Review
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However, this is not an elementary book. The authors say they have aimed this book towards readers who have studied the third year of a mathematics degree and some undergraduate physics. Readers certainly need a comprehensive introductory course in quantum mechanics under their belt. Further, I doubt that three years of undergraduate mathematics study would equip a reader well enough to cope with everything that is in this book. In my experience, some of the mathematics needed to read this book is not taught until the fourth year of a degree or at masters level.

The book outlines the mathematical concepts upon which the theory of quantum mechanics is built. It is not a comprehensive book upon quantum mechanics. The authors expect readers to know about perturbation theory, spin, atoms and molecules, and other elementary topics which they openly acknowledge are omitted from the book. They also expect familiarity with quite profound mathematical concepts like operators on a Hilbert Space and Banach spaces, but they do append clear tuition upon these matters – these mathematical supplements are some of the best I've ever seen. The book provides an introduction to quantum field theory and quantum electrodynamics, but it is not a text upon those subjects. This book does not contain exercises.

In my opinion, this is not a book for undergraduates either in mathematics or physics. It is a first class study book for graduates wishing to gain a very deep understanding of the mathematical theory that underlies quantum mechanics, but they will require more than a three-year mathematics degree to tackle all of it. As such a study book, it is first class – well done to Gustafson and Sigal.

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The colours of infinity, by Ian Stewart, Arthur C Clarke, Benoit Mandelbrot, Michael and Louisa Barnsley, Will Rood, Gary Flake, David Pennock, Robert Prechter and Nigel Lesmoir-Gordon. Pp. 176. £19.95 (pbk). 2004. ISBN 1 904555 05 5 (Clear Books).

In 1994 Nigel Lesmoir-Gordon produced and directed a TV documentary about fractal geometry; this book is published to mark both the 10th anniversary of this film and the 80th birthday of Benoit Mandelbrot. The price covers a DVD of the original documentary which also includes a half-hour 'fractal animation chill-out movie' with music by Pink Floyd's David Gilmour.

Ian Stewart sets the scene with a non-technical introduction to fractals, placing this new geometry in a historical context and explaining the concept of fractal dimension with particular reference to the British coastline. He also looks at applications to turbulence, Brownian motion, vibration and digital communications technology. Arthur C Clarke, who was anchorman in the film, focuses on iteration and the Mandelbrot set, which he calls 'the most extraordinary discovery in the history of mathematics'. Mandelbrot's own chapter is based on a lecture he delivered at a Nobel conference in 1990; it examines the ubiquity of fractals in nature and art. Michael Barnsley, who has specialized in the creation of fractal images, explains the principles of transformation using an analogy from soccer.

Those four writers also appear in the documentary, but, for the purpose of this celebratory volume, other authors have been invited to participate. The mathematician and fractal animator Will Rood looks at the possibilities for colouring the M-set in various ways and draws analogies between it and Escher's tessellations. An essay which models the World Wide Web as an example of fractal organization is contributed by Yahoo! researchers Gary Flake and David Pennock. The 'socionomist' Robert Prechter describes the behaviour of stock prices in the financial market in terms of fractal self-affinity. The final two chapters are devoted to the documentary, with the producer offering a behind-the-scenes account of how it was made as well as the film script itself. The book is lavishly illustrated in colour and would be a welcome addition to any school library, and there is, of course, the bonus of the DVD as an opportunity to liven up the occasional lesson.

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Synch: the emerging science of spontaneous order, by Steven Strogatz. Pp. 338. £8.99. 2004. ISBN 0 141 00763 X (Penguin).

The tendency of systems spontaneously to synchronise or 'self-organise' is observed in a wide range of phenomena in the physical, biological and human worlds. It is encountered on a wide range of scales, from the quantum behaviour of photons in lasers and electrons in superconductors to the simultaneous flashing of fireflies and the 'biological clocks' controlling heartbeats and circadian rhythms.

Synch is a very readable popular account of the efforts of applied mathematicians over the last 30 years or so to formulate mathematical models that exhibit, and thus begin to explain, synchronicity: this 'order in time' may be contrasted with the 'order in space' familiar in crystal structures. The range of potential applications means that this field has an attractive inter-disciplinary character and the mathematical aspects typically involve large systems of coupled non-linear differential equations and the heavy use of computers to discern the behaviour of their solutions.

Parts I and II of *Synch* deal with examples from biology and physics that are modelled by networks of mutually coupled oscillators; Part III is more speculative and looks at synchronicity in chaotic systems and networks where the connectivity is less regular – the so-called 'small-world' networks. From the plethora of clearly presented examples, I shall unpick a thread from Parts I and II which, I hope, gives a flavour of this new area of mathematics.

In one of his last pieces of research, Norbert Wiener hypothesised that the 10 Hz alpha rhythm seen in brain waves might function as a biological clock and that the 10 Hz frequency was maintained by the spontaneous synchronisation of a myriad of coupled neural oscillators. The data, and Wiener's modelling, were inconclusive and it was left to Art Winfree to pick-up the reins after Wiener's death in 1964. Winfree proposed a broad-brush model of biological oscillators involving an 'influence function' governing an oscillator's emission of signals, a 'sensitivity function' governing an oscillator's response to signals and the notion of connectivity – which oscillators respond to which signals. Early models assumed equal and mutual connectivity between oscillators and computer experiments pointed to a 'phase transition' favouring synchronised behaviour once the variance of the distribution of the frequencies of the oscillators was sufficiently small. In the 1970s, Yoshiki Kuramoto gave an analytical solution of a special case of Winfree's model (when the coupling is sinusoidal in form) and Strogatz in the 1980s supplied a proof that the non-synchronised state was unstable above the phase transition threshold. When the variance is zero (so all the oscillators are identical) Strogatz and Mirollo were able to build on Peskin's work on the pacemaker cells of the heart to prove a general theorem guaranteeing synchronicity for a wide class of influence and sensitivity functions. Then, in the 1990s, came the realisation that Kuramoto's model – a solution waiting for a problem – was, in fact, the right model for an array of Josephson junctions. (As an aside, it is typical of Strogatz's eye for the human side