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Exotic quantum holonomy induced by degeneracy hidden in complex parameter space

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ABSTRACT

An adiabatic change of a bound state along a closed circuit in the parameter space can induce holonomies not only in the phase of the state, but also in the associated eigenspace and eigenvalue. The former is the well-known Berry phase while the latter, namely the exotic holonomy, is found a decade ago and its origin has not been understood yet. By extending the parameter into the complex number, the correspondence of the exotic holonomies and the degeneracy of the non-Hermitian Hamiltonian, or the exceptional points, is revealed. We show that this explains all the known non-trivial characteristics of the exotic holonomies.

An adiabatic change of a quantum system along a closed path C in the parameter space may induce discrepancies. Among them, the phase holonomy [1,2] has been thoroughly investigated. Suppose the system is initially prepared at an eigenstate $|\xi_n\rangle$ and evolved adiabatically along C, where spectral degeneracy is assumed to be absent. If the dynamical phase is eliminated from the adiabatic time evolution, the final state coincides with $|\xi_n(C)\rangle$ that is obtained by the parallel transport from $|\xi_n\rangle$ along C [2]. The phase holonomy or the geometric phase resides in the phase of $\langle\xi_n|\xi_n(C)\rangle$ [3]. In the presence of the spectral degeneracy, Wilczek and Zee pointed out that $|\xi_n(C)\rangle$ and $|\xi_n\rangle$ need not be parallel, where the noncommutative extension of the geometric phase is introduced [4].

A decade ago, a somewhat new kind of holonomies, referred to as the exotic holonomies, has been reported in Ref. [5]. Even if the spectral degeneracy is absent so that no Wilczek–Zee's phase holonomy takes place, $|\xi_n\rangle$ and $|\xi_n(C)\rangle$ are not only different from each other but also found to be orthogonal. In fact, $|\xi_n(C)\rangle$ is parallel with another initial eigenstate, namely $|\xi_{n'}\rangle$ ($n' \neq n$). This is nothing but the so-called *eigenspace holonomy*, which is characterized by the matrix $M_{mn}(C) \equiv \langle\xi_m|\xi_n(C)\rangle$. The holonomy matrix $M(C)$ describes both the phase and the eigenspace holonomy in a unified way [6]. It is shown that $M(C)$ is divided into two factors, namely a permutation matrix, which is the signature of the eigenspace holonomy, and a diagonal matrix consisting of phase factors so as to describe the usual phase holonomy. The eigenspace holonomy gives rise to the corresponding *eigenvalue* holonomy since the eigenvalues and the eigenvectors have a one-to-one correspondence.

It has been mysterious why and how such exotic holonomies, the eigenspace and the eigenvalue holonomies, emerge [5–10]. According to Johansson–Sjöqvist's theorem, which is a generalization of Longuet-Higgins' theorem [11], the spectral degeneracies should take place in a surface S enclosed by C when a non-trivial phase holonomy exists along C [12]. Furthermore, if C lies near a degeneracy point, the Berry phase is proportional to the solid angle subtended by C at the degeneracy point in the parameter space [1]. It is natural to expect that the Johansson–Sjöqvist theorem is also applicable to the exotic holonomies. However, it does not seem to be the case since the parameter space of the minimal model giving rise to the exotic holonomies described by 2×2 matrix consists of only a one-dimensional loop, i.e. S^1, so that it offers no room for the theorem [9]. It is also worth noting that the so-called off-diagonal geometric phase is distinguished from the exotic holonomy since it mainly concerns $\langle\psi_j(X_i)|\psi_k(X_f)\rangle$ for $X_i \neq X_f$, where $\psi_{j(k)}$ and $X_{i(f)}$ are the $j(k)$th eigenstate and the initial (final) external parameter, respectively [3]. In contrast, our main interest lies at the case $X_i = X_f$ (see also [10]).

In order to understand the origin of the exotic holonomy we extend the parameter space into complex regime. This allows us to access the *hidden* degeneracy of the *unphysical* complex eigenvalues, which is known as the exceptional points (EPs) in the context of the non-Hermitian Hamiltonian describing open quantum sys-

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tems or the hidden crossing [13–23]. An EP forms a branch point in a parameter space implying eigenfunctions and eigenvalues are no longer single-valued. We find that the resulting structure of a Riemann surface explains all the key characteristics of the exotic holonomies. It is noted that the geometric phase around an EP is defined in a conventional way along a closed path in the Riemann surface, e.g. encircling the loop *twice* in the case that the EP forms a square-root branch point [17]. It is known that when encircling a closed path including an EP *once*, the two eigenstates are exchanged with each other, so do the eigenvalues. Considering the Riemann sheet constructed from a square-root branch point, the holonomy associated with an *open* path, namely encircling an EP *once*, is relevant to our study. Such an open path induces a permutation with possible sign changes among eigenvectors.

The aim of this Letter is to reveal the role of the EPs (square root branch points) hidden behind the exotic holonomies by extending the parameter into the complex number. It is shown, in particular, that the non-trivial topology of the parameter space around the EPs essentially governs the holonomy matrix $M(C)$, which substantiates a unified theory of quantum holonomies including the exotic ones introduced in Ref. [6]. It gives us an important message that *one might observe the unexpected non-trivial holonomies if he does not consider the degeneracy hidden in the complex parameter region*. Note that even a simple one-dimensional nonlinear oscillator may possess an infinite number of branch points [24], which implies the exotic holonomies seem to be ubiquitous and hidden everywhere.

We explain our idea through an analysis of a periodically kicked spin-$\frac{1}{2}$ system, which offers the simplest example of the exotic holonomies [9,6]. Note that the applicability of our idea is not limited into this particular case. The Hamiltonian of the model system is given as

$$\hat{H}(t;\lambda) = \mu\hat{P}(\boldsymbol{e}_z) + \lambda\hat{P}(\boldsymbol{n})\sum_{m=-\infty}^{\infty}\delta(t-m), \tag{1}$$

where μ and λ describe the strength of an applied static external field and that of the time periodic perturbation, respectively. Note that λ should be real in physical situation to keep the resultant time evolution unitary. Later we intentionally extend it into the complex number. $\hat{P}(\boldsymbol{v}) \equiv (1+\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{v})/2$ is a projection operator parameterized by a normalized vector $\boldsymbol{v}$, in which $\hat{\sigma}_j$ $(j = x, y, z)$ represents the Pauli matrices, and we set $\hbar = 1$. The direction of the perturbed magnetic field is parameterized as $\boldsymbol{n} = \cos\phi\sin\theta\boldsymbol{e}_x + \sin\phi\sin\theta\boldsymbol{e}_y + \cos\theta\boldsymbol{e}_z$. The important physical properties of a periodically driven system are characterized by the so-called Floquet operator, which is a time evolution operator during one period. In particular, an eigenvector of the Floquet operator forms a stationary state, and the corresponding eigenvalue is given as $\exp(-i\gamma)$ with a real γ called as a quasienergy. This is nothing but a time domain analogue of Bloch theorem for spatially periodic systems.

For the kicked spin, the Floquet operator is

$$\hat{U}(\lambda) = e^{-i\mu\hat{P}(\boldsymbol{e}_z)/2}e^{-i\lambda\hat{P}(\boldsymbol{n})}e^{-i\mu\hat{P}(\boldsymbol{e}_z)/2}, \tag{2}$$

where the fundamental period of the time evolution is chosen as $-1/2 \leqslant t \leqslant 1/2$. A detailed analysis of $\hat{U}(\lambda)$ is given in Ref. [6]. Since $\hat{U}(\lambda)$ is periodic in λ with a period 2π, the parameter space of λ is identified with S^1. We focus on the closed path C where λ is varied from 0 to 2π in S^1 to examine the quantum holonomies associated with the adiabatic change of the eigenvalues and eigenvectors of $\hat{U}(\lambda)$.

First, we examine the eigenvalues of the Floquet operator. The eigenvalues of $\hat{U}(\lambda)$ are given as

$$z_{\pm}(\lambda) \equiv \exp\{-i[\mu + \lambda \pm \Delta(\lambda)]/2\}, \tag{3}$$

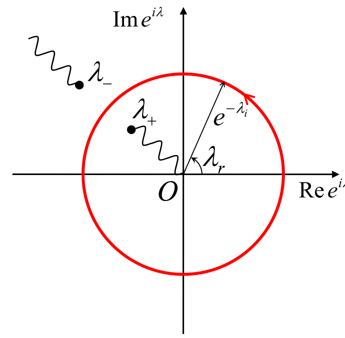

Fig. 1. The parameter space extended into complex λ for the system described by the unitary matrix (2) by using $e^{i\lambda} = e^{-\lambda_i}e^{i\lambda_r}$, where λ_r and λ_i represent the real and imaginary part of λ, respectively. Two EPs $\lambda_{\pm} \equiv \alpha + i\beta$ (black dots) and the corresponding branch cuts of $\Delta(\lambda)$ (curvy lines attached to the dots) are presented, where $\sin\theta\sin(\mu/2) > 0$ (i.e., $\beta > 0$) is assumed. A circle represents the integration contour, in which the angle from the positive x axis and the radius are given as λ_r and $e^{-\lambda_i}$, respectively.

where the gap of the two quasienergies is

$$\Delta(\lambda) \equiv 2\cos^{-1}\left[\cos\left(\frac{\lambda-\alpha}{2}\right)\Big/\cosh\frac{\beta}{2}\right] \tag{4}$$

with two real parameters

$$\alpha \equiv 2\tan^{-1}\left[-\cos\theta\tan\frac{\mu}{2}\right], \tag{5a}$$

$$\beta \equiv 2\tanh^{-1}\left[\sin\theta\sin\frac{\mu}{2}\right]. \tag{5b}$$

The degeneracy then takes place when $\lambda = \lambda_{\pm} + 2\pi k$ is satisfied, where k is an integer and

$$\lambda_{\pm} \equiv \alpha \pm i\beta. \tag{6}$$

Note that the degeneracy resides in the complex parameter regime if $\beta \neq 0$. We ignore the case $\beta = 0$ at which the problem becomes trivial [25]. Near the degeneracy one approximates Eq. (4) to

$$\Delta \sim 2\sqrt{\pm\frac{i}{2}\tanh\frac{\beta}{2}(\lambda-\lambda_{\pm})}, \tag{7}$$

which implies the degeneracies form square-root branch points of $\Delta(\lambda)$, whose configuration and branch cuts are depicted in Fig. 1. This unambiguously reveals the non-Hermitian EPs hidden behind the exotic holonomy of eigenvalues. It is well known that the two eigenvalues are interchanged when the parameters are varied so as to encircle the EP *once* [15,19], which is the key feature of the exotic holonomy.

One needs to carefully take into account a closed loop that the parameter is varied. Naively thinking a semi-infinite rectangular path in $(\mathrm{Re}\,\lambda, \mathrm{Im}\,\lambda)$ containing the real axis from 0 to 2π may be considered as a loop. However, the path from $0 + i0$ to $0 + i\infty$ is exactly identical to that from $2\pi + i0$ to $2\pi + i\infty$ due to the 2π periodicity of λ. In fact it is not necessary to depart from the real axis of λ to $+i\infty$ in order to encircle the EP. To understand what is happening here let us redefine the complex parameter space in the polar coordinate, $e^{i\lambda} = e^{-\lambda_i}e^{i\lambda_r}$, where λ_r and λ_i represent the real and imaginary part of λ, respectively. The EPs and the branch cut may then be represented in Fig. 1. The exotic holonomy has been found when the *real* λ is varied from 0 to 2π, which in fact forms a closed loop encircling the EP, i.e. λ_+ as is clear from Fig. 1. It is reemphasized that *although a single real parameter λ is varied along a closed loop containing no degeneracy on itself this exactly corresponds to encircling a degeneracy hidden in physically inaccessible domain*. Note that at least two independent external parameters are necessary in order to observe the EP (in more technical terms it is a codimension-two object).

Next, we proceed to the analysis of eigenspace holonomy. To obtain eigenvectors, $\hat{U}(\lambda)$ is casted into a normal form

$$\hat{U}(\lambda) = \exp\{-i[\mu + \lambda + \Delta(\lambda)\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{l}(\lambda)]/2\}, \tag{8}$$

where $\boldsymbol{l}(\lambda) = (\cos\phi \boldsymbol{e}_x + \sin\phi \boldsymbol{e}_y)\sin[2\Theta(\lambda)] + \cos[2\Theta(\lambda)]\boldsymbol{e}_z$ and

$$\Theta(\lambda) = \frac{1}{2}\tan^{-1}\frac{\sin\theta}{\sin(\mu/2)\cot(\lambda/2) + \cos\theta\cos(\mu/2)}. \tag{9}$$

Let $|\xi_\pm(\lambda)\rangle$ be the normalized right eigenvectors of $\hat{U}(\lambda)$ corresponding to $z_\pm(\lambda)$, respectively:

$$|\xi_+(\lambda)\rangle = \cos\Theta(\lambda)|\uparrow\rangle + e^{i\phi}\sin\Theta(\lambda)|\downarrow\rangle, \tag{10a}$$

$$|\xi_-(\lambda)\rangle = -\sin\Theta(\lambda)|\uparrow\rangle + e^{i\phi}\cos\Theta(\lambda)|\downarrow\rangle, \tag{10b}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are normalized eigenvectors of $\hat{\sigma}_z$. To incorporate the biorthogonality of the eigenvectors of $\hat{U}(\lambda)$ with a *complex* λ, we denote the eigenvectors of $\{\hat{U}(\lambda)\}^\dagger$ as $|\xi^{\mathrm{B}}_\pm(\lambda)\rangle$, on which we impose the biorthonormal relation $\langle\xi^{\mathrm{B}}_m(\lambda)|\xi_n(\lambda)\rangle = \delta_{mn}$ [26].

Following the prescription introduced in Ref. [6] a non-Abelian gauge potential is given as

$$A_{mn}(\lambda) = i\langle\xi^{\mathrm{B}}_m|\frac{\partial}{\partial\lambda}|\xi_n\rangle. \tag{11}$$

We denote the diagonal part of $A(\lambda)$ as $A^{\mathrm{D}}(\lambda)$, whose elements provide the Mead–Truhlar–Berry's adiabatic gauge potentials [1, 27]. With an arbitrary choice of the phases of $|\xi_\pm\rangle$, the gauge covariant expression of the holonomy matrix for a given closed path C reads

$$M(C) = \exp_{\rightarrow}\left(-i\int_C A(\lambda)\,d\lambda\right)\exp_{\leftarrow}\left(i\int_C A^{\mathrm{D}}(\lambda)\,d\lambda\right), \tag{12}$$

where $\exp_\leftarrow$ and $\exp_\rightarrow$ indicate the path-ordered and the anti-ordered exponentials, respectively [6]. The first factor in the right-hand side describes the transformation of the basis induced by the transport along C and is expected to be a permutation matrix that reflects the key feature of the eigenspace holonomy. Both the overall phase of the first factor and the second factor describe the Berry phase. It is known that one can always choose the gauge such that satisfies the parallel transport condition, $A^{\mathrm{D}}(\lambda) = 0$,[1] which implies the second factor becomes the identity [19].

The gauge potential for the kicked spin-1/2 is obtained from Eq. (11) as

$$A(\lambda) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\frac{\partial\Theta}{\partial\lambda}, \tag{13}$$

where the parallel transport condition $A^{\mathrm{D}}(\lambda) = 0$ is satisfied by choosing the phases of $|\xi_\pm\rangle$. $M(C)$ is then directly acquired from Eq. (12);

$$M(C) = \begin{bmatrix} \cos(\eta(C)) & -\sin(\eta(C)) \\ \sin(\eta(C)) & \cos(\eta(C)) \end{bmatrix}, \tag{14}$$

where

$$\eta(C) = \oint_C \frac{\partial\Theta}{\partial\lambda}\,d\lambda. \tag{15}$$

It is easily shown that $\eta(C) = \mathrm{sgn}(\beta)\pi/2$ from the residue of the pole of the integrand which is located at the EP enclosed by the loop in Fig. 1, where $\mathrm{sgn}(x) = 1$ for $x > 0$ otherwise -1 (recall that $\beta \neq 0$). It immediately gives

$$M(C) = \mathrm{sgn}(\beta)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{16}$$

which is a permutation matrix, as we expected, except the minus sign. The meaning of the minus sign becomes clear if λ is varied along the loop *twice*. According to the geometry of the Riemann surface around the EP the exact initial condition can be recovered only by encircling the EP twice. The holonomy matrix is then given as

$$M(C^2) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{17}$$

Here we obtain the parallel transport with a phase change, namely π, which is nothing but the geometric phase [19]. It implies the minus sign in Eq. (16) also comes from the geometric phase.

In a mathematical sense, the degeneracies governing the exotic holonomies seem to be just equivalent to the EPs of open quantum systems. However, the exotic holonomy has its own physical meaning. To clarify this, three implications of our finding are discussed below.

First, the usual gauge theory of holonomies is also applicable to the holonomy for an open path in the sense that one should encircle the EP twice to complete the path due to the geometry of the Riemann surface. This feature has been overlooked in the conventional treatise of the phase holonomy although it has already been examined experimentally in the context of open quantum systems [17].

Second, in the physical point of view the degeneracy generating the exotic holonomy is distinguished from the EP. The EP has been discussed in the context of an open quantum system [28], in which an effective model Hamiltonian is no longer Hermitian. Thus the extension of the eigenvalues into the complex domain intrinsically occurs. In our case, however, the Hamiltonian does not include any non-Hermitian part, so that the eigenvalues of the Floquet operator remain in the unit circle to keep the unitarity of the whole adiabatic time evolution. The degeneracy is hidden in the unphysical domain described by the complex parameter space. One can say that the unphysical domain becomes physical when the system is opened. In this sense the unphysical domain remains unphysical in our case. It is thus impossible to reach the degeneracy of the exotic holonomy by merely varying parameters without intentionally complexifying them.

Third, we discuss the possibility for exciting a two level system based upon the eigenspace holonomy of our exotic holonomy. Usually the adiabatic parametric evolution around an avoided level crossing induces excitation of a quantum state with only exponentially small probability according to the well-known Landau–Zener transition [29]. This process is thus meaningful only in the non-adiabatic case. The exotic holonomy, however, allows us to achieve almost perfect excitation by directly exchanging the two interacting eigenstates adiabatically. This may be exploited as a new way to achieve almost perfect excitation of a system in an adiabatic process.

Finally, we briefly mention a possible experimental realization of the exotic holonomy. $\hat{U}(\lambda)$ of Eq. (2) can be regarded as products of phase shifters which are implemented in principle based upon an optical qubit [30]. The initial state, namely an eigenstate of $\hat{U}(\lambda)$, is prepared by applying $\hat{U}(\lambda)$ successively with λ adiabatically increased from zero in a manner $\hat{U}(\lambda)\hat{U}(\lambda - \delta)\cdots\hat{U}(2\delta)\hat{U}(\delta)\hat{U}(0)$, where $\delta \ll 1$, to an eigenstate of $\hat{U}(0)$, $|\uparrow\rangle$

[1] $A^{\mathrm{D}} = 0$ is the parallel transport condition for the individual eigenvectors, $|\xi_+(\lambda)\rangle$ and $|\xi_-(\lambda)\rangle$. This is different from the usual parallel transport condition of the non-Abelian holonomy, i.e. $A = 0$, where the combined eigenspace $[|\xi_+(\lambda)\rangle, |\xi_-(\lambda)\rangle]$ is parallel transported. $A = 0$ cannot be achieved in our case [7].

or $|\downarrow\rangle$. One then observes the eigenspace holonomy by applying $\hat{U}(\lambda)$ to the prepared initial state in a similar manner with adiabatic cyclic evolution, $\lambda \to \lambda + 2\pi$.

In summary, we have investigated the exotic holonomies focusing on its origin using the periodically kicked spin-$\frac{1}{2}$ system. All the non-trivial characteristics of the exotic holonomies are ascribed to existence of the degeneracy hidden in the complex parameter space. Our finding delivers an important message that one might encounter the unexpected non-trivial holonomies originated from the degeneracy hidden in the complex parameter space which is usually ignored. It might play an important role in many areas of physics.

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