haar measures on $\mathrm{GL}(n, \mathbb{R})$. (You can find an answer in many books, and you can easily check that it works, but I recommend you to deduce the formula for the Haar measure by using the method that was discussed at the lecture in the context of arbitrary Lie groups.)

5.8 (*the "$ax + b$" group*)**.** Let G be the group of all matrices of the form $\left(\begin{smallmatrix} a & b \\ 0 & 1 \end{smallmatrix}\right)$, where $a \in \mathbb{R}^\times$ and $b \in \mathbb{R}$ (this group is isomorphic to the group of all affine transformations $x \mapsto ax + b$ of $\mathbb{R}$). Find explicity the left and the right Haar measures on G. (You can find an answer in many books, and you can easily check that it works, but I recommend you to deduce the formula for the Haar measure by using the method that was discussed at the lecture in the context of arbitrary Lie groups.)

5.9 (*the Heisenberg group*)**.** Let G be the group of all matrices of the form $\left(\begin{smallmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{smallmatrix}\right)$, where $a, b, c \in \mathbb{R}$. Find explicity the left and the right Haar measures on G. (You can find an answer in many books, and you can easily check that it works, but I recommend you to deduce the formula for the Haar measure by using the method that was discussed at the lecture in the context of arbitrary Lie groups.)