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An Alternative Proof of the Irrationality of *e*

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For a long time, it has been known that the base e of the natural logarithm is not a rational number. This fact was first shown by Leonhard Euler [1] in 1737 using continued fractions. The standard proof nowadays works with an argument on the "convergence speed" of the exponential series, and is said to be due to Jean-Baptiste Fourier [4]. Another analytic proof, going back to Charles Hermite [2], uses properties of Legendre-type polynomias and the mean value theorem to achieve the irrationality of e^c for all rational powers $c \neq 0$; a result known since 1761 and due to Johann Heinrich Lambert (who also studied continued fractions) [3]. It was actually in the very same work that Lambert established the irrationality of π too.

The following alternative proof of the irrationality of e is based on some simple properties of integrals of the form $\int_0^1 x^n e^{-x}\,dx$. For all $x \in [0,1]$ we have

$$1 \geq \frac{1}{e^x} \geq \frac{x}{e^x} \geq \frac{x^2}{e^x} \geq \ldots \geq 0.$$

This implies that the integral $\int_0^1 x^n e^{-x} dx$ has a value between 0 and 1 for all $n \in \mathbb{N}_0$. With a partial integration of $x^{n+1} \cdot e^{-x}$ we obtain the recursion

$$\int_0^1 \frac{x^{n+1}}{e^x}\,dx = -\frac{1}{e} + (n+1)\int_0^1 \frac{x^n}{e^x}\,dx \tag{1}$$

for all nonnegative integers n. This equation leads to some immediate consequences. As $\int_0^1 e^{-x} dx = 1 - e^{-1}$, we see that the integral $\int_0^1 x^n e^{-x} dx$ is always of the form $a_n e^{-1} + b_n$ with integer numbers a_n and b_n. Dividing [1] by $n+1$ and considering the limit for $n \to \infty$ gives

$$\lim_{n\to\infty} \int_0^1 \frac{x^n}{e^x}\,dx = 0. \tag{2}$$

Hence there is a constant $C > 0$ for which the inequation

$$0 < a_n e^{-1} + b_n < \frac{C}{n} \tag{3}$$

is valid for all n. This already implies the irrationality of e^{-1}–which is equivalent to that of e–since a rational $e^{-1} = p/q$ would lead to the contradicting expression

$$0 < pa_n + qb_n < 1,$$

for all n which are sufficiently large.

Remark. Although the property of the exponential function which we used to prove the irrationality of e is other than that in the above-mentioned standard proof, the two methods are formally equivalent. Using induction over n, we see that formula [1] implies

$$\int_0^1 \frac{x^n}{e^x}dx = -\frac{1}{e}\sum_{\nu=0}^{n}\frac{n!}{\nu!} + n! \tag{4}$$

Multiplying this equation with the quotient $\frac{e}{n!}$ gives

$$e - \sum_{\nu=0}^{n}\frac{1}{\nu!} = \frac{e \cdot \int_0^1 \frac{x^n}{e^x}dx}{n!} \to 0, \tag{5}$$

for $n \to \infty$, i.e., we obtain the exponential series.

References

[1] L. Euler, *De fractionibus continuis*, Commentarii Academia Scientiarum Imperialis Petropolitanae 9, 1737, pp. 98–136

[2] C. Hermite, *Sur la fonction exponentielle*, Comptes Rendus de l'Académie des Sciences de Paris 1873, pp. 18–24; pp. 74–79; pp. 226–233; pp. 285–293

[3] J. H. Lambert, *Mémoire sur quelques propriétés remarquables des quantités transcendentes circulaires et logarithmiques*, Mém. Berlin 17, 1761, pp. 265–322

[4] H. V. Mangoldt, K. Knopp, *Einführung in die höhere Mathematik*, 2. Band, Stuttgart 1948 (9th edn.), pp. 118–119

Prime Numbers

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Abstract. This is an exposition of prime numbers through some of the classical theorems of number theory, chosen on the basis of some fundamental questions on primes. The questions addressed are on the number of primes, the distribution of primes on the number line and on the structure of primes. The proofs presented here are some of the nicest ones found in literature, or in the words of Paul Erdös, proofs from THE BOOK.

Introduction

An integer $p > 1$ is called a *prime* or a *prime number* if p has no divisor d satisfying $1 < d < p$. An integer which is not prime is called *composite*. It follows that, an integer $p > 1$ is prime if and only if $p \mid ab$ implies $p \mid a$ or $p \mid b$, where a, b are integers. The Fundamental Theorem of Arithmetic says that any integer $n > 1$ can be factored into primes uniquely, upto the order of the primes. So, the primes are in some sense the building blocks of the number system. One can ask:

(1) How many primes are there?
(2) How are the primes distributed on the number line?
(3) Can one determine the structure of primes?
(4) Given a natural number $n > 1$, how does one decide the primality of n?

This article is aimed at proving some classical theorems of number theory which would answer questions (1), (2) and (3). The proofs presented here are of varying flavour and taken from the book [AZ]. A nicely written article on question (4) is [G]. It is important in this context that, a polynomial time deterministic algorithm for testing primality has been discovered by Manindra Agrawal, Neeraj Kayal and Nitin Saxena in the year 2002 (see [AKS]).

Before we start, we fix a few notations: $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ denotes the set of reals and $\mathbb{P}$ denotes the set of primes.

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1. Infinitude of primes

It is well known that the set of primes $\mathbb{P}$ is an infinite set. Euclid's proof is the following : Suppose the first r primes are $p_1, \ldots, p_r$. Consider $n = p_1 \cdots p_r + 1$, which is not divisible by any p_i. By Fundamental Theorem of Arithmetic there exists a prime p such that $p \mid n$ and $p \neq p_i$ for $1 \leq i \leq r$. Therefore, for any natural number r the number of primes is strictly bigger than r. Hence, there are infinite number of primes. This is perhaps the simplest and the most beautiful proof of infinitude of primes. There are several other proofs and we choose two among these, one using topology and the other using analysis. Both are of compelling beauty.

Theorem 1.1. *The set of primes $\mathbb{P}$ is infinite.*

Proof 1 (Using topology). *We construct a topology on $\mathbb{Z}$. For $a, b \in \mathbb{Z}$ and $b > 0$ let*

$$N_{a,b} := \{a + nb \mid n \in \mathbb{Z}\}.$$

We call a set $\mathcal{O} \subset \mathbb{Z}$ open if either $\mathcal{O} = \emptyset$ or given $a \in \mathcal{O}$ there exists an integer $b > 0$ such that $N_{a,b} \subset \mathcal{O}$. It is now easy to verify that this indeed defines a topology on $\mathbb{Z}$. For, if $\{\mathcal{O}_\alpha \mid \alpha \in \Lambda\}$ is a family of open sets in $\mathbb{Z}$ and $a \in \cup_{\alpha\in\Lambda}\mathcal{O}_\alpha$, we have

$$a \in N_{a,b} \subseteq \mathcal{O}_\alpha \subseteq \cup_{\alpha\in\Lambda}\mathcal{O}_\alpha,$$

for some $\alpha \in \Lambda$, $0 < b \in \mathbb{Z}$, and if $\mathcal{O}_1, \ldots, \mathcal{O}_n$ are finitely many open sets with $a \in \cap_{i=1}^n \mathcal{O}_i$, then $a \in N_{a,b_i} \subseteq \mathcal{O}_i$, for integers $b_i > 0$, $i = 1, \ldots, n$, and therefore

$$a \in N_{a,b_1\cdots b_n} \subseteq \cap_{i=1}^n N_{a,b_i} \subseteq \cap_{i=1}^n \mathcal{O}_i.$$

Moreover, it is clear from the definition that any nonempty open set is infinite. We claim that each $N_{a,b}$ is both open and closed. Each $N_{a,b}$ is open because for any $x \in N_{a,b}$ we have $N_{x,b} = N_{a,b}$. Furthermore,

$$N_{a,b} = \cap_{i=1}^{b-1} N_{a+i,b}$$

because for any $x \in \mathbb{Z}$ one can write $x - a = nb + r$, where $0 \le r \le b - 1$. Hence $N_{a,b}$ is closed. We know that any integer $n \ne 1, -1$ has a prime divisor and therefore $n \in N_{0,p}$ for some prime p. This shows that

$$\mathbb{Z} \setminus \{1, -1\} = \cup_{p \in \mathbb{P}} N_{0,p}.$$

If the set $\mathbb{P}$ is finite then $\mathbb{Z} \setminus \{1, -1\}$ is closed, so $\{1, -1\}$ is open - contradicts that every nonempty open set is infinite. Hence $\mathbb{P}$ is infinite. □

Proof 2 (Using Analysis). *This proof is by Paul Erdös, which shows that $\sum_{p \in \mathbb{P}} \frac{1}{p}$ is divergent and therefore $\mathbb{P}$ is infinite.*

Let $p_1, p_2, \ldots$ be the sequence of primes in the ascending order. We assume that $\sum_{p \in \mathbb{P}} \frac{1}{p}$ is convergent. Then, there exists a natural number k such that

$$\sum_{i \ge k+1} \frac{1}{p_i} < \frac{1}{2}$$

We call $p_1, \ldots, p_k$ the small primes *and $p_{k+1}, \ldots$ the big primes. Now for any natural number N we have*

$$\sum_{i \ge k+1} \frac{N}{p_i} < \frac{N}{2} \tag{6}$$

Let N_b denote the number of integers in between 1 and N which has at least one big prime divisor and N_s denote the number of integers in between 1 and N which has all small prime divisors. For a suitable choice of N we will show that $N_b + N_s < N$, whereas by definition $N_b + N_s = N$ for all natural numbers N. This gives a contradiction.

Estimation of N_b: Note that[1] *$\left\lfloor \frac{N}{p_i} \right\rfloor$ counts the integers n such that $1 \le n \le N$ and $p_i \mid n$. Therefore by (2.2)*

$$N_b \le \sum_{i \ge k+1} \left\lfloor \frac{N}{p_i} \right\rfloor < \frac{N}{2} \tag{7}$$

Since $0 \le \left\lfloor \frac{N}{p_i} \right\rfloor \le \frac{N}{p_i}$ and $\sum_{i \ge k+1} \frac{N}{p_i}$ is convergent therefore $\sum_{i \ge k+1} \left\lfloor \frac{N}{p_i} \right\rfloor$ is convergent and the inequality is preserved.

[1] $\lfloor x \rfloor$ (respectively $\lceil x \rceil$) denotes the greatest integer less than or equal to (respectively the smallest integer bigger than or equal to) x.

Estimation of N_s : Let n be an integer such that $1 \le n \le N$ and n has only small prime divisors. We write $n = a_n b_n^2$, where are a_n is square-free part[2]*. Every a_n is thus a product of different small primes and therefore there can be exactly 2^k different square-free parts. Furthermore, $b_n \le \sqrt{n} \le \sqrt{N}$ and we find that there are at most $\sqrt{N}$ different square parts. Therefore,*

$$N_s \le 2^k . \sqrt{N}$$

Since (2.3) holds for any natural number N, we have to find an N such that $2^k \sqrt{N} \le \frac{N}{2}$, i.e., $2^{k+1} \le \sqrt{N}$. We can take $N = 2^{2k+2}$, then $N_b + N_s < N$ - contradiction. Therefore $\sum_{p \in \mathbb{P}} \frac{1}{p}$ is divergent and hence $\mathbb{P}$ is infinite. □

2. Distribution of primes

How are the primes distributed on the number line? What we mean is that: can one say something about the gap between two consecutive primes? Also, given a real number $x > 1$, what is the number of primes less than or equal to x? First let us mention a simple fact which shows that there can be arbitrarily large gap between primes.

Proposition 2.1. *Given any positive integer k, there exist k consecutive composite integers.*

Proof. Consider the integers

$$(k+1)! + 2, (k+1)! + 3, \ldots, (k+1)! + k + 1.$$

If $2 \le j \le k + 1$ then $j \mid (k+1)! + j$. Therefore every one of the above k integers are composite. □

In this connection, perhaps the most important theorem is

Theorem 2.1 (Prime Number Theorem).

$$\lim_{x \to \infty} \frac{\#\{p \le x \mid p \text{prime}\}}{\frac{x}{\log x}} = 1.$$

The above theorem is hard and a proof can be found in [A]. This was first proved by Hadamard and Ch. de la Vallée-Poussin in 1896. Selberg and Erdös found an elementary proof (without complex analytic tools), but still long and involved, in 1948. Now there exist several proofs.

Theorem 2.2 (Bertrand's Postulate). *For every integer $n \ge 1$, there is a prime p with $n < p \le 2n$.*

[2] a_n and b_n are not necessarily coprime.

In this section we prove Theorem 2.3, which is due to Joseph Bertrand. It is called Bertrand's postulate because it was conjectured and verified empirically for $n < 3000000$ by Bertrand. It was first proved for all n by Pafnuty Chebyshev in 1850. A much simpler proof was given by Ramanujan. The proof we discuss here is by Paul Erdös, appeared in 1932 in his first published paper.

We need a preparation before we prove Bertrand's postulate. The proof is split into 5 parts and uses estimation of binomial coefficients.

Binomial Coefficients

The binomial coefficient $\binom{n}{k}$ is defined as the number of k-subsets of an n-set. Therefore $\binom{n}{k}$ is an integer and we know that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Below listed are some basic properties of the binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$.

Proposition 2.4.

(1) $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

(2) *The sequence* $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ *is symmetric and unimodal:*

$$1 = \binom{n}{0} < \binom{n}{1} < \cdots < \binom{n}{\lfloor \frac{n}{2} \rfloor}$$

$$= \binom{n}{\lceil \frac{n}{2} \rceil} > \cdots > \binom{n}{n-1} > \binom{n}{n} = 1.$$

(3) $\binom{n}{\lfloor \frac{n}{2} \rfloor} \geq \frac{2^n}{n}$ *for* $n \geq 2$ *and the equality holds for* $n = 2$.

(4) $\binom{2}{n}n \geq \frac{4^n}{2n}$ *for* $n \geq 1$.

(5) $\binom{n}{k} \leq \frac{n^k}{2^{k-1}}$.

Proof. We leave (1)–(3) as exercise.

4. Note that, for $n \geq 2$ we have,

$$\binom{n}{\lfloor \frac{n}{2} \rfloor} \geq \binom{n}{0} + \binom{n}{n}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n} - 1.$$

Therefore,

$$n\binom{n}{\lfloor \frac{n}{2} \rfloor} \geq \left[\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} - 1 + \binom{n}{n}\right]$$

$$= 2^n.$$

Hence, $\binom{n}{\lfloor \frac{n}{2} \rfloor} \geq \frac{2^n}{n}$ for $n \geq 2$ and obviously the equality holds for $n = 2$.

5. Follows from (4), take $n = 2k$.

6. $\binom{n}{k} = \dfrac{n(n-1)\cdots(n-k+1)}{k!} \leq \dfrac{n^k}{k!} \leq \dfrac{n^k}{2^{k-1}}$. □

Theorem 2.5 (Legendre's Theorem). *The number* $n!$ *contains the prime factor* p *exactly* $\sum_{k\geq 1} \lfloor \frac{n}{p^k} \rfloor$ *times.*

Proof. Exactly $\lfloor \frac{n}{p} \rfloor$ of the factors of $n!$ are divisible by p, which accounts for $\lfloor \frac{n}{p} \rfloor$ p-factors. Next, $\lfloor \frac{n}{p^2} \rfloor$ of the factors of $n!$ are even divisible by p^2, which accounts for the next $\lfloor \frac{n}{p^2} \rfloor$ prime factors P of $n!$, and so on. □

Proof of Bertrand's Postulate

We will estimate the size of the binomial coefficient $\binom{2}{n}n$ carefully and show that if there is no prime p in the range $n < p \leq 2n$, then $\binom{2}{n}n$ would be too small. Our argument is in five steps.

(I) We first prove Bertrand's postulate for $n < 4000$. This is called Landau's trick. Consider the primes

2, 3, 5, 7, 13, 23, 43, 83, 163, 317, 631, 1259,
2503, 4001

with the property that each is smaller than twice the previous one. Hence every interval $\{y \mid n < y \leq 2n\}$, with $n \leq 4000$, contains one of these 14 primes.

(II) Next we prove that

$$\prod_{\substack{p \leq x \\ p \text{ prime}}} p \leq 4^{x-1} \text{ for all real } x \geq 2. \tag{8}$$

If q is the largest prime with $q \leq x$ then

$$\prod_{\substack{p \leq x \\ p \text{ prime}}} p = \prod_{\substack{p \leq q \\ p \text{ prime}}} p \text{ and } 4^{q-1} \leq 4^{x-1}.$$

Therefore it suffices to prove (2.6) for the case $x = q$-a prime number. Suppose $q = 2$, then the proof is obvious. Suppose $q = 2m + 1$-an odd prime. Then

$$\prod_{\substack{p \leq 2m+1 \\ p \text{ prime}}} p = \prod_{\substack{p \leq m+1 \\ p \text{ prime}}} p \cdot \prod_{\substack{m+1 < p \leq 2m+1 \\ p \text{ prime}}} p$$

$$\leq 4^m \cdot \binom{2m+1}{m} \leq 4^m \cdot 2^{2m} = 4^{2m}.$$

The inequality

$$\prod_{\substack{p \le m+1 \\ p \text{ prime}}} p \le 4^m$$

is by induction. Moreover

(i) $\binom{2m+1}{m} = \frac{(2m+1)!}{m!(m+1)!}$ is an integer, and the primes p which satisfy $m+1 < p \le 2m+1$ are all factors of $(2m+1)!$ but not of the denominator $m!(m+1)!$. Therefore,

$$\prod_{\substack{m+1 < p \le 2m+1 \\ p \text{ prime}}} p \Bigg| \binom{2m+1}{m}$$

and therefore

$$\prod_{\substack{m+1 < p \le 2m+1 \\ p \text{ prime}}} p \le \binom{2m+1}{m}.$$

(ii) Since $\binom{2m+1}{m} = \binom{2m+1}{m+1}$ and they appear in $\sum_{k=0}^{2m+1} \binom{2m+1}{k} = 2^{2m+1}$ as summand, therefore

$$\binom{2m+1}{m} \le 2^{2m}.$$

(III) From Theorem (2.5) we get that $\binom{2n}{n} = \frac{(2n)!}{n!n!}$ contains the prime factor p exactly

$$\sum_{k \ge 1} \left(\left\lfloor \frac{2n}{p^k} \right\rfloor - 2 \left\lfloor \frac{n}{p^k} \right\rfloor \right) \quad \text{times.}$$

Here each summand is at most 1 because each summand is an integer satisfying

$$\left\lfloor \frac{2n}{p^k} \right\rfloor - 2 \left\lfloor \frac{n}{p^k} \right\rfloor < \frac{2n}{p^k} - 2\left(\frac{n}{p^k} - 1\right) = 2.$$

Furthermore, the summands are 0 whenever $p^k > 2n$. Thus $\binom{2n}{n}$ contains p exactly

$$\sum_{k \ge 1} \left(\left\lfloor \frac{2n}{p^k} \right\rfloor - 2 \left\lfloor \frac{n}{p^k} \right\rfloor \right) \le \max\{r \mid p^r \le 2n\} \text{times.}$$

Hence the largest power of p that divides $\binom{2n}{n}$ is at most $2n$. In particular, primes $p > \sqrt{2n}$ appear at most once in $\binom{2n}{n}$.

Furthermore, the key fact according to Erdös for his proof is the following: primes p that satisfy $\frac{2}{3}n < p \le n$ do not divide $\binom{2}{n}n$ at all! For, if $3p > 2n$ (for $n \ge 3$ and hence $p \ge 3$) then p and $2p$ are the only multiples of p that appear as factors in the numerator of $\frac{(2n)!}{n!n!}$, while there are two p-factors in the denominator.

(IV) Now we estimate $\binom{2}{n}n$. For $n \ge 3$, we get

$$\frac{4^n}{2n} \le \binom{2n}{n} \le \prod_{\substack{p \le \sqrt{2n} \\ p \text{ prime}}} 2n \cdot \prod_{\substack{\sqrt{2n} < p \le \frac{2}{3}n \\ p \text{ prime}}} p \cdot \prod_{\substack{n < p \le 2n \\ p \text{ prime}}} p$$

and since there are not more than $\sqrt{2n}$ primes $p \le \sqrt{2n}$ we get

$$4^n \le (2n)^{1+\sqrt{2n}} \cdot \prod_{\substack{\sqrt{2n} < p \le \frac{2}{3}n \\ p \text{ prime}}} p \cdot \prod_{\substack{n < p \le 2n \\ p \text{ prime}}} p \quad \text{for } n \ge 3. \tag{9}$$

(V) Now assume that there is no prime p with $n < p \le 2n$, so the second product in (2.7) is 1. Substituting (2.6) in (2.7) we get

$$4^n \le (2n)^{1+\sqrt{2n}} \cdot 4^{\frac{2}{3}n}$$

or

$$4^{\frac{n}{3}} \le (2n)^{1+\sqrt{2n}} \tag{10}$$

which is false if n is very large. Using $a+1 < 2^a$ for integers $a \ge 2$ (can be proved using induction on a) we get

$$2n = (\sqrt[6]{2n})^6 < (\lfloor \sqrt[6]{2n} \rfloor + 1)^6 < 2^{6 \cdot \lfloor \sqrt[6]{2n} \rfloor}$$
$$\le 2^{6 \cdot \sqrt[6]{2n}} \tag{11}$$

and thus for $n \ge 50$ (in which case $18 < 2\sqrt{2n}$) we obtain from (3.8) and (3.9)

$$2^{2n} \le (2n)^{3(1+\sqrt{2n})} < 2^{\sqrt[6]{2n}(18+18\sqrt{2n})}$$
$$< 2^{20\sqrt[6]{2n} \cdot \sqrt{2n}} = 2^{20(2n)^{\frac{2}{3}}}.$$

This implies $(2n)^{\frac{1}{3}} < 20$ and hence $n < 4000$. This contradicts (I), where we have already proved Bertrand's postulate for $n < 4000$. Hence the proof. □

3. Structure of Primes

A prime can be either 2 or of the form $4m+1$, $4m+3$, where m is a positive integer. The following argument shows that there are infinitely many primes of the form $4m+3$. Suppose there are only finitely many primes of the form $4m+3$ and p_k be the largest such prime. Consider $N_k = 2^2 \cdot 3 \cdot 5 \cdots p_k - 1$, where $p_1 = 2, p_2 = 3, p_3 = 5, \ldots$. Then N_k is of the form $4m+3$ and therefore it has to have a prime divisor of the form $4m+3$. This prime factor is larger than p_k-a contradiction.

Also there are infinitely many primes of the form $4m+1$, which is more difficult to prove. But these can be proved from a much stronger and difficult theorem (see [A] for a proof):

Theorem 3.1 (Dirichlet's Theorem). *If a, b are coprime natural numbers, then there are infinitely many primes of the form $am+b, m \in \mathbb{N}$.*

We are mainly interested in those primes which can be expressed as a sum of two squares. Note that, square of an even integer is congruent to 0 modulo 4 and square of an odd integer is congruent to 1 modulo 4. Therefore, sum of two squares is either congruent to 0 or congruent to 1, modulo 4. Hence, a natural number n which is of the form $4m+3 \equiv 3(\mod 4)$, can never be expressed as a sum of two squares. We now prove that every prime of the form $4m+1$ is a sum of two squares, which is a theorem due to Fermat. The proof[3] we present here is by D. Zagier and taken from [AZ].

Theorem 3.2 (Fermat's Two Square Theorem). *Every prime p of the form $4m+1$ is a sum of two squares, that is, $p = x^2 + y^2$ for some $x, y \in \mathbb{N}$.*

Proof. Consider the set

$$S := \{(x, y, z) \in \mathbb{N}^3 \mid x^2 + 4yz = p\}.$$

This set S is non-empty because $(1, m, 1) \in S$ and it is finite because if $(x, y, z) \in S$ then $x, y, z \in \{1, \ldots, p\}$. Think of S as a subset of $\mathbb{R}^3$ and consider the planes $x = y - z$ and $x = 2y$ in $\mathbb{R}^3$. These planes do not intersect S because $(x, y, z) \in S$, $x = y - z$ imply $p = (y+z)^2$ and $(x, y, z) \in S$, $x = 2y$ imply $p = 4y(y+z)$ - both contradict the primality of p. But they divide S into three mutually disjoint subsets S_1, S_2, S_3 given by[4]

$$S_1 := \{(x, y, z) \in S \mid x < y - z\}$$

$$S_2 := \{(x, y, z) \in S \mid y - z < x < 2y\}$$

$$S_1 := \{(x, y, z) \in S \mid 2y < x\}.$$

Now define a map $f : S \to S$ as

$$f(x, y, z) = \begin{cases} (x+2z, z, y-x-z) & \text{if } (x, y, z) \in S_1, \\ (2y-x, y, x-y+z) & \text{if } (x, y, z) \in S_2, \\ (x-2y, x-y+z, y) & \text{if } (x, y, z) \in S_3. \end{cases}$$

Then f is well-defined and it is easy to verify that f is bijective, $f^{-1} = f$, $f(S_1) = S_3$, $f(S_2) = S_2$ and $f(S_3) = S_1$. Therefore, $\#S_1 = \#S_3$ and if f has any fixed point then that should belong to S_2. It follows that $f(x, y, z) = (x, y, z)$ iff $(2y - x, y, x - y + z) = (x, y, z)$ iff $x = y$. Moreover, $f(x, y, z) = (x, y, z)$ implies $x = y$ and therfore $x(x+4z) = p$, which implies that $x = 1, z = \frac{p-1}{4}$. Hence, f has exactly one fixed point in S_2, namely $(1, 1, \frac{p-1}{4})$ and it maps all other points of S to their images. This implies that $\#S_2$ is odd, therefore $\#S$ is odd.

Now consider the map $g : S \to S$ defined as $g(x, y, z) = (x, z, y)$. Then $g^{-1} = g$ and since $\#S$ is odd, g has to have at least one fixed point. Then there exists $(x, y, y) \in S$, *i.e.*, $x^2 + 4y^2 = p$ - this proves the theorem. □

Remark 3.3. *The above proof also shows that the number of representations of p of the form $x^2 + (2y)^2$ is odd for all primes p of the form $4m+1$. In fact, such representation is unique (see [NZM]).*

Remark 3.4. *It is a theorem due to Lagrange (1770) which says that, every positive integer n can be expressed as $n = x_1^2 + x_2^2 + x_3^2 + x_4^2$ where $x_i \in \mathbb{N}$. It can also be proved that fewer than four x_i's is not enough (see [NZM]).*

[3] A one-sentence proof that every prime $p \equiv 1(\mod 4)$ is a sum of two squares, D. Zagier, *American Mathematical Monthly* 77(1990), 144.

[4] For example, if $p = 5$ then $S = \{(1, 1, 1)\}$, $S_1 = \emptyset$, $S_2 = S$, $S_3 = \emptyset$. If $p = 13$, then $S = \{(1, 1, 3), (1, 3, 1), (3, 1, 1)\}$, $S_1 = \{(1, 3, 1)\}$, $S_2 = \{(1, 1, 3)\}$, $S_3 = \{(3, 1, 1)\}$.

References

[AKS] M. Agrawal, N. Kayal, N. Saxena, *PRIMES is in P*, Preprint 2002, http://www.cse.iitk.ac.in/news/primality.html.

[AZ] M. Aigner, G. M. Ziegler, *Proofs from THE BOOK*, Springer Verlag, 1998.

[A] T. M. Apostol, *Introduction to Analytic Number Theory*, Narosa Publishing House, 1989.

[B] D. M. Burton, *Elementary Number Theory*, Second Edition, Universal Book Stall, New Delhi, 1991.

[G] A. Granville, *It is Easy to Determine whether a Given Integer is Prime*, Bulletin(New Series) of the AMS, Volume 42, Number 1, pp. 3–38, 2005.

[HW] G. H. Hardy, E. M. Wright, *An Introduction to the Theory of Numbers*, Fifth Edition, Oxford University Press, 1979.

[NZM] I. Niven, H. S. Zuckerman, H. L. Montgomery, *An Introduction to the theory of numbers*, Fifth Edition, John Wiley & Sons, Inc, 1991.

Another Proof of $\frac{r}{R} = \cos(\alpha) + \cos(\beta) + \cos(\gamma) - 1$

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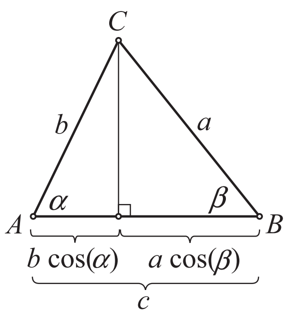
In a talk on glide-reflections, which I gave at IIT Madras, Chennai, on August 24th 2005, I mentioned a result about the ratio $\frac{r}{R}$ of the inradius and circumradius of a triangle. The result is pretty and elegant, but it also seems surprising it had anything to do with the topic of glide-reflections in which it was discovered. In fact it can be proven entirely independently. A direct proof of this theorem is the topic of this article.

Theorem. *Let α, β and γ be the angles of a triangle whose inscribed circle has radius r and whose circumscribed circle has radius R, then*

$$\frac{r}{R} = \cos(\alpha) + \cos(\beta) + \cos(\gamma) - 1$$

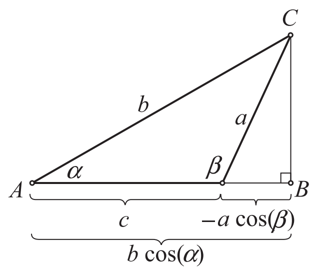
Proof. We will begin with a simple lemma.

Lemma. *If α, β and γ are the angles of a triangle, and a, b and c the lengths of the sides opposite the angles α, β and γ respectively, then*



$$a = b\cos(\gamma) + c\cos(\beta)$$
$$b = c\cos(\alpha) + a\cos(\gamma)$$
$$c = a\cos(\beta) + b\cos(\alpha).$$

The proof of this is elementary. For acute angles it is immediate, and one only needs to realize it works for other angles as well.



From the lemma it now follows that

$$a + b + c = b\cos(\gamma) + c\cos(\beta) + c\cos(\alpha) + a\cos(\gamma) + a\cos(\beta) + b\cos(\alpha).$$

Hence

$$\begin{aligned}(a+b+c)&(\cos(\alpha)+\cos(\beta)+\cos(\gamma)-1)\\ &= a\cos(\alpha) + b\cos(\beta) + c\cos(\gamma) + b\cos(\gamma) + c\cos(\beta)\\ &\quad + a\cos(\gamma) + c\cos(\alpha) + a\cos(\beta) + b\cos(\alpha)\\ &\quad - (a+b+c)\\ &= a\cos(\alpha) + b\cos(\beta) + c\cos(\gamma)\end{aligned}$$

Next note that if h is the altitude from C, i.e., $h = a\sin(\beta) = b\sin(\alpha)$, then

$$\begin{aligned} & a\cos(\alpha) + b\cos(\beta) + c\cos(\gamma) \\ &\quad = a\cos(\pi - \beta - \gamma) + b\cos(\pi - \alpha - \gamma) + c\cos(\gamma) \\ &\quad = a\sin(\beta)\sin(\gamma) - a\cos(\beta)\cos(\gamma) \\ &\qquad + b\sin(\alpha)\sin(\gamma) - b\cos(\alpha)\cos(\gamma) + c\cos(\gamma) \\ &\quad = 2h\sin(\gamma) - (a\cos(\beta) + b\cos(\alpha))\cos(\gamma) + c\cos(\gamma) \\ &\quad = 2h\sin(\gamma) \end{aligned}$$

where in the last step the lemma was used again.
Thus we find

$$\begin{aligned} & (a+b+c)(\cos(\alpha) + \cos(\beta) + \cos(\gamma) - 1) \\ &\quad = 2h\sin(\gamma) \end{aligned}$$

$$\begin{aligned} &= \frac{hc}{R} \quad \text{(by the extended law of sines)} \\ &= \frac{2 \cdot \text{Area}(\Delta ABC)}{R} \\ &= \frac{2rs}{R} \quad \left(\text{where } s = \frac{a+b+c}{2}\right) \\ &= \frac{r(a+b+c)}{R} \end{aligned}$$

which proves the theorem.

One might recall Euler's famous result: $d^2 = R(R - 2r)$, where d is the distance between the circumcenter and incenter, which implies e.g., $0 \le \frac{r}{R} \le \frac{1}{2}$. The theorem proven here gives an explicit formula for $\frac{r}{R}$.

Riemann Zeta Function – I

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The Riemann zeta function play an important role in the study of prime numbers. We shall see some nice relationship between the zeta function and prime numbers in the sequel. This function essentially is a particular kind of Dirichlet series–a series of the form

$$\sum_{n=1}^{\infty} a_n e^{-\lambda_n s}, \quad s \in \mathbb{C},$$

$a_1, a_2, \dots$ are complex numbers and $\lambda_1 < \lambda_2 < \dots$ is a sequence of real numbers converging to ∞. There are a number of texts which deal with Dirichlet series in detail (eg. [1]). The zeta function, usually denoted by $\zeta(s)$, has its origin in the identity expressed by the series formula

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} e^{-s\ln n}. \tag{1}$$

In number theory, it is customary to use s to represent a complex number instead of z. By Weierstrass M–test for convergence of the series of complex valued functions (see [4,5]), we can see that $\zeta(s)$ converges absolutely and uniformly on compact subsets of $\{s \in \mathbb{C} : \operatorname{Re} s > 1\}$. Indeed, we observe that, for $s = \sigma + it$,

$$|n^{-s}| = |e^{-s\ln n}| = e^{-\sigma \ln n} = \frac{1}{n^\sigma}, \quad n \in \mathbb{N}.$$

From a well known result from real analysis on σ-harmonic series, it follows that $\sum_{n=1}^{\infty} 1/n^\sigma$ converges whenever $\sigma > 1$, we obtain that $\zeta(s)$ converges for $\sigma = \operatorname{Re} s > 1$. Now the following result is a simple consequence of the above observation as $|a_n e^{-\lambda_n s}| = |a_n| e^{-\lambda_n \operatorname{Re} s}$

Theorem 1. *If a Dirichlet series converges absolutely at the point $s_0 = \sigma_0 + i\tau_0$, then it converges absolutely and uniformly in the closed half-plane* $\operatorname{Re} s \ge \sigma_0$.

Thus, $\zeta(s)$ defined by the (1) represents an analytic function in the domain $\{s : \operatorname{Re} s > 1\}$.

In this note we present seven methods of finding the value of $\zeta(2)$ and the connection between zeta function and Bernoulli numbers. We see that Bernoulli numbers can be used to evaluate the zeta function for some integer arguments. We start discussing the value of ζ function at some special points. For instance,

Theorem 2. $\zeta(2) = \pi^2/6$.

Our aim is to discuss seven different proofs of this theorem some of which might be familiar to many readers. Since $\sum_{n=1}^{\infty} n^{-2}$ converges absolutely, it follows easily that

$$\frac{3}{4}\zeta(2) = \sum_{n=1}^{\infty}\frac{1}{n^2} - \sum_{n=1}^{\infty}\frac{1}{(2n)^2} = \sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}$$

so that

$$\zeta(2) = \frac{4}{3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}.$$

Thus, we have the equivalent formulation of Theorem 2.

Theorem 3. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$

To present our first proof, we recall the definition of Fourier series.

Definition 1 (Fourier Series). *Let X denote the space of all complex-valued functions of real variable $f : [a, b] \mapsto \mathbb{C}$ that are piecewise continuous on the interval $[a, b]$. We recall that $f : [a, b] \mapsto \mathbb{C}$ is called* piecewise continuous *if it has at most a finite number of points of discontinuity, and in addition, the one-sided limits exist at each point of discontinuity.*

For $f \in X$ we have

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2nx\pi}{b-a}\right) + b_n \sin\left(\frac{2nx\pi}{b-a}\right)\right) \tag{2}$$

where

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\left(\frac{2nx\pi}{b-a}\right)dx$$

and

$$b_n = \frac{2}{b-a}\int_a^b f(x)\sin\left(\frac{2nx\pi}{b-a}\right)dx, \quad n = 0, 1, 2\ldots.$$

The series in (2) associated with f is called the Fourier series of f and a_n, b_n are called Fourier coefficients of f.

A well-known result for functions in this space is that each f has a Fourier series expansion [2] and the Fourier series converges to $f(x)$ at points continuity.

More often our interest lies in the case when $[a, b] = [-\pi, \pi]$ or $[a, b] = [0, 2\pi]$ or $[a, b] = [0, \pi]$ and the choice of the interval depends on the problem. When the domain of f is $[-\pi, \pi]$, we have

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx, \quad n \geq 0$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx, \quad n = 1, 2\ldots.$$

More precisely, we have the following standard result due to Dirichlet.

Theorem 4. *Let $f \in X$ with $a = -\pi$ and $b = \pi$. Suppose that f possesses one-sided derivatives of f at each point in $[-\pi, \pi]$. Then for each $x \in (-\pi, \pi)$ the Fourier series of f converges to the value*

$$\frac{f(x_-) + f(x_+)}{2}.$$

At the endpoints $x = \pm\pi$ the series converges to

$$\frac{f(\pi_-) + f(\pi_+)}{2}$$

where the terms $f(x_-)$ and $f(x_+)$ represent the left and the right limits of f at x, respectively.

By changing the set of functions $\{1\} \cup \{\cos nx, \sin nx\}_{n\geq 1}$ to the system $\{e^{inx}\}_{n=-\infty}^{\infty}$, we have the appropriate Fourier series associated with the new system

$$f(x) \sim \sum_{n=-\infty}^{\infty} c_n e^{inx} \tag{3}$$

where

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\,dx.$$

Another important result of Fourier Analysis is that each $f \in X$ satisfies Parseval's identity

$$\frac{1}{\pi}\int_{-\pi}^{\pi}(f(x))^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2). \tag{4}$$

Now we are in a position to present the first proof which depends on the Fourier theory.

Proof 1. *Consider $f(x) = |x|$. Using the Fourier coefficients defined in (1), we easily have $a_0 = \pi$,*

$$a_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ -\frac{4}{n^2\pi} & \text{if } n \text{ is odd} \end{cases}$$

and $b_n = 0$ because f is even (note that $f(x) = f(-x)$). Therefore, the Fourier series converges to $f(x)$ on $[-\pi, \pi]$ and so

$$|x| = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\cos(nx)}{(2n+1)^2}, \quad |x| \leq \pi.$$

In particular, for $x = 0$ we have

$$\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

and we are done. □

Remark. Some other choice of $f(x)$ can also lead to the conclusion. For example one can check this by choosing $f(x) = x^2$.

Remark. As a by-product of the method of Fourier series, the value of $\zeta(4)$ can be obtained directly from Parseval's identity. In fact, for $f(x) = |x|$,

$$\frac{1}{\pi}\int_{-\pi}^{\pi}(f(x))^2\,dx = \frac{2\pi^2}{3}$$

and, since $a_0 = \pi$ and

$$a_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ -\frac{4}{n^2\pi} & \text{if } n \text{ is odd} \end{cases}$$

(4) becomes

$$\frac{2\pi^2}{3} = \frac{\pi^2}{2} + \frac{16}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^4},$$

whence

$$\sum_{n=0}^{\infty}\frac{1}{(2n+1)^4} = \frac{\pi^4}{96}. \qquad (5)$$

Again, as

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \sum_{n=1}^{\infty}\frac{1}{(2n+1)^4} + \sum_{n=1}^{\infty}\frac{1}{(2n)^4},$$

it is easy to see from (5) that

$$\zeta(4) = \frac{\pi^4}{90}. \qquad (6)$$

Exercise 1. *Using Theorem 4, show that*

$$\zeta(6) = \frac{\pi^6}{945}$$

by choosing a proper $f(x)$.

Proof 2. *Our second proof uses area integral. Recall that*

$$\frac{1}{n^2} = \int_0^1\int_0^1 (xy)^{n-1}\,dx\,dy = \left(\int_0^1 x^{n-1}\,dx\right)\left(\int_0^1 y^{n-1}\,dy\right).$$

By the monotone convergence theorem [2, p. 307], we obtain

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \int_0^1\int_0^1\frac{dxdy}{1-xy}.$$

By change of variables

$$u = \frac{x+y}{2} \quad \text{and} \quad v = \frac{x-y}{2}$$

we get

$$\zeta(2) = 2\iint_S \frac{dudv}{1-u^2+v^2},$$

where S is a square with vertices

$$(0,0), (1/2,1/2), (1,0), (1/2,-1/2).$$

From the symmetry of the region, we get

$$\zeta(2) = 4\int_0^{1/2}\int_0^u\frac{dvdu}{1-u^2+v^2} + 4\int_{1/2}^1\int_0^{1-u}\frac{dvdu}{1-u^2+v^2}.$$

Integration of the above expression yields the required result [3]. □

Proof 3. *As in the previous proof, we can get another representation as*

$$\zeta(2) = \frac{4}{3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = \frac{4}{3}\int_0^1\int_0^1\frac{dxdy}{1-x^2y^2}. \qquad (7)$$

Consider the transformation

$$u = \tan^{-1}\left\{x\sqrt{\frac{1-y^2}{1-x^2}}\right\}$$

and

$$v = \tan^{-1}\left\{y\sqrt{\frac{1-x^2}{1-y^2}}\right\}$$

so that

$$x = \frac{\sin u}{\cos v} \text{ and } v = \frac{\sin v}{\cos u}.$$

The Jacobian is given by $1 - x^2y^2$ *and, therefore, (7) is equivalent to*

$$\frac{3}{4}\zeta(2) = \int\int_{\Sigma} du dv$$

where $\Sigma = \{(u, v) : u > 0, v > 0, u + v < \frac{\pi}{2}\}$ *where the area of* Σ *is* $\pi^2/8$. *Hence the result follows.* □

Proof 4. *Recall that the Taylor expansion of the inverse sine function (which follows by integrating the series expansion for* $\frac{1}{\sqrt{1-x^2}}$*)*

$$\sin^{-1} x = \sum_{n=1}^{\infty} \frac{1.3\ldots(2n-1)}{2.4\ldots(2n)} \frac{x^{2n+1}}{(2n+1)}, |x| \le 1,$$

which for $x = \sin t$ *becomes*

$$t = \sum_{n=1}^{\infty} \frac{1.3\ldots(2n-1)}{2.4\ldots(2n)} \frac{(\sin t)^{2n+1}}{(2n+1)}$$

where $|t| \le \frac{\pi}{2}$. *Integrating the last equation from* 0 *to* $\frac{\pi}{2}$, *we get*

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}.$$

□

Proof 5. *As in the Fourier series method of proof, by using the Fourier series of* $f(x) = x(1-x)$ *we get*

$$a_n = \begin{cases} \frac{1}{6} & \text{if } n = 0 \\ \frac{1}{n^2\pi^2} & \text{if } n \ge 1 \end{cases}$$

and $b_n = 0$, *therefore, we have*

$$x(1-x) = \frac{1}{6} - \sum_{n=1}^{\infty} \frac{\cos 2xn\pi}{(n\pi)^2}.$$

The required conclusion follows if we put $x = 0$ *in the last equation.* □

Remark. If $f(x) = x^{2n}$, then by Fourier series method one can easily see that

$$x^{2n} = \frac{\pi^{2n}}{2n+1} + \sum_{m=1}^{\infty} a_m \cos mx$$

where

$$a_m = \frac{2}{\pi}\int_0^{\pi} x^{2n}\cos(mx)\,dx$$
$$= (-1)^{m+n}2(2n)!\sum_{k=1}^{n} \frac{(-1)^k}{(2k-3)!m^{2n-2k+2}}\pi^{2k-2}.$$

If $n = 1$, then

$$a_m = \frac{4(-1)^m}{m^2}$$

so that the Fourier expansion of the function is

$$x^2 = \frac{\pi^2}{3} + 4\sum_{m=1}^{\infty} \frac{(-1)^m \cos mx}{m^2}.$$

The proof for Theorem 2 follows if we let $x = \pi$ so that $\cos m\pi = (-1)^m$.

Proof 6. *We use the infinite product [4, 5]*

$$\sin \pi x = \pi x \prod_{n=1}^{\infty}\left(1 - \frac{x^2}{n^2}\right)$$

for the sine function. Comparing coefficients at x^3 *in the Maclaurin series immediately yields the required result.* □

Proof 7. *For a proof using calculus of residues see [4, 5] . Indeed we can evaluate* $\zeta(2)$ *by using the Cauchy residue formula by considering the meromorphic function*

$$f(z) = \pi z^{-2}\cot(\pi z).$$ □

Remark. Another proof follows by considering the coefficients at x in Mittag-Leffler's expansion

$$\pi \cot \pi x = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2x}{x^2 - n^2}.$$

We can also use the inequalities $\sin x < x < \tan x$ and $\cot^2 x < x^{-2} < 1 + \cot^2 x$ for $0 < x < \frac{\pi}{2}$ to find $\zeta(2)$. We leave this as a simple exercise to the reader.

For an another proof, we refer to [2]. In addition, there are a number of different proofs for evaluating $\zeta(2)$. For example, proofs involving iterated integrals, identity of Fejér Kernel, Gregory's formula are also available in the literature.

Bernoulli Numbers: The sequence of Bernoulli numbers $\{B_n\}_{n\ge 0}$ is defined as

$$(n+1)B_n = -\sum_{j=0}^{n-1}\binom{n+1}{j}B_j,\ B_0 = 1.$$

It is well known that these numbers are described as the coefficients in the Maclaurin expansion of $z/(e^z - 1)$. It follows that

$$g(z) := \frac{z}{e^z - 1} + \frac{z}{2} := \sum_{n=0, n\ne 1}^{\infty} \frac{B_n z^n}{n!} \text{ for } |z| < 2\pi. \quad (8)$$

Here $B_1 = -1/2$ where g has a removable singularity at $z = 0$. As g is obviously an even function of z. This observation shows that $B_{2n+1} = 0$ for $n \ge 1$. Now, we prove the following

Theorem 5. *If n is a positive integer, then* [6]

$$2(2n)!\zeta(2n) = (-1)^{n+1}(2\pi)^{2n}B_{2n}. \tag{9}$$

Proof. Recall that [4]

$$\sin x = x\prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2\pi^2}\right).$$

Logarithmic differentiation of the expression yields

$$\cot x = \frac{1}{x} - 2\sum_{n=1}^{\infty}\frac{x}{n^2\pi^2 - x^2}.$$

Expanding each term in the sum as a geometric series, we see that

$$\begin{aligned} x\cot x &= 1 - 2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{x}{n\pi}\right)^{2t} \\ &= 1 - 2\sum_{t=1}^{\infty}\zeta(2t)\left(\frac{x}{2\pi}\right)^{2t}. \end{aligned}$$

By the definition of the cotangent, we have

$$\cot x = \imath\frac{(e^{\imath x}+e^{-\imath x})}{(e^{\imath x}-e^{-\imath x})} = \imath\left(1+\frac{2}{e^{2\imath x}-1}\right)$$

and therefore by (8) we have

$$x\cot x = ix + \frac{2\imath x}{e^{2\imath x}-1} = 1 + \sum_{n=2}^{\infty}\frac{B_n(2\imath x)^n}{n!}.$$

As $B_{2n+1} = 0$, by (9) we equate the coefficients of x^{2n}, $n > 0$, to get

$$-2\zeta(2t)/\pi^{2t} = (-1)^t 2^{2t}B_{2t}/(2t)!$$

and the proof is complete. □

In the second part we shall continue our discussion by proving the Euler product representation.

Theorem 6. *For $s \in \Omega$, the infinite product $\prod_{p,\text{prime}}(1-p^{-s})$ converges and*

$$\zeta(s) = \prod_{p,\text{prime}}\left(1-\frac{1}{p^s}\right)^{-1}. \tag{10}$$

We shall later show that the two definitions of the zeta function, namely the Dirichlet series representation (1) and the Euler product representation (10) are equivalent. We shall also present the contour integral formulation of zeta function and its various properties such as its functional equation. Consequently, we would get that $\zeta(-2n) = 0$ when n is a positive integer.

References

[1] T. M. Apostol, Intoduction to Analytic Number Theory, Undergraduate Texts in Mathematics, Springer-Verlag, New York-Heidelberg, 1976.

[2] T. M. Apostol, Mathematical Analysis, Addison-Wesley Publishing company, 1997.

[3] T. M. Apostol, A proof that Euler missed: evaluating $\zeta(2)$ the easy way, *Math. Intelligencer* 5 (3)(1983), 59–60.

[4] S. Ponnusamy, Foundations of Complex Analysis, Second Edn., Narosa Publishing House, India, 2005, 524 pp. (Also by Alpha Science International Publishers, UK).

[5] S. Ponnusamy and H. Silverman, Complex Variables with Applications, Birkhäuser, Boston (In print), 2006, 495 pp.

[6] H.E. Rose, A Course in Number Theory, Oxford Science Publications, 1996.

12-day Instructional Workshop on Convexity in Discrete Structures

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Convexity is basically a branch of Geometry. It has numerous connections with the other areas of mathematics such as Analysis and Linear Algebra. One of the most attracting aspects of convexity theory is the large number of easily stated and intuitively appealing unsolved problems that it still contains. The significance of convexity theory is that it deals analytically with global properties of objects and enables us to deal with extremal problems. Many problems in convexity are cumbersome to formulate algebraically and the subject tends to encourage non-constructive methods. There is a vast amount of literature on convexity theory from different perspectives.

The 12-day instructional workshop aims to take stock of the latest developments on Convexity theory in Discrete structures, starting from fundamentals, by bringing many of the experts working in different aspects such as *convexity in graphs* and other related discrete structures, the notion of convexity in relationship with those of combinatorial optimization and computational convexity. The objective of the workshop is to motivate and equip the young scholars in India to do intensive research on various aspects of *Discrete Convexity* and to attempt solving challenging open problems and new problems in this area by collaborative research. Also, we intend to publish the proceedings of the workshop with articles on the latest developments in Discrete Convexity theory from leading experts, in addition to creative lecture notes.

25–30 outstation participants will be selected on an all-India basis. The selection will be based on the nature of the work already done in the field of Discrete Convexity and related areas as well as the deep interest shown by the prospective participants so as to encourage their interaction with the attending experts in the field. The idea of group discussion is to generate frontline research problems and identify active researchers who have a research agenda before them would be a part of the academic plan of the Workshop.

The following faculty has been tentatively drawn up for the programme:

1. Prof. H. M. Mulder, Erasmus University, Rotterdam, The Netherlands
2. Prof. Sandy Klavzar, University of Maribor, Slovenia
3. Prof. Victor Chepoi, Université de la Méditerranée, Marseille, France
4. Prof. S. B. Rao, Indian Statistical Institute, Kolkatta
5. Prof. C. Pandu Rangan, Indian Institute of Technology (Madras), Chennai
6. Prof. S. A. Choudum, Indian Institute of Technology (Madras), Chennai
7. Dr. B. D. Acharya, Department of Science and Technology, New Delhi
8. Prof. P. Shanmugaraj, Indian Institute of Technology, Kanpur
9. Prof. S. S. Sane, University of Bombay, Kalina, Mumbai
10. Prof. V. Swaminathan, Srinivasa Ramanujan Mathematical Research Center, S. N. College, Madurai
11. Prof. A Vijayakumar, Cochin University of Science & Technology
12. Dr. T. N. Janakiraman, NIT, Trichi
13. Dr. Manoj Changat, University of Kerala
14. Dr. Joseph Mathews, S.B. College, Changanaserri
15. Dr. K. S. Parvathy, St. Mary's College, Trichur
16. Prof. E. Sampathkumar, University of Mysore, Mysore

The main topics to be covered are:

1. Classical convexity
2. Abstract convexity
3. Combinatorial aspects of convexity
4. Transit functions and associated convexities
5. Betweenness
6. Convexities on graphs
7. Special classes of graphs in connection with convexities in graphs
8. Convexity and centrality notions in graphs
9. Computaional aspects of convexity in discrete structures
10. Convexity and optimization problems

Applications are invited for participation in the above Instructional Workshop as per format attached/provided below. Faculty and Research scholars from Universities, Colleges and Institutes interested/doing research in areas of Discrete Mathematics (those working on convexity theory and related areas will be given preference), are encouraged to apply. Candidates selected for participation will be paid TA (as per actuals limited to 3-tier AC train fare) subject to their being present throughout the duration of the Workshop.

The last date for receiving the applications is **20th February 2006**. Applications may be sent by email in advance. However, one copy of the application duly signed by the candidate and recommended by the Head/forwarding authority should be sent to

Dr. Manoj Changat,
Department of Futures Studies,
University of Kerala,
Trivandrum 695 034.
E-mail: changat@vsnl.com,
mchangat@gmail.com,
Tel:(Office) 0471-2305321, (Resi): 0471-2742309
(Mobile): 09847721782.

Registration Form

12-day Instructional Workshop
Convexity in Discrete Structures

(22 March–2 April, 2006)

1. Name:
2. Sex:
   Date of birth:
3. Designation:
   Official address:

   Phone: Fax:
   E-mail:
4. Mailing Address:
5. Academic qualifications (Bachelor's degree onwards; if you are having a Ph.D degree, mention the title of the thesis as well as the year of award of the degree):
6. Field of Interest:
7. Details of Research Work done if any:
   (Attach a list of publications with the name of the Journal, coauthors and other details)
8. Number of years of Teaching experience and research experience:
9. Whether studied Discrete Convexity theory or related areas: Yes/No
10. a) Do you wish to present a paper on this topic? Yes/No
    b) If 'yes' then give the following details:
    b.1: Title
    b.2: Abstract
    b.3: Do you wish to submit it for consideration of its inclusion in the proceedings to be published? Yes/No
11. Do you need accommodation? Yes/No
12. Expected date & time of arrival:
13. Expected date & time of departure:

Date:

Place:

Signature of the Applicant.

Recommendations of the Head/Forwarding authority
Signature of the Head/Forwarding authority

(With seal, designation and address)

An International Conference on Nanoscience & Technology (ICONSAT 2006) is being organized at India Habitat Centre, New Delhi during March 16–18, 2006 under 'Nano Science and Technology Initiative (NSTI)' programme of the Department. The conference will bring together well known scientists from India and abroad for an in depth discussion regarding the growth direction of this field and will provide a unique opportunity to the young scientists to interact with front-ranking scientists. It is an important occasion to exchange policies and identifying areas of future support.

For further details please log on to:

http://www.iconsat2006.com

Report on the International Workshop in Commutative Algebra and Algebraic Geometry Conducted from July 18–23, 2005

Center for Post-graduate Studies and Research in Mathematics, St. Joseph's College, Irinjalakuda and Kerala Mathematical Association jointly organized a six-day International Workshop on Commutative Algebra and Algebraic Geometry from July 18–23, 2005. The programme was sponsored by National Board of Higher Mathematics (DAE). The Workshop was well-attended by teachers, research-scholars and PG students from Universities and affiliated colleges all over India.

The Workshop was inaugurated formally by Prof. P. Jothilingam, formerly Head of the Department of Mathematics, Pondicherry Central University in a meeting presided over by Dr. Sr. Ranjana, the Principal of the college. Prof. T. Thrivikraman, formerly Head of the Department of Mathematics, Cochin University of Science and Technology offered felicitations. The proceedings of the National Seminar conducted by the Mathematics Department in March 2003 was released on this occasion.

The following is a list of the experts and the topics they dealt with in the Workshop:

(1) Professor M. I. Jinnah (Kerala University): Introduction to rings and modules

(2) Professor P. Jothilingam (Formerly of Pondicherry University): Local Cohomology, sheaf cohomology

(3) Dr. T. Duraivel (Pondicherry University): Completion, dimension theory and integral dependence

(4) Dr. S. Mangayarcarassy (Pondicherry Engineering College): Primary decomposition, chain conditions and Noetherian rings

(5) Professor Markus Brodmann (Institut fur Mathematik, Universitat Zurich): Introduction to local cohomology with geometric examples

(6) Professor R. Sridharan (Chennai Mathematical Institute): Riemann-Roch theorem

(7) Professor K. R. Nagarajan (Chennai Mathematical Institute): Riemann-Roch theorem

(8) Professor R. S. Chakravarthi (Cochin University of Science and Technology): Localisation in non-commutative rings

(9) Professor H. N. Ramaswamy (Mysore University)

(10) Dr. Tony Joseph Puthenpurakal (IIT Bombay): On a filtration of the canonical modules

(11) Dr. C. V. R. Harinarayanan: Complementary Domination Complexes and Homology

The Workshop concluded on July 23, 2005 with an evaluation session in which the participants and resource persons expressed their suggestions and comments. Dr. Sr. Lilly P. L, local convenor of the workshop proposed a formal vote of thanks.

Twentieth Annual Conference of the Ramanujan Mathematical Society

—A Report

The Department of Mathematics, University of Calicut, had the privilege to host the **Twentieth Annual Conference of the Ramanujan Mathematical Society** from 27th to 30th July 2005. The theme chosen for this conference was **Algebraic Topology**. As is customary, the Conference was preceded by a two-days Workshop on the same topic on 25th and 26th July 2005.

The Ramanujan Mathematical Society (RMS) was founded in 1985 with the purpose of promoting mathematics at all levels. Since 1986 the society has been holding Annual Conferences. They also publish a Journal on Mathematics called the "Journal of Ramanujan Mathematical Society". Further, the society brings out a Mathematics Newsletter periodically.

The Conference was inaugurated by Prof. Syed Iqbal Hasnain, the Honourable Vice Chancellor of the University. In his speech Vice-Chancellor offered to present a portrait of Srinivasa Ramanujan to the RMS, to be kept at the proposed office of the Ramanujan Mathematical Society at Chennai, as a gift from the University of Calicut. The Key-note Address was delivered by the Patron of the Society Prof. M. S. Raghunathan, FRS (TIFR, Mumbai). The Technical Address in the inaugural session was delivered by the President of the Society Prof. R. Parimala (TIFR, Mumbai). There was a special lecture, called the Ramanujan Lecture, by the eminent Mathematician Prof. C. S. Seshadri, FRS (CMI, Chennai). There were four invited lectures, by Prof. R. Sridharan (CMI, Chennai), Prof. T. Thrivikraman (Cochin), Prof. Kumar Murty (Canada) and Prof. R. Tandon (University of Hyderabad). There were five endowment lectures. The Prof. M.N. Gopalan Endowment Lecture was delivered by Prof. Manjul Bhargava (Princeton). Prof. J. N. Kapur Memorial Endowment Lecture was delivered by Prof. Adimurthy (TIFR, Bangalore). Prof. C. S. Venkataraman Memorial Lectures I & II were delivered by Prof. Hans Gill (Punjab University, Chandigarh) and Prof. Venketesh Raman (IMSc, Chennai) respectively. Prof. S. G. Dani (TIFR, Mumbai) delivered Prof. W. H. Abdi Memorial Lecture.

Through the seven lectures delivered at the Workshop on 25 & 26 July 2005 a first course on Algebraic Topology was unfolded at a rapid pace by **Prof. Datta** (IISC, Bangalore), **Prof. Satya Deo** (HRI, Allahabad) and **Prof. Naolekar** (ISI, Bangalore). These lectures served has a background for the Thematic Lectures at the Conference. The Thematic Lectures were delivered by **Prof. P. Sankaran** (IMSc, Chennai), **Prof. Parameswaran** (TIFR, Mumbai) and **Prof. H. K. Mukherjee** (NEHU, Meghalaya) during the Conference. In this series there were nine lectures devoted to the theme and of advanced nature.

In addition to these lectures there were three paper presentation sessions by researchers in which 16 papers in different areas of Mathematics were presented.

The details of the entire programme of lectures and paper presentations are appended below.

During the Conference the Annual General Body Meeting of the Ramanujan Mathematical Society was held on 28th July at 5.30 pm. At this meeting the activities of the past were reviewed and suggestions for the future activities were discussed. It was decided to hold the 21st Ramanujan Mathematical Society Annual Conference at the University of Hyderabad during June–July, 2006. The theme chosen for the Conference is **Number Theory**. It was also decided to have a change in the duration of the Workshop and Thematic Lectures. Instead of the present practice of holding two days of Workshop and nine Thematic Lectures during the Conference henceforth there will be three days of Workshop devoted to the theme and three days of Conference. There will be no specific theme for the Conference as such. A new set of office bearers for the Society was also finalised.

This Department proposes to publish the lectures delivered at the Conference at the earliest.

In connection with the Conference a cultural programme was organised on 27th July 2005 evening. Shri. Harigovindan, a noted "Sopana Sangeetham" exponent rendered Kritis from Bhajagovindam. This was followed by a dance programme by Smt. Jayasree and her party.

The Conference was finaced by the National Board for Higher Mathematics (NBHM), the Department of Science and Technology (DST), the Council of Scientific and Industrial Research (CSIR), the Kerala State Council for Science Technology & Environment, Kerala (KSCST), and the University of Calicut.

On behalf of the Ramanujan Mathematical Society and the Department of Mathematics, University of Calicut, we thank the NBHM, DST, CSIR, KSCST and the University of Calicut for the generous financial support which helped make the Conference a grand success.

We also place on record the generous help and encouragement received from the Vice-Chancellor, Administration of the University of Calicut and the teachers and the students towards the conduct of the Conference.

V. Krishna Kumar,
Local Organising Secretary,
20th Annual Conference of the
Ramanujan Mathematical Society

Prof. M. S. Raghunathan,
Differentiable Structures on Manifolds
TIFR, Mumbai
E-mail: msr@math.tifr.res.in (Held on 27.07.2005)

Ramanujan Lecture

Prof. C. S. Seshadri,
Geometric Reductivity
Chennai Mathematical Insti.
E-mail: css@cmi.ac.in (Held on 27.07.2005)

President's Lecture

Prof. R. Parimala,
TIFR, Mumbai
E-mail: parimala@math.tifr.res.in (Held on 27.07.2005)

Endowment Lectures

CSV Endowment Lecture 1

Prof. Hans Gill,
Sphere Packings, Coverings
Punjab University and View-obstruction
E-mail: hansgil@pu.ac.in (Held on 28.07.2005)

MNG Endowment Lecture

Prof. Manjul Bhargava,
The representation of integers
Princeton University by quadratic forms
E-mail: bhargava@math.princeton.edu (Held on 27.07.2005)

JNK Endowment Lecture

Prof. Adimurthy,
Conservation Laws
TIFR Centre, IISC Campus, Bangalore
E-mail: aditi@math.tifrbng.res.in (Held on 29.07.2005)

W. H. Abdi Endowment Lecture

Prof. S. G. Dani,
Cumulative Effect of Random
TIFR, Mumbai Linear Transformation
E-mail: dani@math.tifr.res.in (Held on 30.07.2005)

CSV Endowment Lecture 2

Prof. Venkatesh Raman,
Graph Theoretical Structural Institute of Mathematical
Sciences Results and Efficient Algorithms for Chennai
Cycle Hitting Set Problems
E-mail: vraman@imsc.res.in (Held on 30.07.2005)

Invited Lectures

Prof. R. Sridharan,
Bilinear Forms
IMS, Chennai (Held on 27.07.2005)

Prof. Rajat Tandon,
Distinguished Representations
University of Hyderabad
E-mail: rtsm@uohyd.ernet.in (Held on 28.07.2005)

Prof. T. Thrivikraman,
Kerala's Contribution to Mathematics
Cochin University of Science & Technology
E-mail: thekkedath@vsnl.net (Held on 28.07.2005)

Prof. Kumar Murty,
Ramanujan Graphs & Isogens of Elliptic Curves
Canada (Held on 30.07.2005)

Thematic Lectures

Prof. Parameswaran Sankaran,
Some Problems in Algebraic IMS,
Chennai and Differential Topology
E-mail: sankaran@imsc.res.in
(Held on 27, 28 & 29 July 2005)

Prof. H. K. Mukherjee,
Why Algebraic Topology & How?
North Eastern Hill University, Meghalaya
E-mail: himadri@nehu.ac.in (Held on 28 & 29 July 2005)

Prof. A. J. Parameswaran,
Topological Techniques in Algebraic
School of Mathematics Geometry and Singularity Theory
TIFR, Mumbai
E-mail: param@math.tifr.res.in
(Held on 28, 29 & 30 July 2005)

Workshop Lectures

Prof. Satya Deo,
Homology and Cohomology
Harish Chandra Research Institute
Allahabad
E-mail: sdeo@mri.ernet.in (Held on 25 & 26 July 2005)

Prof. B. Datta,
Fundamental Groups
IISC, Bangalore & Triangulations
E-mail: dattab@math.iisc.ernet.in
(Held on 25 & 26 July 2005)

Dr. A. Naolekar,
Manifold Theory
Indian Statistical Institute
Bangalore
E-mail: murty@math.toronto.edu
(Held on 25 & 26 July 2005)

M. T. & T. S. 2006
14th Mathematics Training and Talent Search Programme

Funded by

National Board for Higher Mathematics

Aim: The aim of the programme is to expose bright young students to the excitement of doing mathematics, to promote independent mathematical thinking and to prepare them for higher aspects of mathematics.

Academic Programme: The programme will be at three levels: Level O, Level I and Level II. In Level O there will be courses in Linear Algebra, Analysis and Number Theory/ Discrete Mathematics. In Levels I and II there will be courses on Algebra, Analysis and Topology. There will be seminars by students at all Levels.

The faculty will be active mathematicians with a commitment to teaching and from various leading institutions. The aim of the instructions is not to give routine lectures and presentation of theorem-proofs but to stimulate the participants to think and discover mathematical results.

Eligibility

Level O: Second year undergraduate (B. Sc./B. Stat./B. Tech. etc.) students with Mathematics as one of their subjects. Good first year students may also apply.

Level I: Final year undergraduate (B. Sc./B. Stat./B. Tech. etc.) students with Mathematics as one of their subjects.

Level II: First year postgraduate (M. Sc./M. Stat./M. Tech. etc.) students with Mathematics as one of their subjects.

Level O of the Programme will also be conducted at the Department of Mathematics, Jadavpur University, Kolkata during the period May 22–June 17, 2006 and at the Department of Mathematics, Panjab University, Chandigarh during the period May 29–June 24, 2006.

How to Apply?

Details and application forms can be had from the Head, Department of Mathematics, of your Institution. They can also be downloaded from MTTS Websites given below.

In case of difficulties, write to Professor S. Kumaresan by sending a self addressed and stamped (Rs. 10) envelope of size 10 cm × 2 cm to the address given below. Write "MTTS-2006 Application Form" on the cover of your letter. The completed application form should reach the Programme Director latest by 25th February, 2006.

Selection: The selection will be purely on merit, based on consistently good academic record and the recommendation letter from a mathematics professor closely acquainted with the candidate.

Only selected candidates will be informed of their selection by the end of March 2006. The list of selected candidates will be posted on the Websites of MTTS.

Candidates selected for the programme will be paid sleeper class return train fare by the shortest route and will be provided with free board and lodging for the duration of the course.

Venues and Duration: There are three venues for MTTS 2006. All the three levels will be held at the Department of Applied Mathematics, University Institute of Chemical Technology, University of Mumbai, Mumbai, during the period May 15–June 10, 2006.

For More Details Contact/Visit/Write

Professor S. Kumaresan
Programme Director, MTTS
Department of Mathematics
University of Mumbai
Vidyanagari, Kalina, Mumbai 400 098
http://math.mu.ac.in/mtts/index.html
http://www.geocities.com/mttsprogramme
E-mail: mttsprogramme@gmail.com

JAM–Joint Admisssion Test to M.Sc

JAM-Joint Admission Test to M.Sc is a common admission test for candidates seeking admission to the 2-year M.Sc programme in physics, chemistry & mathematics at IITS; and a 3-year post-B.Sc programmes (MCA & M.Tech in Applied Geology/Applied Geophysics) at IIT Roorkee & M.Sc.-Ph.D dual degree programme in Physics at IIT Kanpur. More details are available at: http://jam.iitd.ac.in

Mathematical Conference Announcements by Date

Details may be obtained from
http://www.math.helsinki.fi/EMIS/conf/by-date.html.

January 2006

January 02–05 Mathematics in the Twentieth Century
Site: Mathematical Sciences Foundation,
St. Stephen's College, Delhi (more...)

January 02–February 18 Arithmetic Algebraic Geometry
Site: Erwin Schrödinger International Institute
for Mathematical Physics (more...)

January 16–18 5th Annual Hawaii
International Conference on Statistics,
Mathematics and Related Fields
Site: The Renaissance Ilikai Waikiki Hotel (more...)

January 23–26 Recent Progress in
Wavelet Analysis and Frame Theory
Site: Bremen, Germany (more...)

January 23–28 C*-algebras and elliptic theory. II
Site: Banach Center, Bedlewo, Poland (more...)

January 25–February 05
The Russian winter diffiety school 2006
Site: Kostroma, Russia (more...)

February 2006

February 05–09 Winter School in
Complex Analysis and Operator Theory
Site: Antequera (Málaga), Spain (more...)

February 06–10 Recent Trends in Nonlinear Science 2006
Site: Gijon (Spain) (more...)

February 12–15 Fractal 2006
Site: Vienna (more...)

February 23–23 The Fourth International Conference on
Computer Application (ICCA 2006)
Site: Yangon, Myanmar (more...)

February 24–27 Lie groups: Dynamics,
Rigidity, Arithmetic (in honor of Gregory Margulis)
Site: Yale University, New Haven, USA (more...)

February 27–May 12
Diophantine Approximation and Heights
Site: Erwin Schrödinger International Institute
for Mathematical Physics (more...)

March 2006

March 01–July 31 Stochastic Analysis, Stochastic Partial
Differential Equations and Applications to Fluid Dynamics
and Particle Systems
Site: Centro Di Ricerca Matematica Ennio De Giorgi
(Pisa, Italy) (more...)

March 06–10 GAEL–Géométrie algébrique en liberté
Site: Bedlewo, Poland (more...)

March 13–17 3–manifolds After Perelman
Site: Heriot Watt University, Edinburgh (more...)

March 27–31 Mathematical Population Genetics
Site: Edinburgh (more...)

April 2006

April 02–09 Formal theory of partial
differential equations and their applications
Site: Mekrijärvi Research Station,
University of Joensuu, Finland (more...)

April 05–07 PICOF'06–Inverse Problems,
Control and Shape Optimization (Third edition)
Site: Nice, France (more...)

April 10–13 58th British Mathematical Colloquium
Site: University of Newcastle,
Newcastle upon Tyne, UK (more...)

April 23–May 07 Rigidity and Flexibility
Site: Erwin Schrödinger International Institute
for Mathematical Physics (more...)

May 2006

May 05–12 Mathematical Models of the Heart
Site: Longyearbyen, Svalbard (more...)

May 06–11 International conference on Fourier and Complex analysis, classical problems–current view
Site: Protaras, Cyprus (more...)

May 08–July 31
Gerbes, Groupoids, and Quantum Field Theory
Site: Erwin Schrödinger International Institute for Mathematical Physics (more...)

May 08–19
CANT' 2006: International School and Conference on Combinatorics, Automata and Number Theory
Site: University of Liége, Belgium. (more...)

May 17–19 Conference of Applied Statistics in Ireland
Site: Cork, Ireland (more...)

May 22–27 Lie days in Martina Franca
Site: Martina Franca, Italy (more...)

May 30–June 06 NAFSA 8–8th
International Spring School on Nonlinear Analysis, Function Spaces and Applications
Site: Prague, Czech Republic (more...)

June 2006

June 01–03 International Conference on Computers and Communications
Site: Baile Felix Spa, Oradea,Romania (more...)

June 04–10 Workshop on Commutative Rings
Site: Cortona, Italy (more...)

June 07–09 Boltzmannś Legacy 2006
Site: Erwin Schröedinger Institute for Mathematical Physics (more...)

June 07–10 Recent advances in nonlinear partial differential equations and applications: A workshop in honor of Peter D. Lax and Louis Nirenberg
Site: Toledo, Spain (more...)

June 09–14 Eighth International Conference on Geometry, Integrability and Quantization
Site: Sts. Constantine and Elena resort (near Varna), Bulgaria (more...)

June 11–14 ICMSE 2006 International Congress in Maths, Sciences and Science Education
Site: University of Aveiro, Aveiro, Portugal (more...)

June 12–16 10th Asian Solid State Ionics Conference
Site: Kandy, Sri Lanka (more...)

June 12–16 Workshop on Current trends in Nonlinear Analysis, dedicated to Professor Dino Fortunato on the occasion of his 60th birthday
Site: Otranto, Italy (more...)

June 12–16 Function Theories in Higher Dimensions
Site: Tampere University of Technology, Tampere, Finland, (more...)

June 13–16 SDS 2006–Structural Dynamical Systems: Computational Aspects.
Site: Hotel Porto Giardino, Capitolo, Monopoli, Italy (more...)

June 19–23 Modern stochastics: theory and applications
Site: Kyiv, Ukraine (more...)

June 19–23 Formal Power Series and Algebraic Combinatorics 2006
Site: San Diego, California, USA (more...)

June 19–23 Quantile Regression, LMS Method and Robust Statistics in the 21st Century
Site: Edinburgh, UK (more...)

June 21–23 Mathematical Problems in Engineering and Aerospace Sciences
Site: Budapest, Hungary (more...)

June 26–30 Applied Asymptotics and Modelling
Site: Edinburgh, UK (more...)

July 2006

July 02–07 ICOTS 7,
International Conference on Teaching Statistics
Site: Salvador (Bahia), Brazil (more...)

July 05–08 Numerical Analysis and Approximation Theory NAAT 2006
Site: Cluj–Napoca, Romania (more...)

July 06–08 4th Portuguese Finance Network (PFN) Finance Conference
Site: Porto, Portugal (more...)

July 07–10 2nd International Conference "From Scientific Computing to Computational Engineering" (2nd IC–SCCE 2006)
Site: ATHENS, GREECE (more...)

July 09–22 Horizon of Combinatorics
Site: Budapest, Hungary (more...)

July 10–14 New Directions in Applied Probability: Stochastic Networks and Beyond
Site: Edinburgh, UK (more...)

July 10–14 CMPI–2006 Campus Multidisciplinar en Percepcióne Inteligencia
Site: Albacete, Spain (more...)

July 10–15 Conference on Recent Developments in the Arithmetic of Shimura Varieties and Arakelov Geometry
Site: Centre de Recerca Matemática (Bellaterra, Barcelona) (more...)

July 17–19 Geometric Aspects of Integrable Systems
Site: Coimbra–Portugal (more...)

July 17–21 Extremal Kähler Metrics and Stability
Site: Edinburgh, UK (more...)

July 24–28 2nd SIPTA School on Imprecise Probabilities
Site: Madrid, Spain (more...)

August 2006

August 07–11 Partial Differential Equations on Noncompact and Singular Manifolds
Site: University of Potsdam, Germany (more...)

August 07–12 Algebraic Theory of Differential Equations
Site: Edinburgh, UK (more...)

August 13–19 Workshop on Triangulated Categories
Site: University of Leeds (more...)

August 14–18 International Conference on Spectral Theory and Global Analysis
Site: Carl von Ossietzky University, Oldenburg, Germany (more...)

August 22–30 International Congress of Mathematicians (ICM 2006)
Site: Madrid, Spain (more...)

September 2006

September 01–04 Topics in Mathematical Analysis and Graph Theory
Site: Belgrade, Serbia and Montenegro (more...)

September 01–04 Conference on Mathematical Neuroscience
Site: Universitat d'Andorra. Principat d'Andorra (more...)

September 02–05 37th Annual Iranian Mathematics Conference
Site: Azarbaijan University of Tarbiat Moallem (more...)

September 04–08 Barcelona Analysis Conference (BAC06)
Site: Barcelona (Spain) (more...)

September 04–29 The Painlevé Equations and Monodromy Problems
Site: Isaac Newton Institute for Mathematical Sciences, Cambridge, UK (more...)

September 05–08 CDDE 2006, Colloquium on Differential and Difference Equations
Site: Brno, Czech Republic (more...)

September 11–15 Groups of Diffeomorphisms 2006
Site: University of Tokyo, Japan (more...)

September 19–21 Credit Risk under Lévy Models
Site: Edinburgh, UK (more...)

September 21–24 Fifth International Conference on Applied Mathematics (ICAM5). In Honour of Professor Ioan A. Rus with the occasion of his 70th birtday
Site: North University of Baia Mare, Baia Mare, Romania (more...)

October 2006

October 23–December 15
Stochastic Computation in the Biological Sciences
Site: Isaac Newton Institute for Mathematical Sciences, Cambridge, UK (more...)

2007

January 2007

January 08–June 29
Analysis on Graphs and its Applications
Site: Cambridge, UK (more...)

January 15–July 06 Highly Oscillatory Problems: Computation, Theory and Application
Site: Cambridge, UK (more...)

July 2007

July 23–December 21
Strong Fields, Integrability and Strings
Site: Cambridge, UK (more...)

September 2007

September 03–December 21
Phylogenetics
Site: Cambridge, UK (more...)

National Conference on Recent Advances in Analysis and its Applications (Conducted under UGC, DRS-SAP-I)

March 22–24, 2006

Announcement and Call for Papers

Organised by

Department of Mathematics
Karnatak University
Dharwad 580 003

Venue

Department of Mathematics
Karnatak University
Dharwad 580 003

Theme and Objectives: The national conference will have invited talks and paper presentations. it provides an opportunity to acquaint with recent advances in analysis and its applications and it will be an occasion for youngsters to learn from eminent mathematicians doing pioneering work in their respective fields.

Invited Speakers: Eminent research workers from TIFR, Bombay, Bangalore and IIT Centers and other reputed Universities will be delivering invited talks.

Call for Papers: Abstracts of Contributed Papers not exceeding 300 words should be sent by February 15, 2006.Young researchers working in the fields of Real Analysis, Complex Analysis, Functional Analysis, Differential Geometry, Differential Equations etc. are encouraged to send their research articles for presentation at the National Conference. However, researchers in other fields can also send their research papers for presentation to:-

Dr. S. S. Bhoosnurmath
Organizing Secretary
National Conference on Research Advances in Analysis and it's Applications
Department of Mathematics
Karnatak University,
Dharwad 580 003

Registration: Those who are interested in participating in the National Conference are requested to fill up the enclosed registration form and send it to the Organizing Secretary before February 15, 2006, along with the Registration Fee of Rs. 500/- through D.D. in the name of the Organizing Secretary, "National Conference on Recent Advances in Analysis and its Applications", payable at Dharwad.

Accommodation

Accommodation will be arranged in the University Guest House or Hostel or in a nearby Hotel.

Financial Assistance

With meager financial assistance we are conducting the National Conference. So the participants are advised to get financial assistance from their respective institutions. However a few deserving participants may get partial support from the organizers.

Dharwad is well connected by road and rail. It is about 500 kms by rail from both Poona and Bangalore. University Guest House is around 4 kms from both Bus Stand and Railway Station. Delegates are requested to make their return journey reservations at the starting station itself.

Registration Fee:

Rs. 500/- for participants.
Rs. 400/- for accompanying persons.

Registration Form

1. Name ....
2. Official address ....

   ....

   ....

   Ph No. .... E-mail ....

   ....

3. Residential address ....

   ....

   ....

   Ph. No. .... Mobile No. ....

4. Paper presentation: Yes/No. If Yes

   (i) Name(s) of joint author(s) ....

   (ii) Title of the paper ....

5. Accommodation required: Yes/No ....
6. (i) Expected date and time of arrival at Dharwad

   (ii) Mode of Journey: Train/Bus

7. Whether return journey reservation required?

   Yes/No. Amount sent Rs. ....

8. Accompanying persons:

   Name:

   Relationship:

Signature

Please send the Registration form along with the Registration fee to the Organizing Secretary Dr. S. S. Bhoosnurmath on or before February 28, 2006.

Enquires and Address for Correspondence

Prof. S. S. Bhoosnurmath
Coordinator, UGC/SAP
Organizing Secretary
National Conference on Recent Advances in Analysis and its Applications
Department of Mathematics
Karnatak University, Dharwad 580 003
Telephone: 0836-2215222
Mobile: 9845277305
Fax: 91-0836-2747884
E-mail: ssbmath@yahoo.com

(and)

Prof. V. A. Hiremath
Dy. Coordinator, UGC/SAP
Convener (address as above)
E-mail: va_hiremath@rediffmail.com

International Conference on Special Functions and their Applications

February 21–23, 2006

Department of Mathematics
University of Pune
Pune 411 007 (India)

About the University and the Department: The University of Pune, established in 1948, is one of the leading centers for research and teaching in the country. The beautiful 400 acre campus is located in the North Western part of Pune. The placid environs and state of the art facilities provide its numerous students with ideal atmosphere to pursue research in various areas of Science, Arts, Commerce and Language. The university houses 40 departments which provide a wide array of academic programs.

The Department of Mathematics, established in 1950, offers M.A./M.Sc, M. Phil and Ph. D. Programs in Mathematics. The Department has made significant contributions in the field of Analysis, Algebra, Relativity and Discrete Mathematics.

The Department has its own rich Library and has electronic access to MathScinet as well as all Mathematics Journal of Academic Press.

About the Conference: The three day International Conference on Special Functions and their Applications will bring together the Mathematicians working in the area of special functions for interaction and exchange of ideas. In addition, it will inspire young researchers to peruse research in this important branch of mathematics.

The conference is organized in the honour of Prof. T. T. Raghunathan who is completing 68 years in April 2006. Prof. Raghunathan has been a very influential teacher and has devoted more than three decades of his life for the enrichment of the Department of Mathematics.

Invited Speakers

R. P. Agarwal (Lucknow University, Lucknow)
P. N. Rathie (Brazil)
P. E. Ricci (Italy)
N. K. Thakare (Dhule)
M. A. Pathan (A. M. U., Aligarh)
R. S. Pathak (B. H. U., Varanasi)
K. Shrinivas Rao (Inst. Math. Sc., Chennai)
P. K. Banerji (JNV Uni., Jodhpur)
K. C. Gupta (MNIT, Jaipur)
Arun Varma (IIT, Roorkee)
S. Bhargava (Mysore Uni., Mysore)
T. B. Jagtap (Dean, Science faculty, Shivaji Uni., Kolhapur)
D. K. callebaut (University of Antwerp, Belgium)

Call for Papers: The organizers of the Conference invite full length papers for presentation in the Conference. The papers may be sent (preferably by E-mail) to:

Prof. M. A. Pathan
5, Kabir Colony
Anupshehar Road
Aligarh 202 002 (India)
E-mail: mapathan@gmail.com
on or before February 7, 2006.

The Papers are also Invited for the Following Prizes

A. K. Agarwal Prize for Best Publication of the Year carrying a certificate and a cash award of Rs. 1,000/- will be awarded to an Indian mathematician (below 45 year of age) working at an Indian University/Institute for his paper published/accepted in the preceding year of the Conference in the areas influenced by Ramanujan which includes special functions, number theory, q-series, mock theta functions etc. Applicants for this prize should send five copies of their papers to Prof. A. K. Agarwal, Center for Advance Study in Mathematics, Punjab University, Chandigarh 160 014 (aka@pu.ac.in) along with the copy of the acceptance letter (if the paper has been accepted but not published) by February 5, 2006.

Aruna Gupta Prize and M. I. Qureshi Prize for **Best Paper Presentation**, each carrying a certificate and a cash award of Rs. 500/- will be given to an Indian mathematician (below 35 years of age). Three copies of the paper to be considered for the prize along with the abstract may be sent to Prof. M. A. Pathan by February 5, 2006. At the top of each copy of the paper the name of the prize should be indicated. The papers to be presented for consideration for prize should be under single authorship. The stress will be given on both quality of research as well as performance during presentation.

Registration Fee

Students/Research Scholars	:	Rs. 500/-
Delegates (Indian)	:	Rs. 700/-
(Foreign)		US $ 100
Accompanying persons	:	Rs. 500/- per person

The Registration fee should be paid by demand draft drawn in favour of "The Registrar, University of Pune, Pune". The Registration form along with the Registration fee must reach the Local Organizing Secretary by January 30, 2006 so as to enable the organizers to make proper arrangements. The Registration fee includes boarding and lodging which will be arranged on the campus of the University.

For any other information or inquiry regarding the conference, participants may contact:

Dr. M. M. Shikare
Local Organizing Secretary (ICSFA–2006)
Department of Mathematics
University of Pune, Pune 411 007
Phone: 020-25601272
Mobile: 9423206784
E-mail: mms@math.unipune.ernet.in

Registration Form

International Conference on Special Functions and their Applications

Department of Mathematics
University of Pune, Pune 411 007

(To be returned by January 30, 2006)

1. Name :
   Designation : M/F :
2. Address for correspondence :
3. Telephone No :
   E-mail :
4. Title of the talk/Contributory paper :
5. Whether Accommodation is required : Yes/No
   (i) Date of Arrival:
   (ii) Date of Departure:
   (iii) Mode of Travel:
6. Names of the accompanying persons (if any) :
7. Details of Demand Draft enclosed as Registration Fee :
   (i) Demand Draft No:
   (ii) Amount Rs:
   (iii) Charges for accompanying person : Rs.
   Place:

   Date: Signature

International Conference on Management Sciences

Optimization Models and Applications in Honor of Prof. Suresh Sethi, Ashbel Smith Professor of the School of Management of Universty of Texas, Dallas May 20 – 22, 2006 UT Dallas, Richardson, Texas.

Keynote Speaker of the Conference: Prof. Harry Markowitz, 1990 Economics Nobel laureate Prof. Suresh Sethi, a graduate of Indian Institute of Technology Bombay and Carnegie Mellon University, and a Fellow of the Royal Society of Canada, has made fundamental contributions to operations management, finance, marketing, and industrial engineering.

In his 60th year, his students, postdocs and colleagues, and UTD are hosting a conference in his honor. It starts on May 20 (Saturday) evening and continues to the afternoon of May 22, 2006 (Monday).

The conference will include plenary lectures and paper presentations on optimization models in management science. Topics include optimal control theory and applications, control and game models in marketing, economics and financial engineering, flexible manufacturing, scheduling, inventory and supply chain management.

Please see the Conference Web Page:

http://www.se.cuhk.edu.hk/icoso/ for further details and the schedule.

Name of the Candidate	:	Rahul Jain
Degree Registered	:	Integrated Ph.D.
Department	:	Department of Mathematics, Indian Institute of Science, Bangalore, INDIA.
Research Supervisors	:	Dr. B.R.Nagaraj, Dr. Sameer Jalnapurkar
Title of the Thesis	:	Regularity and Propagation Phenomenae in some Linear and Non-Linear Partial Differential Equations with Particular Reference to Microlocal Analysis.

Synopsis

This thesis is addressed to solving some challenging questions relating to solutions of some very important Partial Differential Equations which arise both in theory and practice. The thesis solves the important question (as this relates to image processing as well) of whether the unique viscosity

Lipschitz solution to the Dirichlet Problem for Laplacian infinity in n-dimensions is C^1, by showing that it has infact $C^{(1,1/3)}$-Hölder regularity. In the process, the thesis obtains a a fundamental result of a valid product structure between L^2 functions and distributions in the convolution space θ'_c, wherein the Hörmander product condition is not necessarily verified. We demonstrate the profitable application of this fact to non-linear PDEs. The thesis obtains the 'doubling phenomenon' in the Sobolev regularity, compared to the $H^{1/2}$ 'smoothing' of P. L. Lions et al for moments/velocity average of solutions to the Boltzmann transport equation $(\partial_t + \xi \cdot \nabla_x)u = Q(u, u)$; the procedure to obtain the extra smoothing on a scale of Sobolev exponents uses a novel method of allowing a principal axes transformation on the velocity variable ξ. The thesis also obtains the cancellation of the propagation of a newly formed singularity in some semi-linear hyperbolic problems by second order Fuchsian perturbation. Techniques of linear PDEs and methods of Microlocal Analysis have been used for the above solution procedures. The thesis also obtains new propagation results for the pseudodifferential operator class $M_{\lambda,\beta} = tD_t^2 + \lambda(x, D_x)D_t + \beta(x, D_x)$ under the 'interaction condition' $\mathrm{WF}(\lambda) \cap \mathrm{WF}(\beta) \neq \phi$. Microlocal arguments are also given for a unification procedure under which the 'reduction theorems' of M. Kuranishi and Ju. V. Egorov are naturally related.

The following papers are written based on the material embodied in this thesis:

(1) (With Dr. B. R. Nagaraj) $C^{(1,1/3)}$-regularity in the Dirichlet problem for Δ_∞ (submitted for publication)

(2) (With Dr. B. R. Nagaraj) Propagation and Cancellation of Singularities in a class of Fuchsian operators and their Perturbations (submitted for publication)

(3) (With Dr. B. R. Nagaraj) On distribution products without Hörmander's condition and applications to non-linear PDEs (submitted for publication)

(4) (With Dr. B. R. Nagaraj) Microlocal Non-Elliptic Regularity in the Moments of Solution to the Boltzmann Transport Equation (submitted for publication)

(5) (With Dr. B. R. Nagaraj) On the link between the reduction theorems of Kuranishi and Egorov in Microlocal Analysis (submitted for publication)

DST Sponsored IV SERC School on Special Functions and Functions of Matrix Argument: Recent Advances and Applications in Stochastic Processes, Statistics and Astrophysics

(6th March–7th April, 2006)

Organised by

Centre for Mathematical Sciences,
Thiruvanathapuram and Pala Campuses
(CMS)

All India Selection: 30 Seats

All Expenses Met By
Department of Science & Technology
Government of India
New Delhi

Eligibility: Young faculty below 35 years at any college or university or other institutions in India, Ph.D. degree holders, Post-doctoral fellows, others interested in Research.

Minimum Qualification:

B. Sc. (Mathematics), M. Sc. in Mathematics/Statistics/ Theoretical Physics – first classes throughout.

Closing Date of Application: 25 January 2006

Theory of special functions or transcendental functions was originally developed for tackling problems in applied mathematics, physics and engineering. Generalized hypergeometric functions, G and H-functions were developed for dealing with applications in Statistics, Econometrics, Physics, Engineering and other areas, such as energy generation in stars, communication problems, radar, tracking devices, signal suppression, input-output models, reaction-diffusion problems and so on.

Later developments include multivariable and matrix variable functions and their applications to many areas.

CMS had conducted one 6-week SERC School in 1995, another 5-week SERC School in 2000 and a third 5-week SERC School in 2005. The next one is the **4th SERC School**, a 5-week School to be held from **6th March to 7th April 2006.**

Topics to be covered will be the following: gamma, beta, zeta functions; hypergeometric functions, generalized hypergeometric functions, Meijer's G and Fox's H-functions, q-hypergeometric functions and Ramanujan's work. Functions of matrix argument and Jacobians of matrix transformations. Fractional derivatives, fractional integrals, Mittage-Leffler functions, reaction-diffusion problems. Applications in statistical distribution theory, stochastic processes and time series, astrophysics problems such as stellar models, energy generation, gravitational instability etc.

(It is a very seriously run School and all participants of the first three Schools were highly benefited – a good number of them finishing Ph.D.'s and publishing many research papers. Only serious persons who would like to learn need to apply for admission.)

Faculty

Top research workers in these areas will be the faculty.

Main Faculty for the 4th SERC School

Dr. H. J. Haubold (United Nations)
Dr. A. M. Mathai (Canada)
Dr. S. Bhargava (Mysore, India)
Dr. R. K. Saxena (Jodhpur, India)

Local Faculty Include

Dr. A. Sukumaran Nair (CMS)
Dr. K. S .S. Nambooripad (CMS)
Dr. R. N. Pillai (CMS)
Dr. K. K. Jose (M. G. University)
Dr. R. Y. Dennis (Gorakhpur University)
Dr. Yageen Thomas (Univ. of Kerala)
Dr. K. Jayakumar (Calicut University)

Lectures

(Monday through Friday)

First lecture	08.30–10.30
Second lecture	14.00–16.00
First problem session	10.30–13.00
Second problem session	16.00–18.00

Note: No Lectures on Saturdays and Sundays. Attendance in every lecture and every problem session is compulsory. No part-time attendance. Class tests every week. Cumulative grades will appear on the certificate. The best five are usually given DST pre-doctoral or post-doctoral fellowships. For Indian participants all expenses will be met by CMS, including to and fro second class train travel, local accommodation, food, study materials, stationery etc. Foreign participants must come with return international air tickets. Their local hospitality and study materials will be met by CMS.

The **4th SERC School** will be held in the picturesque Pala (Kerala) area, in calm and quiet atmosphere. Admission is open to all with the minimum qualifications irrespective of nationality, sex, caste or creed.

Apply on plain paper with all the following details (if details are incomplete such applications will not be considered): Name, age, male/female, marital status, full address, E-mail, phone number, copies (not originals) of all certificates and mark lists, one paragraph detailing why you wish to participate plus **permission certificate if employed** and a self addressed empty envelope with Rs. 5/- stamp affixed if acknowledgement is required.

Last date for receiving the application is 25-01-2006.

Address for Correspondence

Centre for Mathematical Sciences Pala Campus
St. Thomas College, Pala
Arunapuram P. O., Pala
Kottayam District, Pincode 686 574
Kerala State, India
Phone: 91+ 4822-201288,
(04822-201288, 216493)
E-mail: statpala@sancharnet.in, statpala@yahoo.com
Fax: 91+ 4822-216313

University of Pune
Department of Mathematics
National Workshop on Connectivity in Graphs

(18th February–21st February, 2006)

(Announcement)

A National Workshop on Connectivity in Graphs is going to be held at the department of Mathematics. University of Pune during February 18–21, 2006.

Aim of the Workshop: Connectivity is a fundamental, core and very important concept in Graph Theory. The structure theory for n-connected graphs is developed by Whitney, Mader, Tutte, Slater etc. The workshop aims to focus on recent developments and research problems in this area.

Topics to be Covered: n-connected graphs, operations on graphs to preserve connectivity, critically and minimally connected graphs, removable cycles in n-connected graphs and related results in graphs and matroids.

Speakers:

Prof. E. Sampathkumar (University of Mysore),
Prof. A. Vijayakumar (Cochin University of Science and Technology),
Prof. M. K. Srinivasan (I.I.T. Bombay),
Prof. B. N. Waphare and Prof. M. M. Shikare (University of Pune).

There is **no registration fee** for the participants. The organizers will provide lunch. Free accommodation, on request, will be arranged to the outstation participants. **No TA/DA** will be provided to the participants.

Applications are invited from research scholars, faculty members in colleges and universities. Eligible candidates intending to participate in the workshop should send their applications, with resumes, by Post or by E-mail to the following contact address on or before **25th January 2006.** The organizers will send formal invitation to the selected participants.

Contact Address

Mr. Y. M. Borse
Department of Mathematics
University of Pune
Ganeshkhind, Pune 411 007
E-mail: ymborse@math.unipune.ernet.in
Phone: 020 25601272

XIII Ramanujan Symposium on Nonlinear Differential Equations

February 8–10, 2006

The Ramanujan Institute for
Advanced Study in Mathematics
University of Madras
Chepauk, Chennai 600 005

Prof. S. Paravthi Director and Head Coordinator (UGC-SAP)	Prof. R. Sahadevan Convenor, Deputy Coordinator (UGC-SAP)

Funded by

University Grants Commission (UGC-SAP)
New Delhi

About the Host Institution: The Ramanujan Institute for Advanced Study in Mathematics (RIASM), named after the mathematical luminary Srinivasa Ramanujan, is located on the Wallajah Road, Chepauk, Chennai 600 005 (near Marina beach) and is part of the University of Madras. The University of Madras is celebrating its 150th year in 2006.

The Ramanujan Institute of Mathematics was originally established as a memorial to Srinivasa Ramanujan by the philanthropist late Dr. Rm. Alagappa Chettiar in April 1950. The Department of Mathematics at the University of Madras was started as early as 1927. During the period 1957–66, the Department of Mathematics and the Ramanujan Institute of Mathematics were functioning as two independent bodies under the University of Madras. In 1967 these two were amalgamated to form the UGC Center for Advanced Study in Mathematics and named as the Ramanujan Institute for Advanced Study in Mathematics and has been receiving grant from UGC continuously. The RIASM has also been identified by the Department of Science and Technology, Government of India for grant under FIST programme. The present RIASM owes it growth to the vision and far reaching policies of the UGC and the positive attitude of the University of Madras.

Objectives of the Symposium: The primary objective of XIII Ramanujan Symposium on Nonlinear Differential Equations is to provide a forum for bringing together the various research groups working with different backgrounds in the area of Nonlinear Differential Equations on a single platform for reporting research findings and discussing their investigations.

Tentative List of Speakers

Prof. K. T. Joseph
(Tata Institute of Fundamental Research, Mumbai)

Prof. P. Kandaswamy
(Bharathiyar University, Coimbatore)

Prof. V. Kannan
(University of Hyderabad, Hyderabad)

Prof. S. Kesavan
(Institute of Mathematical Sciences, Chennai)

Prof. A. Kundu
(Saha Institute of Nuclear Physics, Kolkotta)

Prof. M. Lakshmanan
(Bharathidasan University, Tiruchirapalli)

Dr. B. Mayil Vaganan
(Madurai Kamaraj University, Madurai)

Prof. K. Nandakumaran
(Indian Institute of Science, Bangalore)

Prof. S. Ponnusamy
(Indian Institute of Technology Madras, Chennai)

Prof. Radha Balakrishnan
(Institute of Mathematical Sciences, Chennai)

Prof. N. Ramanujam
(Bharathidasan University, Tiruchirapalli)

Prof. M. Sitaramayya
(Acharya Aryabhata University, Hyderabad)

Prof. G. Srinivasan
(Indian Institute of Technology Madras, Mumbai)

Prof. N. Srinivasan
(Indian Institute of Technology Madras, Gwahathi)

Prof. P. L. Suchdev
(Indian Institute of Science, Bangalore)

Prof. N. Sthanumoorthy
(RIASM, Chennai)

Prof. K. M. Tamizhamani
(Pondicherry University, Pondicherry)

Call for Papers: All the participants in the conference must be registered. Registration fee is Rs. 500/-. Maximum number of participants for paper presentation (oral) is 10. Participants are requested to fill up the registration form and send it along with an abstract as well as a Demand Draft, payable at any Nationalized Bank at Chennai in favour of *Convenor, XIII Ramanujan Symposium* on or before 06 January 2006 to the following address.

Prof. R. Sahadevan
Convenor, XIII Ramanujan Symposium
The Ramanujan Institute for Advanced Study in Mathematics
University of Madras
Chepauk, Chennai 600 005
Phone: (044) 25360357 (Off); (044) 24725350 (Res)
Fax: (044) 2536 6693 (University)
25384870 (RIASM)
E-mail: ramajayamsaha@yahoo.co.in

XIII Ramanujan Symposium on Nonlinear Differential Equations

February 8–10, 2006

Registration Form

Name: ....

Designation: ....

Institute: ....

Mailing address: ....

....

E-mail: ....

Fax: ....

DD Details: ....

I will participle (please tick appropriate box)

( ) without presenting a paper

( ) and present a paper

Title: ....

Author(s): ....

Accommodation required: Yes/No

Arrival Date: ....

Departure Date: ....

Date: Signature

Indian Institute of Science

Bangalore 560 012, India

IISc Young Science Fellowship Programme (YSFP) for Toppers in PUC/12th Standard Supported by Sir Ratan Tata Trust, Bombay Indian Institute of Science, (IISc) Bangalore invites applications from rank holders (within top 20 ranks or equivalent) in the 12th Standard/PUC Examination who wish to pursue studies/research in Science/Mathematics.

Eligibility Criteria

1. Student must have passed PUC/12th Standard or equivalent examination conducted in 2005 and must be a topper (top 20 ranks or equivalent) in the Board/University. In cases where ranks are not declared like CBSE/ICSE, he/she must have obtained a high level of distinction of the Board or be within the top 1% of the students taking the examination. In the absence of this data, suitable criterion will be evolved by IISc.
2. Student must have studied Mathematics as one of the subjects at +2 level.
3. Student must have joined B.Sc/Integrated M.Sc programme in any branch of Science at any College/ University in India.

Note: Candidates studying Bachelor degree in Veternery/ Engineering/Dental/Medicine/Pharmacy are not eligible for this Fellowship.

Relaxation Criteria for SC/ST/PH candidates: There will be suitable relaxation in the eligibility criteria for SC/ST/PH candidates.

Summer Contact Programme: The Fellowship holders will be invited during May-June every year for 2 years, starting from 2006 onwards, to spend one month at IISc and interact with the faculty of IISc. In the Summer Programme, there will be organized lectures, problem solving sessions, visits to labs and visits to other academic institutions. The participants are expected to stay on campus at IISc.

Financial Support: The participants are eligible for the following financial assistance on successful completion of the Summer Contact Programme

1. Second class rail/bus to and fro fare from college/residence to IISc.
2. Free board and lodging (on a shared basis) at IISc.
3. Towards fees at actuals or Rs. 5,000/- per annum whichever is less (no deposits/donations).
4. A grant of Rs. 1,500/- per annum towards purchase of books.

Only selected candidates will be asked to produce following documents through the Principal of the College:

1. Copy of the PUC/+2 /CBSE/ICSE Marks Card
2. Copy of SC/ST/PH Certificate
3. Study certificate from the Principal.

Submission of Application: Application can be submitted on-Line using the following Web form.

Details and the application form may be obtained from http://www.iisc.ernet.in/ysfp/

National Seminar on Stochastic Process Modeling, Distribution Theory and Order Statistics and Annual Conference of Kerala Statistical Association

March 16–18, 2006

Organised by

Department of Statistics
University of Kerala
P. O. Kariavattom, Trivandrum 695 581

We are happy to inform that a National Seminar on "STOCHASTIC PROCESS MODELING, DISTRIBUTION THEORY AND ORDER STATISTIC" is proposed to be held during 16 to 18 March 2006 at the Department of Statistics, University of Kerala, Kariavattom, Trivandrum. Eminent Statisticians working in the above areas are invited to deliver talks in the Conference. One session is the Seminar is scheduled for presentation of papers for selection of Prof. R. N. Pillai Young Statistician Award for the year 2006 [for young researchers who are members of Kerala Statistical Association (KSA) only] which has been instituted by the KSA. Contributory paper presentation sessions in Stochastic Process Modeling. Distribution Theory, Order Statistics and related areas such as Statistical Inference, Bayesian Inference, Biostatistics, Exploratory Data Analysis etc. are also scheduled in the Seminar. Hence papers are invited in the above areas for presentation in the Seminar.

Registration and Fee

All those desirous of attending the conference are required to register using the form with this brochure. The Registration fee for the Seminar is as given below.

Registration Fee

General	Rs. 600
PG & MPhil Students	Rs. 400

The Registration fee covers cost of seminar materials, moderate hostel accommodation and local hospitality during the days of the Seminar (16 to 18 March 2006). Those who don't require accommodation, the Registration fee in the two categories are Rs. 400 and Rs. 200 respectively. Those who require moderate hotel accommodation (the daily rent ranges from Rs. 350 to Rs. 900) should inform the Organising Secretary in advance and send a DD for the expenses of hotel accommodation

Important Dates

Last date for submission of abstract	– 15 Jan., 2006
Last date for registration and payment of fee	– 15 Jan., 2006
Last date for submission of full text of the paper	– 15 Feb., 2006

The Registration form, Registration fee and hotel accommodation charges (by DD drawn in favour of the Organising Secretary – National Seminar from any nationalized bank payable at Trivandrum) shall be sent to Dr. C. Satheesh Kumar (Organising Secretary) Department of Statistics, University of Kerala, P. O. Kariavattom, Trivandrum, Pincode 695 581 [Ph: 0471-2418905 (O), 9447134758 (R)]. Please submit your abstract/full paper by post to the Organising Secretary or by on mail. E-mail: ukstatseminar@yahoo.co.in; ukstatseminar@rediffmail.com Delegates are advised to avail TA from their own institutions. Financial support may be given to delegates subject to availability of funds.

During March the climate in Trivandrum is quite pleasant, with an average temperature of 30°C. Several tourist centres and places of important historical interest are situated in and around Trivandrum, the capital of God's own country.

For details see official website:

http://www.keralatourism.org/

Registration Form

Name	:
Address	:
E-mail	:
University/Institution	:
Are you presenting paper	:
Title of the paper	:
Do you need accommodation	:
Amount sent	:
D. D. No. & Date	:
Bank	:
Branch	:
Date	:
Place	:
Signature	:

Tata Institute of Fundamental Research, Mumbai School of Mathematics Summer Programme for Students – 2006

This programme is intended to orient students towards research in Mathematics. Students doing Masters degree with a good academic and motivation for research are invited to apply. Exceptionally good and motivated students from B. Sc/B. Tech may also apply.

Students selected under the programme are invited to visit the Institute, and are guided through advanced Reading Course (or other appropriate project). There will also be special lectures aimed at introducing the students to advanced areas of mathematics. The students will have access to the Institute's library and certain other facilities, during the visit.

The programme will be in two batches of one month each between May 15th to July 15th. Please indicate in your application which batch you would prefer, if selected.

Selected students will be paid a stipend (Rs. 4000/- for the month) and a travel allowance (1.33 times the second class return railway fare). Accommodation will be provided.

Application Procedure: Application forms may be downloaded from here (applications may also be made on plain paper).

The application should include the following information:

- Name
- Date and Place of Birth
- Affiliation (College, University)
- Academic Qualifications (Degree, Institution, Marks, Year. Attach copies of mark-lists)
- Details of Projects Undertaken, any Extra-curricular Mathematical Studies, Published Work etc., if any

Please include the following along with your application:

- Mailing address (and E-mail address if available);
- One recent passport-size photo;
- Recommendation letters from two teachers acquainted with your mathematical aptitude.

Applications should be marked 'VSRP (Mathematics)' and sent to:

Assistant Registrar (Academic),
Tata Institute of Fundamental Researchm,
Homi Bhabha Road, Colaba,
Mumbai 400 005

Closing date for receipt of applications:
February 10, 2006

In addition, the text of the applications may also be sent by E-mail (see address below).

Address

Tata Institute of Fundamental Research,
Homi Bhabha Road, Colaba,
Mumbai 400 005
Fax: +91-22-2280 4610/11
Telephone: +91-22-2280 4545
E-mail: vsrp@math.tifr.res.in
Web: http://www.math.tif.res.in/ vsrp

Visiting Fellowships Academic Positions and Professional visits may be sent to the Dean, Mathematics Faculty.

MATHEMATICS NEWSLETTER

Volume 15 December 2005 No. 3

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