A CONJECTURE REGARDING THE LOGARITHMIC COEFFICIENTS OF UNIVALENT FUNCTIONS

I. M. Milin

UDC 517.54

One considers the class S of functions, regular and univalent in $|z|<1$ and normalized by the expansion $f(z)=z+c_2z^2+\ldots$. By the logarithmic coefficients of the function $f(z)\in S$ one means the coefficients of the expansion

$$\log\frac{f(z)}{z}=\sum_{k=1}^{\infty}2\gamma_k z^k,\quad |z|<1 .$$

Earlier, the author had formulated the following conjecture: for any function $f(z)\in S$, for each $r\in(0,1)$ one has the inequality

$$\sum_{k=1}^{\infty}k|\gamma_k|^2 r^{2k}\leqslant\max_{|z|=r^2}\operatorname{Re}\sum_{k=1}^{\infty}\gamma_k z^k .$$

In this paper this conjecture is proved for spiral-shaped functions and for functions from S with real coefficients and under some additional assumptions.

1°. Formulation of the Conjecture

Let S be the class of functions $f(z)=z+c_2z^2+\ldots$, regular and univalent in the circle $|z|<1$. By the logarithmic coefficients of function $f(z)\in S$ one means the coefficients of the expansion

$$\log\frac{f(z)}{z}=\sum_{k=1}^{\infty}2\gamma_k z^k,\quad |z|<1.$$

For a function $\Psi(z)=\sum_{k=0}^{\infty}c_k z^k$, regular in $|z|<1$ and for $r\in(0,1)$, we denote

$$M(r,\Psi)=\max_{|z|=r}|\Psi(z)|,$$

$$\sigma(r,\Psi)=\frac{1}{\pi}\iint_{|z|<r}|\Psi'(z)|^2 d\sigma=\sum_{k=1}k|c_k|^2 r^{2k}.$$

In [1], the author has stated the following conjecture: for any function $f(z)\in S$, for each $r\in(0,1)$ one has the inequality

$$\sigma\left(r,\log\frac{f}{z}\right)=\sum_{k=1}^{\infty}4k|\gamma_k|^2 r^{2k}\leqslant 2\log\frac{M(r^2,f)}{r^2}. \tag{1}$$

Equality holds only for functions $f(z)=z$ and

$$f(z)=z(1-e^{-i\beta}z)^{-2/m}\quad (m=1,2,\ldots;\ 0\leqslant\beta<2\pi).$$

Since

$$\log\frac{M(r,f)}{r}=\max_{|z|=r}\log\left|\frac{f(z)}{z}\right|=\max_{|z|=r}\operatorname{Re}\sum_{k=1}^{\infty}2\gamma_k z^k,$$

Translated from Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR, Vol. 125, pp. 135-143, 1983.

0090-4104/84/2606-2391$08.50

inequality (1) can be represented in the equivalent form:

$$\sum_{k=1}^{\infty} k|\gamma_k|^2 r^{2k} \leq \max_{|z|=r^2} \operatorname{Re} \sum_{k=1}^{\infty} \gamma_k z^k. \tag{1'}$$

For bounded functions $f(z) \in S$, i.e., for functions satisfying the condition $|f(z)| < M$ in $|z| < 1$, Lebedev [2] has obtained first the generalized area theorem, from which there follows the inequality

$$\sigma(1, \log \frac{f}{z}) = \sum_{k=1}^{\infty} 4k|\gamma_k|^2 \leq 2 \log M, \tag{2}$$

confirming hypothesis (1) for bounded functions in the limiting case $r = 1$. In [1], inequality (1) has been proved for star-shaped functions $f(z) \in S$ with the aid of the integral representation known for such functions and also for an arbitrary function $f(z) \in S$ for sufficiently small values of $r, 0 < r < r_f$.

It is useful to note that the difference between the left- and the right-hand sides in (1') varies in a very simple manner when passing to k-symmetric functions of the class S. Namely, for $r, R \in (0,1)$ we consider

$$T(r, R, f) = \sum_{k=1}^{\infty} k|\gamma_k|^2 r^{2k} - \max_{|z|=R} \operatorname{Re} \sum_{k=1}^{\infty} \gamma_k z^k.$$

If from the function $f(z) \in S$ we construct the symmetric function

$$f_m(z) = [f(z^m)]^{1/m} \quad (m = 2, 3, \ldots),$$

then we have the relation

$$T(r, R, f) = m T(r^{1/m}, R^{1/m}, f_m). \tag{3}$$

Below we prove some new results regarding conjecture (1).

2°. Area Formulas

LEMMA 1. Assume that the function $\Psi(z) = \sum_{k=0}^{\infty} c_k z^k$ is regular in the circle $|z| < 1$ and let $u(z)$ and $v(z)$ be the real and the imaginary parts of $\Psi(z)$. Assume that for every $r \in (0,1)$ one selects numbers r_1 and r_2 satisfying the conditions

$$0 < r_1, r_2 < 1, \quad r_1 r_2 = r^2. \tag{4}$$

Then we have the area formulas

$$\sigma(r, \Psi) = \frac{1}{\pi} \int_0^{2\pi} u(z_1) \frac{dv(z_2)}{d\vartheta} d\vartheta \tag{5}$$

$$= -\frac{1}{\pi} \int_0^{2\pi} v(z_1) \frac{du(z_2)}{d\vartheta} d\vartheta, \tag{5'}$$

where

$$z_1 = r_1 \zeta, \quad z_2 = r_2 \zeta, \quad \zeta = e^{i\vartheta}, \quad 0 \leq \vartheta < 2\pi. \tag{6}$$

Proof. First we derive formula (5). Taking into account the expansion $\Psi(z) = \sum_{k=0}^{\infty} c_k z^k$, we find

$$u(z) = \frac{1}{2} \left(\sum_{k=0}^{\infty} c_k z^k + \sum_{k=0}^{\infty} \bar{c}_k \bar{z}^k \right),$$

$$\vartheta(z)=\frac{1}{2i}\Big(\sum_{k=0}^{\infty} c_k z^k-\sum_{k=0}^{\infty}\bar{c}_k\bar{z}^k\Big),$$

$$\frac{du(z)}{d\vartheta}=\frac{i}{2}\Big(\sum_{k=1}^{\infty} k c_k z^k-\sum_{k=1}^{\infty} k\bar{c}_k\bar{z}^k\Big)=-\operatorname{Im}\left[z\Psi'(z)\right],$$

$$\frac{d\vartheta(z)}{d\vartheta}=\frac{1}{2}\Big(\sum_{k=1}^{\infty} k c_k z^k-\sum_{k=1}^{\infty} k\bar{c}_k\bar{z}^k\Big)=\operatorname{Re}\left[z\Psi'(z)\right].$$

Inserting the obtained expressions under the integral sign in (5), we obtain

$$\frac{1}{\pi}\int_0^{2\pi}u(z_1)\frac{d\vartheta(z_2)}{d\vartheta}d\vartheta=\frac{1}{4\pi}\int_0^{2\pi}\Big[\sum_{k=0}^{\infty}c_k r_1^k\zeta^k+\sum_{k=0}^{\infty}\bar{c}_k r_1^k\bar{\zeta}^k\Big]\Big[\sum_{k=1}^{\infty}k c_k r_2^k\zeta^k+\sum_{k=1}^{\infty}k c_k r_2^k\zeta^k\Big]d\vartheta=\frac{1}{4\pi}\Big[\sum_{k=1}^{\infty}k|c_k|^2(r_1r_2)^k+\sum_{k=1}^{\infty}k|c_k|^2(r_1r_2)^k\Big]\cdot 2\pi=\sum_{k=1}^{\infty}k|c_k|^2(r_1r_2)^k,$$

which, taking into account (4), gives

$$\frac{1}{\pi}\int_0^{2\pi}u(z_1)d\vartheta(z_2)=\sum_{k=1}^{\infty}k|c_k|^2r^{2k}=\sigma(r,\Psi).$$

Formula (5') is derived from (5) by applying integration by parts.

We denote by Γ_r the image of the circumference $|z|=r$ under the mapping by the function $\Psi(z)$. Formulas (5), (5') are the generalizations of the well-knwon formulas

$$\sigma(r,\Psi)=\frac{1}{\pi}\int_{\Gamma_r}u\,d\vartheta=-\frac{1}{\pi}\int_{\Gamma_r}\vartheta\,du.$$

<u>Remark 1.</u> For any function $\Psi(z)=\sum_{k=1}^{\infty}c_k z^k$, regular in $|z|<1$, one can derive in a similar manner the area formula

$$\sigma(r,\Psi)=\frac{1}{\pi}\int_0^{2\pi}\operatorname{Re}[\Psi(z_1)]\Big\{\operatorname{Re}\left[z_2\Psi'(z_2)\right]+\lambda_0+\lambda\operatorname{Im}\left[z_2\Psi'(z_2)\right]\Big\}d\vartheta, \tag{7}$$

where λ_0 and λ are arbitrary real numbers.

Now we select an arbitrary function $f(z)\in S$ and we apply the area formulas (5) and (7) to the function $\Psi(z)=\log\frac{f(z)}{z}$. We set

$$z=re^{i\vartheta},\quad f(z)=Re^{i\theta}.$$

Since

$$u(z)=\operatorname{Re}\log\frac{f(z)}{z}=\log\left|\frac{f(z)}{z}\right|=\log\frac{R}{r},$$

$$\vartheta(z)=\operatorname{Im}\log\frac{f(z)}{z}=\arg\frac{f(z)}{z}=\theta-\vartheta,$$

$$z\Psi'(z)=\frac{zf'(z)}{f(z)}-1,$$

it follows, using the notations

$$R(r_1,\vartheta)=R_1,\quad R(r_2,\vartheta)=R_2,$$

$$\theta(r_1,\vartheta)=\theta_1,\quad \theta(r_2,\vartheta)=\theta_2,$$

that the area formula can be represented in the form

$$\sigma\Big(r,\log\frac{f}{z}\Big)=\frac{1}{\pi}\int_0^{2\pi}\log\frac{R_1}{r_1}\,d(\theta_2-\vartheta)= \tag{8}$$

$$= -\frac{1}{\pi}\int_0^{2\pi}(\theta_1-\vartheta)\,d\log\frac{R_2}{r_2} = \frac{1}{\pi}\int_0^{2\pi}\log\frac{R_1}{r_1}\left\{\mathrm{Re}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]+\lambda_0+\lambda\,\mathrm{Im}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]\right\}d\vartheta.$$

If we make use of the equality for harmonic functions

$$\int_0^{2\pi}\log\frac{R_1}{r_1}\,d\vartheta = 0,$$

then from the first formula (8) we obtain the working formula

$$\sigma\left(r,\log\frac{f}{z}\right)=\frac{1}{\pi}\int_0^{2\pi}\log\frac{R_1}{r_1}\,d\theta_2 \tag{9}$$

needed in the sequel.

It is useful to emphasize two particular cases of formula (9).

1. Considering the limiting values of the function $f(z)\in S$ on the circumference $|z|=r$ and setting $r_1=r^2$ we obtain

$$\sigma\left(r,\log\frac{f}{z}\right)=\frac{1}{\pi}\int_0^{2\pi}\log\frac{R(r^2,\vartheta)}{r^2}\,d\theta(1,\vartheta).$$

2. For any function $f(z)\in S$ setting $r_1=r_2=r$ we obtain the well-known area formula in the form of a line integral along the contour Γ_r , the image of $|z|=r$ under the mapping by function $f(z)$:

$$\sigma\left(r,\log\frac{f}{z}\right)=\frac{1}{\pi}\int_{\Gamma_r}\log\frac{R}{r}\,d\theta. \tag{10}$$

In this case the line integral in (10) can be expressed by a double integral over some annular set. For this, we consider a circumference of arbitrary radius $\rho\geq M(r,f)$ and we consider the annular set $K_{r,\rho}$ as the difference between the circle $|w|<\rho$ and the closed interior of Γ_r . By the Green–Ostrogradskii formula, we have

$$\int_{\Gamma_r}\log\frac{R}{r}\,d\theta = 2\pi\log\frac{\rho}{r}-\iint_{K_{r,\rho}}\frac{dR\,d\theta}{R},$$

whence

$$\sigma\left(r,\log\frac{f}{z}\right)=2\log\frac{\rho}{r}-\frac{1}{\pi}\iint_{K_{r,\rho}}\frac{dR\,d\theta}{R}. \tag{11}$$

Formula (11), in the case of an arbitrary ρ , has been encountered previously by Teichmüller [3] and by Grinshpan [4].

3°. Some Results Regarding the Conjecture

We recall that a function $f(z)\in S$ is said to be spiral-shaped of order α , if in the circle $|z|<1$ one has the inequality

$$\mathrm{Re}\left[e^{i\beta}\frac{zf'(z)}{f(z)}\right]\geq\alpha \qquad (0\leq\alpha\leq\cos\beta,\ -\pi/2<\beta<\pi/2). \tag{12}$$

THEOREM 1. For each spiral-shaped function $f(z)\in S$ of order α , for any $r\in(0,1)$ one has the inequality

$$\sigma\left(r,\log\frac{f}{z}\right)\leq 2\left(1-\frac{\alpha}{\cos\beta}\right)\log\frac{M(r^2,f)}{r^2}\ (0\leq\alpha\leq\cos\beta,\ -\pi/2<\beta<\pi/2). \tag{13}$$

Equality in (13) for $\beta=0$ prevails in the case of the functions $f(z)=z$ and $f(z)=z(1-\eta z^m)^{-2(1-\alpha)/m}$ ($|\eta|=1$; $m=1,2,\ldots$).

Proof. To the function $\log\frac{f(z)}{z}$ we apply the third of formulas (8), setting for sufficiently small $\varepsilon>0$ $r_1=r^{2-\varepsilon}$, $r_2=r^{\varepsilon}$:

$$\sigma\left(r,\log\frac{f}{z}\right)=\frac{1}{\pi}\int_0^{2\pi}\log\frac{R_1}{r_1}\left\{\mathrm{Re}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]+\lambda_0+\lambda\,\mathrm{Im}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]\right\}d\vartheta.$$

Rewriting condition (12) in the equivalent form

$$\mathrm{Re}\left[\frac{zf'(z)}{f(z)}\right]-\mathrm{tg}\,\beta\,\mathrm{Im}\left[\frac{zf'(z)}{f(z)}\right]-\frac{\alpha}{\cos\beta}\geq 0,\quad |z|<1, \tag{12'}$$

we set

$$\lambda_0=-\frac{\alpha}{\cos\beta},\quad \lambda=-\mathrm{tg}\,\beta.$$

Then for the area we shall have the formula

$$\sigma\left(r,\log\frac{f}{z}\right)=\frac{1}{\pi}\int_0^{2\pi}\log\frac{R_1}{r_1}\left\{\mathrm{Re}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]-\mathrm{tg}\,\beta\,\mathrm{Im}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]-\frac{\alpha}{\cos\beta}\right\}d\vartheta. \tag{13'}$$

According to (12), the expression in the curly brackets in the integrand is nonnegative on the entire interval $[0,2\pi]$,

$$\sigma\left(r,\log\frac{f}{z}\right)\leq\max_{|z|=r_1}\log\frac{R_1}{r_1}\cdot\frac{1}{\pi}\int_0^{2\pi}\left\{\mathrm{Re}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]-\mathrm{tg}\,\beta\,\mathrm{Im}\left[\frac{z_2 f'(z_2)}{f(z_2)}\right]-\frac{\alpha}{\cos\beta}\right\}d\vartheta=2\left(1-\frac{\alpha}{\cos\beta}\right)\log\frac{M(r^{2-\varepsilon},f)}{r^{2-\varepsilon}}. \tag{14}$$

From (14), letting $\varepsilon\to 0$, we derive (13). By verification we can see that for the functions indicated in the theorem we actually have equality in (13).

Remark 2. From (13) we conclude that conjecture (1) holds for the subclass of spiral-shaped functions $f(z)\in S$, and setting $\beta=0$ we can see the validity of the conjecture for star-shaped functions of order α. As far as all star-shaped functions $f(z)\in S$ are concerned, the simplest inequality (1) follows directly from the area formula (9) if we set in it $r_1=r^{2-\varepsilon}$, $r_2=r^{\varepsilon}$ we take into account that for star-shaped functions on any circumference $|z|=r$ we have $d\theta(r,\vartheta)>0$, we estimate the integral in (9) and we let $\varepsilon\to 0$.

LEMMA 2. Assume that the function $\Psi(z)=\sum_{k=1}^{\infty}c_k z^k$ is regular in the circle $|z|<1$, continuous in $|z|\leq 1$ and has real coefficients c_k ($k=1,2,\ldots$). If the equation

$$\mathrm{Im}\,\Psi(z)=0$$

has on the unit circumference only two real roots, then on any circumference $|z|=r$, $0<r<1$, this equation will have only two real roots.

Geometrically, Lemma 2 has the following meaning: if the real axis intersects the curve $\Gamma=\{\Psi(z):|z|=1\}$ only in two points, then this axis intersects the image Γ_r of each circumference $|z|=r$ under the mapping by the function $\Psi(z)$ also only in two points.

Proof. By the assumption of Lemma 2, $\mathrm{Im}\,\Psi(z)$ preserves its sign both in the upper and in the lower semicircumference $|z|=1$. In addition, at each point of the segment $[-1,1]$ we have $\mathrm{Im}\,\Psi(z)=0$ due to the fact that the coefficients c_k are real. Therefore, by virtue of the maximum principle for harmonic functions, $\mathrm{Im}\,\Psi(z)$ does not vanish inside the upper and the lower semicircles of the circle $|z|<1$. Consequently, on any circumference $|z|=r$, $0<r<1$, $\mathrm{Im}\,\Psi(z)=0$ only at the points $z=\pm r$. Lemma 2 is proved.

THEOREM 2. For each function $f(z) \in S$ with real coefficients, continuous together with $f'(z)$ in $|z| \leq 1$, and satisfying the condition: the equation

$$\operatorname{Im} \frac{z f'(z)}{f(z)} = 0 \tag{15}$$

has only real roots on the circumference $|z| = 1$, conjecture (1) is true. Equality sign prevails in (1) for the functions $f(z) = \frac{z}{(1 \pm z)^2}$.

Proof. We take an arbitrary $r \in (0,1)$. Obviously, also the function $zf'(z)/f(z)$ has real coefficients and, therefore, by Lemma 2, on the circumference $|z| = r$ its imaginary part at the points $z = \pm r$ is equal to zero and at the other points of this circumference it is different from zero. From the obvious identity

$$\frac{z f'(z)}{f(z)} = \frac{d\theta}{d\vartheta} - i\,\frac{d \log R}{d\vartheta} \tag{16}$$

we can see that R as a function of ϑ has strictly two extrema on the curve Γ_r, the image of the circumference $|z| = r$ under the mapping by the function $f(z)$. Consequently, Γ_r represents a curve, symmetric with respect to the real axis, for which the distance $R(r, \vartheta)$ is a strictly monotone function of ϑ in the intervals $[0, \pi]$ and $[\pi, 2\pi]$.

Now for a sufficiently small ε we select $r_1 = r^{2-\varepsilon}, r_2 = r^{\varepsilon}$ and we apply formula (9) to the logarithmic area:

$$\sigma\left(r, \log \frac{f}{z}\right) = \frac{1}{\pi} \int_0^{2\pi} \log \frac{R_1}{r_1}\, d\theta_2.$$

We note that the curve L, defined by the parametric equations

$$\begin{aligned} \theta_2 &= \theta(r_2, \vartheta) = \arg f(r_2 e^{i\vartheta}), \\ R_1 &= R(r_1, \vartheta) = |f(r_1 e^{i\vartheta})|, \quad \vartheta \in [0, 2\pi], \end{aligned} \tag{17}$$

is a simple closed curve, symmetric with respect to the real axis: the strict monotonicity of R_1, as a function of ϑ in the intervals $[0, \pi]$ and $[\pi, 2\pi]$ does not admit that the curve L should then intersect itself. We shall consider the definite integral from the right-hand side of (9) as a line integral along the contour L, namely,

$$\int_0^{2\pi} \log \frac{R_1}{r_1} \frac{d\theta_2}{d\vartheta}\, d\vartheta = \int_L \log \frac{R_1}{r_1}\, d\theta_2, \tag{18}$$

and we express the latter in terms of a double integral by the Green–Ostrogradskii formula (11) for $\rho = M(r, f)$:

$$\frac{1}{\pi} \int_L \log \frac{R_1}{r_1}\, d\theta_2 = 2 \log \frac{M(r_1, f)}{r_1} - \frac{1}{\pi} \iint_K \frac{d\theta\, dR}{R}, \tag{19}$$

where the annular set K complements the closed interior to a circle of radius $M(r, f)$. Taking into account (18) and (19), from (9) we obtain

$$\sigma\left(r, \log \frac{f}{z}\right) \leq 2 \log \frac{M(r_1, f)}{r_1} = 2 \log \frac{M(r^{2-\varepsilon}, f)}{r^{2-\varepsilon}}$$

and letting $\varepsilon \to 0$, we arrive at (1). By verification we can see that for the functions $f(z) = z(1 \pm z)^{-2}$ we actually have equality in (1).

COROLLARY. For each function $f(z)\in S$ with real coefficients and for which the function $zf'(z)/f(z)$ is univalent in $|z|<1$, conjecture (1) is true. Equality sign prevails for the functions $f(z)=z(1\pm z)^{-2}$.

Indeed, for a univalent function $\frac{zf'(z)}{f(z)}$ with real coefficients, Eq. (15) has on each circumference $|z|=\rho$, $0<\rho<1$, only real roots. Therefore, applying Theorem 2 to the function $f_\rho(z)=f(\rho z)/\rho$, $0<\rho<1$, for any $r\in(0,1)$ we obtain

$$\sigma(r,\log\frac{f_\rho}{z})\leq 2\log\frac{M(r^2,f_\rho)}{r^2}. \tag{20}$$

It remains to pass in inequality (20) from the function $f_\rho(z)$ to the function $f(z)$, namely, to write (20) in the form

$$\sigma(r\rho,\log\frac{f}{z})\leq 2\log\frac{M(r^2\rho,f)}{r^2\rho}$$

and to let $\rho\to 1$.

LITERATURE CITED

1. I. M. Milin, "On a certain property of the logarithmic coefficients of univalent functions," in: Metric Questions of the Theory of Functions [in Russian], Naukova Dumka, Kiev (1980), pp. 86-90.
2. N. A. Lebedev, "The application of the area principle to problems on nonoverlapping domains," Tr. Mat. Inst. Akad. Nauk SSSR, 60, 211-231 (1961).
3. O. Teichmüller, "Untersuchungen über konforme und quasikonforme Abbildungen," Deutsche Nath., 3, No. 6, 621-678 (1938).
4. A. Z. Grinshpan, "An application of the arc principle to Bieberbach—Eilenberg functions," Mat. Zametki, 11, No. 6, 609-618 (1972).

EXPANSION OF AUTOMORPHIC FUNCTIONS ON $SL_3(\mathbb{C})/SU(3)$

N. V. Proskurin

UDC 511.3

One constructs the expansion for automorphic functions on $SL_3(\mathbb{C})/SU(3)$ using the method presented by the author in a previous paper [Zap. Nauch. Sem. Leningr. Otd. Mat. Inst., 116, 119-141 (1982)].

In this paper, by the method we have presented in [3], we construct the expansion of the function $F:X\to\mathbb{C}$ on the homogeneous space $X\cong SL_3(\mathbb{C})/SU(3)$, automorphic with respect to the group $SL_3(\mathcal{O},\mathfrak{q})$, the principal congruence subgroup $\bmod\,\mathfrak{q}$ in $SL_3(\mathcal{O})$; $\mathcal{O}$ is the ring of integers of the imaginary quadratic field $\mathbb{F}$, $\mathfrak{q}$ is an ideal in $\mathcal{O}$. The fundamental results of the paper are formulated in Sec. 2 in the form of two theorems. The first of these theorems furnishes a basis in the space $L^2(N,\Delta)$ (see below the notations), while the second one gives an expansion of F with respect to this basis.

Notations. $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$ are the ring of rational integers, the field of real and the field of complex numbers; $\mathbb{R}_+=\{x\in\mathbb{R}, x>0\}$.

Translated from Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR, Vol. 125, pp. 144-153, 1983.

