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Review

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factorisation theorem). XI, elimination, and practical solution of equations (Horner's method, etc.); XII, Galois theory. XIII, invariants: the main theorem of the symbolic theory is proved for binary forms; tensor methods of constructing concomitants are also considered in some detail. XIV, algebras: the structure theorems for simple and semi-simple algebras over a general field are proved (existence of a unit element is assumed). XV, group algebras: the orthogonal relations; the formula for the characters of the symmetric group; Young's idempotents; the Littlewood-Richardson rule for the multiplication of S -functions. XVI, the representations of the full linear, and orthogonal, groups (including spin representations); applications to invariant theory.

Though the book has a sound programme, it is very carelessly written. Essential conditions are omitted from the enunciations of theorems; far from obvious assertions are simply written down, as though no proof were required; proofs are often obscure and condensed, and sometimes wrong; and some of the statements are untrue. Here are some examples. (1) The axioms given for an (Archimedean) ordered integral domain (p. 145) do not ensure that the product of two positive elements is positive; e.g., they are satisfied for the field $R_0(i)$ (R_0 the rational field) when we define an order \succ by the rule: $a + ib \succ c + id$ if and only if $a + b\sqrt{2} > c + d\sqrt{2}$ (where $>$ has its usual meaning). Likewise, the axioms for an algebra over a field (p. 224) do not ensure that subtraction is possible, or that the 1 of the field is the unit operator on the algebra. (2) Theorem II on p. 156 is false—if ξ_1, ξ_2 are roots of an irreducible polynomial $g(x)$ over a field F of characteristic zero, it is *not* true in general that there exists an automorphism of $F(\xi_1, \xi_2)$ over F such that $\xi_1 \rightarrow \xi_2, \xi_2 \rightarrow \xi_1$. For example, let ω be a primitive fifth root of unity and take $\xi_1 = \omega, \xi_2 = \omega^2$ and $F = R_0$; then $\xi_1 \xi_2 = 1$ but $\xi_2 \xi_1 = \omega^3 \neq 1$. (3) Theorem III on p. 191 is the theorem that an equation is solvable by radicals if and only if its Galois group is solvable. In its enunciation, the term "quotient group" makes its first appearance in the book (properties of quotient groups have not been dealt with, even implicitly). The proof is vitiated by the tacit assumption that the primitive r_j -th root of unity ω lies in the ground-field (the assumption comes in at the steps, "Hence, clearly $\omega S y = y$ " (line 19) and, "Then also $S^{-1} y = \omega^{-1} y$ " (line 33)). Finally, the last part of the proof (viz. that G_{j-1}/G_j is cyclic) is omitted altogether. (4) The statement (p. 267), "Hence if the complete tensor of rank r is separated into sets which transform independently, the matrices of transformation of the various sets will be the irreducible invariant matrices of degree r " is patently false. Yet it is the only justification which the author gives of the (correct) process by which he decomposes a tensor into irreducible parts. Moreover, the question of full reducibility is not mentioned at all in the discussion.

In conclusion, I feel that this book could be much improved without altering the main lines. At present, its good intentions are not carried into effect.

G. E. WALL.

Introduction to Linear Algebra and the Theory of Matrices. By HANS SCHWERTFEGER. Pp. 280. Fl. 15; cloth Fl. 17.50. 1950. (Noordhoff, Groningen)

This is a truly introductory book, in which the author achieves his aim of preparing the reader for the study of more abstract and advanced books on algebra. It covers very thoroughly the ground of a first or second year course in linear algebra and matrices for students of mathematics, physics, engineering and statistics. Throughout the book the author emphasises the close connection between algebra and geometry, and several geometrical topics are dealt with by means of the newly-acquired algebraical tools.

A brief summary will indicate the scope of the book : I. The notions of linear dependence and rank are developed in geometrical language, column vectors being interpreted as points in a real or complex number space. The theory of linear equations is treated without the use of determinants, which altogether play a rather secondary rôle in this book. Determinants are then defined recursively by expansion with respect to one of their columns. (This method leads fairly quickly to a proof of the chief properties of a determinant, but it lacks symmetry and does not fully bring out the connection with the permutation group.)

II. Linear transformations. Formal laws of matrix algebra.

III. Equivalent and congruent matrices. Elementary transformations. Reduction of bilinear and quadratic forms. Hermitian forms. Skew matrices.

IV. The group concept. Groups of matrices. Similarity. The characteristic polynomial. The orthogonal and unitary groups. Triangular form. The eigen-value problem of a normal matrix. The symplectic group and the Pfaffian invariant.

While the book is essentially of an elementary character, there are numerous notes containing references to classical and modern literature, which give the student a glimpse of the land that lies beyond the horizon. Some of the more advanced subjects are discussed in the form of worked examples, including such topics as Hadamard's inequality, quaternions, vector algebra in three and four dimensions and the Minkowski-Grassmann calculus. There is also an Appendix on projective theory and null systems.

No treatment is given of elementary divisors or the Cayley-Hamilton equation in the case of a general square matrix. These subjects are to be included in a second volume, which is to be regarded as a sequel to the present book.

The style is sufficiently broad to make the book easy for a beginner without making it long-winded. The exposition is very lucid, and there are many exercises and problems for the reader. (The understanding of the book is not seriously hampered by a few odd phrases and a number of unidiomatic sentences.) The work is a valuable addition to the textbook literature, and can be warmly recommended to all students of pure and applied mathematics.

WALTER LEDERMANN.

Differentialgleichungen. Lösungsmethoden und Lösungen. I. By E. KAMKE. 3rd edition, reprinted. Pp. xxvi, 666. \$7. 1948. (Chelsea Company, New York)

The first three German editions of this first volume of Kamke's massive compendium were published in 1940, 1942 and 1944, so that copies were not easily to be obtained in this country, and the great value of the work has in consequence perhaps not been widely recognised. The Chelsea Company's reprint, an early member of that excellent list, has only just reached us; nevertheless, attention should be called to its existence, for the book is certainly a reference work of outstanding importance, which should be in every mathematical library. The mathematician faced with a differential equation for which he must find a solution or characterising properties of that solution should have Kamke at his elbow.

The first section, of about 180 pages, is concerned with methods of solving differential equations, and gives a survey of the whole field of ordinary equations, including recent methods such as that of the Laplace transform, and sections on numerical and graphical integration. The second section, of about 100 pages, deals with boundary value problems and with eigenvalues. This is particularly valuable, for while the main contents of the first section are familiar, this second section collects into a concise form a great deal of

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