Paraconsistent Approaches to Fuzziness and the Sorites Paradox

Marcelo Vásconez

June 2003

ABSTRACT

After a concise characterization of the sorites paradox, four paraconsistent approaches to the nature of fuzziness are presented: subvaluationism (a kind of Jaskowski'sdiscussive logic, a dual of supervaluationism), relevantism asdeveloped by the late Sylvan, vagueness adaptive logic, and a particular version of manyvalued fuzzy logic. The first two proposals constitute a drastic departure from classical logic, for both restrict *modus ponens*. Besides that, the former theory does not treat conjunction truth-functionally, thereby affecting the meaning of the universal quantifier. The vagueness-adaptive logic is criticized for its assimilation of fuzziness to ambiguity. Finally, a solution is offered based on viewing fuzziness as gradual and contradictory, which calls for infinite degrees of truth and of membership to a set. This fuzzy logic accommodates admissible contradictions thanks to a distinction between two kinds of negation, weak and strong. The sorites is considered a fallacy, since it proceeds by way of disjunctive syllogism, which is invalid for weak negation, but the second premise is true: there are no sharp boundaries.

****

> The good news is that the sorites paradox has been solved. The bad news is that the solution comes at the price of believing... contradictions.
>
> Sorensen [2001, p. 20]

1.- Introduction

Bivalent, truthfuncional, classical logic -CL, for short- has come under attack for its allegedly inadequate handling of some problems, two examples of which are those of fuzziness and the sorites paradox. These have prompted varied responses. However the first difficulty one is confronted with is that of an adequate definition of what fuzziness is, and there are many available solutions to the second problem, the most prominent of which are agnosticism (the so called epistemicism), and supervaluationism. Nevertheless, the present paperwill concentrate on proposals which unfortunately have forthe most part been neglected in current discussions: those which appeal to a paraconsistent logic in order to defuse the paradox and to elucidate the underlying phenomenon of fuzziness. Furthermore, solutions allowing for incoherence in this particular domain do not abound; and those which do exist have not received the attention they deserve. As a token of a beforehand refusal attitude, let me quote a passage from Rosanna Keefe (p.197):

> ...many philosophers would soon discount the paraconsistent option (almost) regardless of how successfully it treats vagueness, on the grounds of the... absurdity of p and ¬p both being true...

But, even if, at the end, one is going to reject all paraconsistent perspectives, that dismissal should be the result of a detailed assessment of pro and con reasons; in the absence of such a canvassing, and in the light of arguments given in favour of one of those systems, that dogmatic attitude does nothing to make the current discussion move on. Anyway, in this paper I will give an overview of four approaches tolerating inconsistencies: subvaluationism, Australian relevantism, adaptive logics and infinite valued paraconsistent fuzzy logic. I will critically examine them in turn, finally adopting the last solution.

2.- Fuzziness and the Sorites

The question of the nature of fuzziness is a vexing one. Philosophers debate about whether it involves a failure of the principles of bivalence, excluded middle, or of non contradiction. More generally, does fuzziness present a challenge for bivalent, truth-functional logic? Another contentious issue is whether fuzziness consists in a specific kind of ignorance: our not knowing which determinate truth value a fuzzy sentence happens to have. While some authors deem necessary a functor like Δ -interpreted variously as definitely, clearly, completely- to correctly account for fuzzy cases, other authors deny that. Concerning the extension of fuzzy terms, there are several conflicting ways of conceiving it via: sharp boundaries, replacement of borderlines by border zones, boundarylessness. And so on. Although it would be nice to have a taxonomy of possible alternatives, that is not a task I intend to undertake now. Nor am I going to offer a characterization that is neutral. Rather my purpose was to highlight the difficulty one encounters at the moment of defining fuzziness. Hence no wonderthat a definition by ostension has been advanced to circumvent the whole problem. Nonetheless, I take it that there is one markwhich is almost unanimously accepted by all parties, namely: one expression is fuzzy only if it leads to a sorites paradox. So let us turn to this.

As it is ordinarily presented, the paradox uses the rules of *modus ponens* or mathematical induction. Notwithstanding, for reasons having to do with the proper understanding of the whole issue, I prefer to avoid those versions. Instead I will interpret the argument as involving disjunctive syllogism. Let us imagine a sequence of collections of grains, starting with one containing a very large amount of grains, say 100.000 -constituting a heap-, and ending with "one" composed of zero grains -not a heap at all-, and such that each subsequent aggregate having one less grain than the immediately preceding one. Symbolizing each member of the series by *a* followed by an index expressing the number of grains it comprises, we can represent the major premise of the reasoning as: either a_i and a_{i-1}, are both *F* or none is; alternatively: it is not the case that a_i is *F* but a_{i-1} is not F. By de Morgan and double negation, this last formulation is equivalent to: either a_i is not *F* or a_{i-1} is *F*. The meaning of this major premise is clear: we should judge a pair of adjacent members in the series similarly, either attributing the predicate to each or withholding it from both; what is ruled out is that only one of a pair of indistinguishable grain collections is a heap while the other is not. Thus, this premise is patently true. Nonetheless, it is paradoxical, in the sense that, by reiterated applications of (double negation, universal instantiation and) disjunctive syllogism, it allows us to slide down the slippery slope leading us to the absurd conclusion that a zero grain collection constitutes a heap.

1)	$a_{100.000}$ is *F*.	First Prem.
2)	$a_{100.000}$ is not *F* or $a_{99.999}$ is *F*	Major Prem.
3)	$\therefore a_{99.999}$ is *F*	1, 2 D.S.
4)	$a_{99,999}$ is not *F* or $a_{99.998}$ is *F*	Major Prem.
5)	$\therefore a_{99.998}$ is *F*	3, 4 D.S.
⋮		
n-2)	a_1 is *F*	n-4, n-3 D.S.
n-1)	a_1 is not *F* or a_o is *F*	Major Prem.
n)	$\therefore a_o$ is *F*	n-2, n-1 D.S.

So we are faced with the uneasy dilemma of renouncing the obvious truth of one of the premises or invalidating the rule of inference. The four approaches we will examine all take the second alternative: respect fuzziness but find fault with CL.

3.- Subvaluationism

The application of subvaluationism to the problem of fuzziness and the sorites has been studied by Dominic Hyde, who does not endorse it. The theory shares with supervaluationism the use of precisifications, inheriting thereby its problems. According to it, fuzziness can be clarified by assimilating it to a kind of ambiguity, that is, a fuzzy sentence can be interpreted as expressing several different exact propositions, its truth value being dependent on the admissible ways of disambiguating it, or of making it more precise. A sentence attributing a fuzzy predicate to a borderline case can have a true as well as a false disambiguation; hence, a fuzzy statement can be both true and false, because true in a precisification and false in another. At the root of this conception, there lies a notion of truth weaker than that used by supervaluationism, in that it is not required that a sentence be true in all admissible precisifications in order for it to be true *simpliciter*, just one precisification suffices. So what were indeterminate sentences in supervaluationism become (weak) inconsistencies in subvaluationism: on the one hand,a sentence, p, and, on the other hand, its negation, $\neg p$, are both true, but their conjunction, $p \wedge \neg p$, is not true. Thus fuzziness is rightly seen as incoherent: a predicate applies and does not apply to its borderline case. Although this is so, the adjunction rule fails: from p, q, you cannot infer $p \wedge q$. This is a peculiarity subvaluationism shares with Jaskowski's discussive logic. So properly speaking, there are no true contradictions in the system, but just two contradictory statements, both true, but separately. The theory is weakly inconsistent: it countenances inconsistencies without contradiction.

As for the solution to the sorites, Hyde discusses a form of the paradox in which its major premiss is construed as a conditional, and the inference proceeding by means of *modus ponens*. Precisely it is this rule which is not valid according to subvaluationism. To see this, suppose: a) that the first member of the soritical series is a heap; b) that the removal of a grain does not make the difference between what is a heap and what is not, i.e., that any two adjacent grain piles satisfy the condition that, if the first is a heap, so is the other consisting of one less grain; and c) that the last member of the series, eventually, say, 0 grains, is determinately not a heap. Granting all these three assumptions, there is no way to block the conclusion other than by repudiating the inference rule: *modus ponens* is invalid, for although the premisses are true on some disambiguation, the conclusion is false on all disambiguations.

So far the strategy is clear: the soritical conclusion does not follow because the argument form is invalid. Now, Hyde tries to explain the failure of *modus ponens* by appealing to the conception of vagueness as a kind of ambiguity, as previously seen. What is wrong with the sorites is that it commits the fallacy of equivocation: the premisses are not true on the same disambiguation. The same sophism is incurred as in the following argument: $\Diamond p$, $\Diamond(p \supset q) \vdash \Diamond q$; what is required here for the truth of the conclusion is that the same possible world encompassing both p and p⊃q comprise q also. Analogously, *modus ponens* demands that the same disambiguation make all the premisses and the conclusion true. However, subvaluationism only requests that the premises and conclusion be all true on some disambiguation, but not necessarily the same. But if this is so -Hyde concludes-, the notion of validity in subvaluationism is loose, and should be appropriately strengthened in order to fit the demanded standard.

What do we think of subvaluationism? Well, it seems that to abandon *modus ponens* is too drastic. I do not claim that this rule is unproblematic nor that it is self-evident; nothing in principle escapes the possibility of being revised, as holist Quine has argued for; but dispensing with the rule should be left as an extremely last resource, when all plausible alternative ways out are not available. Notwithstanding, the subvaluationist strategy to the sorites is on the right track: the major premises are true, but the reasoning is not truth preserving.

Moreover, it may be objected that the rule involved here is not *modus ponens* but disjunctive syllogism. Indeed, if we avail ourselves of a distinction between two negations: weak and strong, then the correct definition of the conditional does not involve the former but the latter. Saying "not p or q" is not the same as saying "not p at all or q". Only the latter is equivalent to "if p, then q". Hence it is debatable whether the argument is an instance of *modus ponens*. See the final section for more on this.

Besides, as a consequence of the failure of adjunction, truth-functionality is lost for conjunction: the truth of both "p" and "q" does not determine the truth of "p and q". This failure -Hyde correctly says- might give ground to reject the subvaluational conjunction.

The main drift in Hyde's argumentation is to demonstrate that subvaluationism is on an equal foot relatively to supervaluationism; that none is superior, since the evidence supports gaps and gluts equally, and therefore, that there is no justification for the preferred status of supervaluationism. But looking at the choice from a broader range of alternatives, both options together may seem to be unsatisfactory, in view of the fact that whatever deviance is found at the level of non truth-functional conjunction and disjunction is going to be also transmitted to the meaning of the quantifiers. Therefore, if supervaluationism can be charged with not capturing the sense of the English expression 'there is', a similarcharge may be levelled against the subvaluationist treatment of the universal quantification: "for all x, *F*x" may fail to be true although all its instances are true. This problem seems to disqualify both candidates, supervaluationism as well as subvaluationism, as logics and semantics of natural language. Hyde grants all this, distancing himself from subvaluationism. And I cannot but agree with him on this point.

4.- Relevantism

The second approach to fuzziness and the sorites paradox that I want to review is that of relevantism as developed by Richard Sylvan and Dominic Hyde. They criticize most of the definitions of fuzziness for theirfailing to accountfor its overdetermination, admitting at most its underdetermination. An adequate characterization should acknowledge both. Indeed, due to one of the de Morgan principles, the negation of the principle of excluded middle, $\neg(p \vee \neg p)$, is equivalent to an indetermination, $\neg p \wedge \neg\neg p$, i.e., neither p nor its negation are true; and by double negation, this amounts to a contradiction, $\neg p \wedge p$. So Sylvan and Hyde would accept that those indeterminists who, when queried about a fuzzy predication, affirm that the corresponding object neither is nor ceases to be *F,* but are unwilling to allow for inconsistencies must deny either de Morgan or double negation, the latter being the most likely candidate. Indeterminism comes at a price. By contrast, relevantism is prepared to embrace underdetermination as much as overdetermination.

Furthermore, the other aspect of fuzziness concerns the borders of the extension of fuzzy words. In this regard, Sylvan and Hyde describe the boundary as a region in which the exterior and the interior parts are neither exclusive -allowing for an overlap of *both*- nor exhaustive, including members which are *neither* in nor out.

Clearly, this enlargement of the bivalent possibilities moves in the right direction, and should be welcome by whoever is urging that the very narrow frontiers set once and for all by CL be expanded. However, having in mind that the Neither and the Both areas are logically equivalent, and thus amount to only one, it might seem that to concede just one more case besides the traditional two is not sufficient. Indeed, not all elements in this non standard region enjoy the same status. To make room for gray is good, but not everything in there is 50% black and 50% white. Mixtures of the extremes in different proportions should be permitted; in order to have the full range of cases, an infinity of intermediate situations must be granted. For more on this criticism, see Peña 1996.

Sylvan and Hyde say that fuzzy logic is not a rival of relevantism, since the latter has multi-valued matrices, and they even go as far as claiming that 'the more or less' is an essential feature of fuzziness, and consequently, this functor will be needed as a test of adequacy for any logic of fuzziness (p. 14). But they claim their system can accommodate the functor in the syntax without being committed to a many-valued semantics. There is no need for an appeal to degrees of truth. The authors give an instance of the kind of analysis they have in mind.

That p is approximately true iff 'approximately p' is true.

I assume that they would grant a more general principle, like

p is ... true iff '...p' is true,

in which the blank space should be filled by an adverb of quantity, or intensity. For example,

That France is large is more or less true iff 'More or less France is large' is true.

(The predicate '...is true' of the right member will be treated as redundant, so that the quotation mark device can be dropped, assuming a deflationist theory of truth). But if this is so, we are already embarked on the road to a many-valued logic. In the simple case of an attributive sentence, it is not clear how to interpret the right member of the biconditional other than as expressing the extent to which an object possesses a given property. The more natural way of understanding the functor 'more or less' is to take it as affecting the grade in which the object belongs to the extension of the predicate. Thus,

France is more or less large.

But to accept this is to accept fuzzy sets. Put otherwise, we have found that degrees of truth go hand in hand with degrees of possessing a property. More exactly, "both" kinds of degrees are strictly identical. In general, a sentence predicating *F* of a is true to exactly the same extent as a is *F*. This is how the functor of weak assertion 'more or less' can be handled in a many-valued fuzzy logic. But unfortunately, our authors do not elaborate on this issue in the paper.

As for the sorites, our authors also consider its conditional form; the type of argument using the'indistinguishable'predicate is reducible to that conditional scheme. The difference in relevantism between two kinds of conditionals: the if...then, here represented as $\supset$, and entailment, $\rightarrow$, does not affect the uniformity of their approach, since parallel considerations apply to both. What they want to say in this connection is that *modus ponens* for the conditional $\supset$ is not truth preserving when it is reiterated a large number of times -as it occurs with the sorites-, though it is *locally* valid, i.e., for a small number of reapplications. Just see what happens with the segment of the spectrum of colours flanked by yellow and red, which is so constructed that the members of any pair of contiguous bands are indiscernible from each other:

$$a_i \sim_d a_{i+1}.$$

To be faithful to this fact entails to retain the truth of the major premise. Yet, the first band, a_o, is yellow, but the last one, a_n, is not; a_0 and a_n are discernible:

$$\sim(a_0 \sim_d a_n).$$

Therefore, the relation of discernibility fails to be transitive. And similar considerations apply to the conditional. In a soritical series we have:

$$Fa_o,\ Fa_o \supset Fa_1,\ \ldots,\ Fa_{n\text{-}1} \supset Fa_n,\ Fa_n.$$

But, since all the members of the chain are true, except the last, iterated *modus ponens* does not hold. As we mentioned, the authors distinguish between validity and truth preservation; the former is defined syntactically: whenever p together with p ⊃ q are proven, so is q. The validity of *modus ponens* is not questioned; only its truth preservation in the long run.

This is in a nutshell the position of the Australian philosophers. They both deserve to be praised for their defence of a paraconsistent conception of fuzziness and their demand for a reform of classical logic. This notwithstanding, the failure of *modus ponens* to preserve truth is too high a price. As previously said, insofar as one theory drops a classical rule of inference, it places itself in a disadvantage position in comparison with another theory keeping -under certain interpretation- all rules of CL. The system advocated in the last section, §6, has the advantage of being strongly conservative, in the sense that it retains all classical principles and rules of inference, and in that respect, it is superior to relevantist logic. Additionally, it contains a non classical implication whose properties closely resemble relevantist entailment.

5.- Vagueness-Adaptive Logic

The third contribution to the debate I want to examine is the application of adaptive logics to fuzziness made recently by Guido Vanackere and Bart Van Kerkhove. The latter reinforces the pragmatical approach of the former, who belongs to the team of researchers developing aspects of adaptive logics found by Diderik Batens of Gent. So I first give a rough outline of these logics in general, as a background.

The main idea is that many theories, scientific or otherwise, sometimes at one stage of their development come to collide with a presupposition of their underlying logic, usually CL. The thesis that goes against some of the principles or rules of the logic is called an abnormality, and it always takes the form of an inconsistency, which therefore threatens to renderthe whole theory trivial. In these circumstances, an inconsistency-adaptive logic, IAL, comes to the rescue. IAL is a provisional tool allowing to reason from the incoherent theory in order to locate the inconsistencies, with the ultimate purpose of eliminating them and of arriving at a new theory. But instead of definitely invalidating the rule of inference employed in the derivation of the abnormality, the strategy used is to consider the instantiations of such a rule as incorrect, so that the rule is suspended relative to the set of propositions giving rise to the inconsistency, but this does not prevent the rule from being used in other contexts where no abnormalities arise. Rules are valid as long as and provided that no contradictions are derived with their help; whenever a sentence of the form "p∧¬p" might be deduced, the rule used is no longer valid in that context; therefore, rules are conditionally valid. Adaptive logicians believe that it is preferable to "locally" invalidate the rule rather than to allow for inconsistencies; this is their way of avoiding trivialization. An adaptive logic thus plays its proper role during the transition period. It is weaker than the upper limit logic (ULL), i.e., the intended logic underlying the theory, but it is stronger than the lower limit logic (LLL), which results from entirely dropping the principle conflicting with the abnormality. In the case of the IAL, the LLL invalidates the principle *ex contradictione quodlibet*, and in so doing, LLL becomes a paraconsistent logic. In otherwords, the AL does not have all the consequences of the original LILL, but it is more powerful than the LLL. Inconsistencies are somehow tolerated, at the price of allowing for exceptions to the rules, but consistency is considered a sound methodological requirement.

In order to apply this framework to the treatment of fuzziness and the sorites paradox, our Belgian authors need to consider fuzziness as a kind of ambiguity. The supposition that a fuzzy expression has a unique meaning is -according to them- what causes the paradox. To see this, consider the following version of the sorites which nicely brings to the fore the close relation between fuzziness and degrees:

John is rich; John is a little bit richer than Mary ∴ Mary is rich too.

Leaving aside the comparative, we have two occurrences of the monadic adjective, namely, 'rich' and `rich too', both of which, normally, would be considered as having the same meaning. But our authors think that precisely this would be a mistake; if we want to avoid identifying the rich and the poor, we better distinguish the several occurrences of the monadic predicate by giving each a different index. So, let us symbolize the argument as follows:

$$R^1j, j >^R m \therefore R^2m.$$

(I have substituted the original '<' for `>'). The next argument in the chain is: Mary is rich too; Mary is a little bit richer than Paul; then Paul is rich too. That is:

$$R^2m, m >^R p \therefore R^3p.$$

And so on. Were we to take R^n and R^{n+1} as identical, we will end up being caught by the paradox. So each occurrence of the predicate has a different sense. And this is part of the solution to the paradox. We would be forced to renounce the assumption: "one term, one meaning"; in the presence of vagueness, this must be given up.

Finally, let us see how the vagueness-adaptive logic (VAL) works. We need to identify which abnormality is in play here. It is any sentence of the form: $\neg(R^n\alpha \equiv R^{n+1}\alpha)$. This is abnormal in that it violates the axiom of the ULL of meaning univocity. And precisely this plurivocity is an instance of vagueness. Hence, fuzziness is treated as an abnormality.

Now, the main problem I have with this conception is its considering vagueness as a sort of ambiguity. Ordinary speakers will agree that the sense of the vague predicate 'rich' does not change because the financial situation of the person varies in one cent more or less. We would be more inclined to say that Paul and Mary are rich in the only sense of the word, differing by a certain amount of money, rather than saying that both are rich but each in a different sense of the adjective. If John is $rich_1$, Mary is $rich_2$, Paul is $rich_3$, then the word 'rich' would denote an exact amount, which -I take it- goes against ordinary usage. It seems to be a presupposition of the paradox that the meaning of the predicate remains constant from beginning to end, so that when it is affirmed that a_o is F and that a_n is not F, the meaning of F has not been altered in both assertions, nor in any of the intermediary cases. A central ingredient of the VAL is that a pair of contradictory sentences, p, and not-p, result from ambiguity. This appears questionable from a point of view which sees fuzziness precisely as contradictory.

The merit of any IAL is that it permits to reason within incoherent theories averting triviality. However, there are other systems which fulfil the same function without suspending any classically valid rule of inference, as we will see immediately.

In conclusion, the three theories we have studied so far are correct in allowing inconcistencies produced by fuzziness, but divergencies surface as a result of differing manners of handling contradictions within the frameworks of rival paraconsistent systems. There will come one day when some theory of this type will be appreciated as the closest to satisfy the standards of adequacy.

6.- An Alternative Approach

I cannotfinish this paperwithout briefly sketching myfavourite solution. (Fora detailed presentation of the system, see Peña and Vásconez unpublished). This is a kind of infinite valued paraconsistent fuzzy logic, as has been developed by Lorenzo Peña. I give a summary of its main components.

There are infinite degrees of truth, which are identical with degrees of membership of an element to a set, or with the degree of possession of a property. All degrees greater than zero are designated (true), and all degrees smaller than 1 are antidesignated (false); hence, with the exception of the extreme values, all others are both designated and antidesignated; that is, all intermediate values are contradictory.

It is the graduality of the predication which results in contradiction: the fuzzy predicate both to some extent applies and in some measure does not apply to the object. A borderline case neither completely has a property nor totally lacks it. It is these two features which essentially enter into the meaning of fuzziness: graduality and incoherence.

However, there is no need to renounce either to the principle of excluded middle nor to the principle of non contradiction; both are true, but not completely true. So both are to some extent also false. Furthermore, even a weak version of the principle of bivalence is retained: every sentence has a designated or an antidesignated value. There is no room for indeterminacy (lack of truth value).

What is wrong with classical logic is its use: to take it as being exhaustive, covering all cases; CL is alright for reasoning with sentences having values 1 or 0; but when degrees enter the scene with their ensuing contradiction, CL becomes insufficient.

Contradictions are allowed due to a basic distinction between two kinds of negation: the classical one, ¬, considered as strong or absolute, whose reading must be 'not at all', and a weak, simple or natural negation, ~, the mere 'no', following the rule: /~p/ = 1-/p/. Contradictions formed by means of ¬ are unacceptable, while those involving ~ are true to some extent.

The conditional, ⊃, is defined via ¬, but not via ~. Besides that, a functor of strict implication, →, is introduced satisfying the condition that: /p→q/ takes a designated value iff /p∧ql = /p/. The meaning of "p→q" is: p implies q iff the value of the consequent is at least as great as the value of the antecedent. This functor is suitable to cope with the symbolization of comparative sentences, and allows us to have strict equivalence (identity of truth value), stronger than the mere biconditional.

There is also a functor of overaffirmation: 'H', whose reading is: 'It is completely true that', which may perform the functions of a definitely operator. /Hp/ = 1 iff /p/ = 1. And the functor of weak assertion 'more or less' is symbolized as 'L'. /Lp/ = 1 iff /p/ > 0.

The resulting system is strongly conservative, in the sense that, for ¬, ∧, ∨, ⊃ and ≡, it keeps absolutely every classical tautology and every classical rule of inference, without exception whatsoever.

An argument is defined as valid iff it is impossible for the conclusion to be completely false given the truth, in some extent or other, of the premises.

The sorites is judged defective because it is an instance of an invalid rule: disjunctive syllogism does not hold for weak negation, though it does for total negation. In this manner, the major premises are maintained in harmony with the meaning of fuzzy terms. But they are also partly false. Consequently, fuzzy properties are bounded and have many boundaries, but not sharp.

The advantages of the present solution are that, by upholding the major premisses, a direct account is offered of the little by little transition from *F* to not *F*. A small difference in the input cannot produce a great difference in the output. If the input is multiply valued, the output should not be bivalent. Furthermore, graduality is not eliminated, and the incoherence is explained as supervening on degrees. Additionally, since like cases are treated alike, observational predicates are not threatened, as in agnosticism or supervaluationism. And finally, absolutely all tautologies and rules of inference of CL are preserved. Even a version of the principle of bivalence survives, to wit, any sentence has a designated (true) value or an antidesignated (false) one. But of course, more truths are added. We have everything that is wanted. So a solution along these lines can be shown to be a far better account of fuzziness and the sorites.

****

I would like to tank to Dominic Hyde, Bart van Kerkhove, Guido Vanackere, my promotor Leon Horsten and two anonymous referees for their insightful suggestions.

Bibliography

Diderik BATENS (1997), "Inconsistencies and Beyond. A Logical-Philosophical Discussion", *Revue Intemationale de Philosophie* 200, pp. 259-273.

Id. (1998),"Inconsistency-Adaptive Logics", in Ewa Orlowska (ed.), *Logic at Work. Essays dedicated to the memory of Helena Rasiowa*. Heidelberg, New York, Springer, pp. 445-472.

Id., (1999), "Paraconsistency and its Relation to Worldviews", in *Foundations of Science* 3, pp. 259-283.

Id. (2000), "A Survey of Inconsistency-Adaptive Logics", in A survey of inconsistency-adaptive logics", in D. Batens, C. Mortenson, G. Priest, J.p. Van Bendegem (eds.), *Frontiers of ParaconsistentLogic*, Baldock, Research Studies Press, King's College Publications, pp. 49-73.

Dominic HYDE, (1997b), "From Heaps and Gaps to Heaps of Gluts", in *Mind* 106, pp. 641-660.

Rosanna KEEFE (2000), Theories and Logic of Vagueness, Cambridge, Cambridge University Press.

Lorenzo PEÑA (1993), "Slippery Slope Arguments", *Pragmatics and Cognition* 1, pp. 401-410.

Id. (1996), "Graham Priest's < Dialetheism -- Is It Altogether True?", in Sohtes7, pp. 28-56. URL: (<http://www.sorites.org/issue_07/item5.htm)>

Lorenzo PEÑA, and Marcelo VASCONEZ, (unpublished), "Vagueness or Graduality?"

Richard SYLVAN, and Dominic HYDE (1993), "Ubiquitous Vagueness without Embarrassment: logic liberated and fuzziness defuzzed (i.e. respectabilized)", *Acta Analytica* 10, pp. 7-29.

Roy SORENSEN, *Vagueness and Contradiction*, Oxford, Clarendon, 2001.

Bart Van KERKHOVE and Guido VANACKERE (2003), "Vagueness-Adaptive Logic: A Pragmatical Approach to Sorites Paradoxes", to appear in *Studia Logica* 75, Nº 3, pp. 383-411.

Marcelo Vásconez
University of Cuenca - Ecuador
Katholieke Universiteit Leuven - Belgium