

economic and cultural base provided by the thriving local economy has played a role over the years. One can also point to the moderate climate, the proximity to ocean beaches, and the karma of San Francisco and the Golden Gate.

But a main ingredient emerging from the case study is an unrelenting search for scholarly excellence in making appointments. Even in the earliest days in Oakland, university leaders demanded scholarly excellence of its appointees, at least in areas other than mathematics. There was a recurrent tendency to view mathematics as a service department rather than an independent field of scientific research. However, when the same standards of high quality were applied to mathematics as were being applied to other areas, the stage was set for mathematics to make significant advances.

Evans brought to the department a broad view of mathematics that overlapped with areas of application. This vision of mathematics within the university, reinforced by the extensive use of fractional joint appointments to reach out to other parts of the university, has been an important ingredient of success.

Concluding Comments

Moore has a clear writing style, somewhat reminiscent of a departmental letter in support of a personnel action. He depends heavily on department administration documents such as department letters in support of appointments, reports of visiting committees, and articles memorializing deceased faculty. There is a steady flow of facts, facts, facts, and these are used to buttress occasional summarizing assessments. The author focuses on painting the big picture. There is very little offered about the operational details of the functioning of a mathematics department. The reader will find neither gossip nor much insight into the personalities of the mathematicians in the department (though one is left with no doubt that Berkeley deans regarded Neyman as a pain in the neck).

The history is very well written. I found it an immensely enjoyable read, particularly when set against the backdrop of my own connections with Berkeley. The history has given me insight into the development of my own

UCLA mathematics department and the university as a whole. I particularly enjoyed the discussion related to the long-range planning for Berkeley in the years 1955–1957, including Kerr’s calculations for an optimal size for the student body and the considerations entering into planning for a building for mathematics. As a case study, this history has much to say to mathematicians and to academic leaders of today’s university.

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An Imaginary Tale: The Story of $\sqrt{-1}$

by Paul J. Nabin

REPRINT OF THE 1998 EDITION AND FIRST PAPERBACK PRINTING, WITH A NEW PREFACE AND APPENDICES BY THE AUTHOR, 2007. PRINCETON, NEW JERSEY, PRINCETON UNIVERSITY PRESS, 269 PP., 2007, US \$16.95, PAPERBACK
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REVIEWED BY GENEVRA NEUMANN

One of the first things welcoming a reader to this history of $\sqrt{-1}$ is a *Calvin and Hobbes* cartoon about imaginary numbers. Throughout this book, biographical details and anecdotes abound, along with detailed calculations and examples. The book covers an amazing variety of topics related to complex numbers. This “tale” is written in the first person, in a conversational tone with lots of figures included to help the reader follow the discussion. The author is not shy about expressing his opinions. The stories give the impression that mathematics is a lively and human enterprise, warts and all. Almost all of the mathematics is presented at a level that should be accessible to a student who has completed freshman calculus, although partial derivatives and multiple integrals are used in a few places.

The introduction presents a prehistory of $\sqrt{-1}$, from ancient Egypt to India to the introduction of the term

“imaginary.” The first three chapters describe how $\sqrt{-1}$ evolved from being regarded as an impossible expression appearing in a solution of a cubic equation to an honest-to-goodness, legitimate number. The next two chapters discuss a variety of applications of complex numbers, ranging from a puzzle of G. Gamow’s concerning buried treasure to examples of electronic circuits. The sixth chapter focuses on results of Euler. The last chapter provides a glimpse into function theory. There are six appendices, three of which are new in the paperback edition. The preface to the paperback edition discusses comments the author has received concerning possible errors in the text, but the errors mentioned are not corrected in the body of the text. The book has a section of detailed notes at the end, a name index, and a subject index. In addition to references, the notes often contain additional discussion. The table of contents also includes a short summary of each chapter.

This book is a “tale” that interweaves mathematics with history and applications. I don’t have sufficient expertise to comment on the correctness of the historical and biographical material nor the material on physics and engineering. As with any good tale, there are heroes and villains. The historical discussions include feuds about priority, people who helped the careers of others, and those who made things difficult for others—all told in a chatty and opinionated way. As this is not a textbook, there is no table of symbols and readability trumps rigor. Arguments are often computational, similar in flavor to those found in a typical freshman calculus book, and there are plenty of helpful figures and drawings. The calculations are broken into easily digested steps; one complicated calculation is deferred to Appendix E. Also, some nonstandard notation is used ($r\angle\theta$ represents a complex number in polar form). Readers who are comfortable letting the equations wash over them while enjoying the historical details or who wish to follow along with their calculators will be comfortable with the mathematical presentation.

Some readers will be put off by the use of rounded values in equations; for example, the last equation on page 59 is

$$u = \frac{-1 \pm \sqrt{5}}{2} = 0.618034 \quad \text{and } -1.618034.$$

A more significant example is the calculation on page 58, which is another verification that one of the cube roots of $z = 2 + \sqrt{-121}$ is $2 + i$. De Moivre's formula is used with $\arg(z) = \tan^{-1}(11/2)$, which is then set equal to 79.69515353° . This rounded calculation gives the desired answer, but it's a bit misleading. It would have been nice to use \approx instead of $=$ when rounded values are used.

This is difficult material to present at such an elementary level. The author does point out some of the more egregious sleights of hand (for example, substituting ix for x in the power series for e^x on pages 144–145). However, this is not always the case. A reader might need more reminders than given that (1) the polar representation of a complex number isn't unique and that (2) extra care must be taken with the complex logarithm. For example, this is a source of confusion in Section 6.7's discussion of i^i (for which some corrections are discussed on page xix in the new preface). Also, in Appendix B (second paragraph on page 230), a reader may be misled by the suggestion that the transcendental function under consideration will have an infinite number of zeros, because its power series has all powers of z . An infinite power series is not a polynomial as suggested on page 195 when justifying the analyticity of e^z . Because not all readers will have previous sections firmly in mind, it would have been nice to remind the reader of assumptions made in previous sections when stating equations. For example, the gamma integral is defined in Section 6.12 on page 175, along with the assumption $n > 0$. The reason for this assumption is not given (the in-

tegral is infinite for $n \leq 0$) and the reader is not reminded of this assumption when the integral is reintroduced in Section 6.13 on page 182. The gamma function is extended to negative real numbers by using the recursion relation mentioned on page 176 and not by this integral; no mention is made of the fact that the gamma function is undefined for integers $n \leq 0$. Moreover, the expressions for $\Gamma(n)\Gamma(1-n)$ and for $(-n)!(n!)$ on page 184 involve dividing by $\sin(n\pi)$, and no restriction on n is given. It would have been nice to have included graphs for some of the special functions introduced in Chapter 6, as well as a sketch of a Riemann surface.

This book might frustrate readers who do not read the book linearly or who read the book in small chunks over a long time period. As the story of $\sqrt{-1}$ unfolds, the author revisits earlier topics. He doesn't include page numbers (or equation numbers) when referring to material elsewhere in the book. For example, the calculation on page 58 is another way of verifying that the positive solution of the irreducible cubic $x^3 = 15x + 4$ is given by Cardano's formula (even though it involves square roots of negative numbers) and was settled in another manner in Chapter 1. Instead of referring the reader back to the specific page (page 18), the reader is asked to "recall from chapter 1 the Cardan formula to the irreducible cubic considered by Bombelli" followed by the formula for the solution. For the most part, equations are not numbered; the exceptions are the equations in Appendix E and a boxed equation on page 53. Although this may make the book more welcoming to a general reader, there are several more places where equation numbers on selected equations would make arguments easier to follow. For example, in the second paragraph on page 145, the author points out that the series being

discussed "provides the proof to a statement I asked you just to accept back in section 3.2" and then gives a formula for $\frac{\sin x}{x}$. The table of contents does not give page numbers for specific sections, and Chapter 3 runs over 30 pages. Section 3.2 starts on page 60 and ends on page 65. Going back to section 3.2, the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ is used to go from the equation for $\frac{\sin \theta}{\theta}$ on the bottom of page 63 to an infinite product formula for $2/\pi$ on page 64. Near the top of page 64, the author says that he will derive this limit in Chapter 6 (again, no page number or equation number). Chapter 6 runs over 40 pages.

It's clear that a great deal of research went into preparing this book. An amazing amount of mathematics has been presented in historical context at an elementary level. There are mistakes and careful readers will probably be unhappy with the balance chosen between rigor and readability. I'm disappointed that the corrections mentioned in the new preface weren't either corrected in the text itself or at least listed in a separate section of errata for the sake of the unsuspecting reader who skips prefaces. Because of the lively presentation and variety of topics, I think this book has the potential to convince a general reader that there's interesting (and even useful) mathematics out there beyond freshman calculus. The author makes a strong case that complex numbers are useful and aren't just funny looking solutions from the quadratic formula; mathematics teachers (high-school level and above) might find this book useful as a source of examples and anecdotes.

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