

Contemporary Mathematicians

Fritz Gesztesy
Gilles Godefroy
Loukas Grafakos
Igor Verbitsky
Editors

Nigel J. Kalton

Selecta
Volume 1

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Joseph P.S. Kung

Editor

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Nigel J. Kalton Selecta

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Editors

Fritz Gesztesy
Department of Mathematics
University of Missouri
Columbia, Missouri, USA

Gilles Godefroy
Université Pierre et Marie Curie
Institut de Mathématiques de Jussieu
Paris, France

Loukas Grafakos
Department of Mathematics
University of Missouri
Columbia, Missouri, USA

Igor Verbitsky
Department of Mathematics
University of Missouri
Columbia, Missouri, USA

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Preface

Nigel Kalton's research spans more than 40 years, and his publications represent thousands of pages of original, deep, and concisely written mathematics. Sadly, the Great Book of Analysis composed by Nigel suddenly closed on August 31, 2010, and we are left with the daunting task of continuing its legacy. But Nigel's desire clearly was that, no matter the circumstances, research must go on. These selected publications and their commentaries represent a modest attempt to fulfill these goals.

Every single article that Nigel authored (or co-authored) contains at least one important result, and a fair number of his contributions completely renewed an entire topic. Therefore, it was quite difficult to select the most significant articles in his formidable bibliography. The present volumes aim to provide the mathematical community with some of Nigel's most influential works. They include most of the many fields to which he contributed.

All mathematicians who were privileged to be Nigel's collaborators, colleagues, or students know what a prolific problem solver he was. However, it would not do justice to reduce Nigel's scientific contributions to the very impressive list of open problems that he solved. Indeed, Nigel's vision of mathematics was so far-reaching that it enabled him to use quite exotic objects, such as nonlocally convex spaces, to provide original results in classical and mainstream analysis. Above all, he was a creative mind who would set up the mathematical framework in which a question is best put into its proper context, who would then penetrate it from all possible angles, and who would finally solve it, along with a family of related problems. Therefore, his commanding point of view prepared the ground for future work, and provided his followers with precious new tools.

This volume is a user's guide to the tools, techniques, and ideas Nigel Kalton gave us. Indeed, leading experts kindly provided insightful comments on each article in our selection. We are most grateful to all the contributors to this volume, as they were instrumental in actually realizing the idea of Kalton Selecta. Thanks to their work and expertise, this volume is much more than a mere commemoration of Nigel's achievements, or a tribute to his memory. We should rather understand it as the proceedings of a major scientific event: an attempt to facilitate the understanding of Nigel Kalton's contribution, and a starting point for further research on the problems left open. The influence of Nigel's work is now stronger than ever, and it is our hope that experienced

researchers as well as beginners will benefit from easy access to some of his fundamental articles and ideas.

We invite the reader to delve into Nigel Kalton's work, appreciate every line of it and all that can be found between the lines. When entering this gold mine, one must be prepared to dig deep, but in return, great enjoyment and rewards should be expected.

We now proceed to a short description of the contents of the Kalton Selecta. We decided not to include in these volumes papers co-authored with ourselves or with Nigel's departmental colleagues. Our intention was to pick articles from the various fields to which Nigel Kalton contributed. Each article is preceded by a contribution written by an expert, who is often, but not always, Nigel's collaborator. In some cases, the contributor decided to gather comments on several related articles in a single text. For the convenience of the reader, the comments and the articles have been classified into eight parts: Nonlocally Convex Spaces and Submeasures, Differential Games, Operator Theory, Harmonic Analysis and PDEs, Approximation Theory, Geometry and Banach Spaces, Interpolation Theory, and Probability and Banach Spaces. It should, however, be stressed that this classification is somewhat arbitrary. Tight links exist between these various fields, and these links are frequently due to Nigel Kalton himself. Indeed, Nigel had an outstanding ability to use ideas and techniques in places where they were not expected to be effective, and to relate concepts that appeared disjoint to less penetrating minds. Due to the great size of the work that we have collected, it was necessary to split the chapters into two volumes. Nigel's CV along with the first four parts comprise the first volume, whereas the second volume consists of the remaining four parts.

The first chapter contains Nigel's CV, last updated in February 2015. Part I covers a field in which Nigel Kalton is the undisputed leader: Nonlocally Convex Spaces. This part contains some deep results about the structure of quasi-Banach spaces, the existence of peculiar objects such as minimal quasi-Banach spaces, and submeasures. This voyage through the Kalton zone $0 \leq p < 1$ is commented on by Fernando Albiac, José Luis Ansorena, Per Enflo, David Fremlin, Anna Kaminska, and Bernard Maurey.

Part II is devoted to Optimal Control Theory and its relationship with von Neumann's Game Theory through Differential Games. The contributor to this part is Nigel's co-author Robert Elliott. Part III presents some contributions to Operator Theory: maximal regularity, Hilbertian theory and traces, similarity problems, and the recent and crucial notion of R -boundedness. Contributions were made by Fritz Gesztesy, Tamara Kucherenko, Gilles Lancien, Vladimir Peller, Pierre Portal, and Fedor Sukochev.

Part IV gathers some articles from Harmonic Analysis and Partial Differential Equations and covers various topics, such as lacunarity in harmonic analysis and H^∞ -functional calculus with its applications to partial differential equations. Comments for this part are written by Sergei Ivanov, Nikolai Nikolski, Igor Verbitsky, and Lutz Weis. The original article "The H^∞ -functional calculus and square function estimates" by N. Kalton and

L. Weis is published as the seventh chapter of this part. This article has been extensively quoted in the literature and we are grateful to Lutz Weis for his decision to publish this article in the *Selecta*.

The other set of Parts I–IV is contained in the second volume. Part I (volume 2) concerns Approximation Theory, in the sense of applied mathematics, namely approximation by smooth functions or by vectors with finite support with a given rate of convergence. Contributors to this section are Yuri Brudnyi, Stephen Dilworth, and Denka Kutzarova. Part II (volume 2) focuses on the Geometry of Banach Spaces, a topic to which Nigel contributed in every possible way: isomorphic theory, isometric theory, nonlinear geometry... and of course interpolation theory, but the relevant articles have a section of their own. Contributors to Part II (volume 2) are Yoav Benyamini, Jesús Castillo, Garth Dales, Anna Kaminska, Gilles Lancien, Mikhail Ostrovskii, Gilles Pisier, and Dirk Werner.

Part III (volume 2) is relevant to interpolation theory and the fascinating links between interpolation lines, twisted sums, quasi-linear maps, weaker distances between non-isomorphic Banach spaces and the nonlocally convex world. Nigel's contribution to this field is such that it is fair to designate the corresponding theory as the Kalton calculus. Contributors to this section are Michael Cwikel, Mario Milman, Richard Rochberg, and Stephen Dilworth. Finally, Part IV (volume 2) is devoted to certain aspects of the interplay between Probability Theory and Banach Spaces: decoupling property, spaces generated by a sequence of independent random variables, and Rademacher series. Contributors to this section are Stephen Dilworth and Stephen Montgomery-Smith.

We conclude this introduction by expressing our deepest gratitude to all the contributors to this volume. Their comments provide the best tribute they could pay to Nigel's memory: a mathematical analysis of his works. Moreover, we thank them, and numerous colleagues and friends, mathematicians, and publishers, for their encouragement, generous advice, hands-on help, and steadfast support in the preparation of these Kalton *Selecta*. In particular, we sincerely thank each of the publishers involved for granting us permission to include their pdf files in these volumes. We are particularly indebted to Michael Cwikel not only for spearheading the extensive commentaries on interpolation theory in Part III of the second volume, but also for supplying us with numerous comments and corrections throughout the *Selecta*. His level of involvement with this project went far beyond anything we could have possibly expected. In addition, we sincerely thank each of the publishers involved for granting us permission to include their pdf files in these volumes.

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Finally, we dedicate this book to Nigel Kalton. Thank you Nigel, for a life devoted to mathematics.

Columbia, Missouri, USA

Paris, France

Columbia, Missouri, USA

Columbia, Missouri, USA

Fritz Gesztesy

Gilles Godefroy

Loukas Grafakos

Igor Verbitsky

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- N. J. Kalton and J. W. Roberts, Uniformly exhaustive submeasures and nearly additive set functions, *Trans. Amer. Math. Soc.* **278** (1983), 803–816.
- N. J. Kalton, Banach envelopes of non-locally convex spaces, *Can. J. Math.* **38** (1986), 65–86.
- N. J. Kalton, The basic sequence problem, *Studia Math.* **116** (1995), 167–187.
- R. J. Elliott and N. J. Kalton, Values in differential games, *Bull. Amer. Math. Soc.* **78** (1972), 427–431.
- R. J. Elliott and N. J. Kalton, The existence of value in differential games, *Mem. Amer. Math. Soc. No.* **126** (1972), 67 pp.
- N. J. Kalton, A note on pairs of projections, *Bol. Soc. Mat. Mexicana* **3** (1997), 309–311.
- N. J. Kalton and T. Kucherenko, Operators with an absolute functional calculus, *Math. Ann.* **346** (2010), 259–306.
- N. J. Kalton and G. Lancien, A solution to the problem of L_p -maximal regularity, *Math. Z.* **235** (2000), 559–568.
- N. J. Kalton and G. Lancien, L_p -maximal regularity on Banach spaces with a Schauder basis, *Arch. Math.* **78** (2002), 397–408.
- N. J. Kalton and C. LeMerdy, Solution of a problem of Peller concerning similarity, *J. Operator Theory* **47** (2002), 379–387.
- N. J. Kalton, A remark on the H^∞ -calculus, *CMA/AMSI Research Symposium, “Asymptotic Geometric Analysis, Harmonic Analysis, and Related Topics”*, 81–90, *Proc. Centre Math. Appl. Austral. Nat. Univ.*, 42, *Austral. Nat. Univ.*, Canberra, 2007.
- N. J. Kalton and P. Portal, Remarks on ℓ_1 and ℓ_∞ -maximal regularity for power-bounded operators, *J. Aust. Math. Soc.* **84** (2008), no. 3, 345–365.
- N. J. Kalton, Unusual traces on operator ideals, *Math. Nachr.* **134** (1987), 119–130.
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- K. J. Dykema and N. J. Kalton, Spectral characterization of sums of commutators II, *J. Reine Angew. Math.* **504** (1998), 127–137.
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- S. Ivanov and N. Kalton, Interpolation of subspaces and applications to exponential bases, *St. Petersburg Math. J.* **13** (2002), 221–239.
- N. J. Kalton and L. Rubel, Gap-interpolation theorems for entire functions, *J. Reine Angew. Math.* **316** (1980), 71–82.

- N. J. Kalton and L. Tzafriri, The behaviour of Legendre and ultraspherical polynomials in L_p -spaces, *Canad. J. Math.* **50** (1998), 1236–1252.
- N. J. Kalton and L. Weis, The H^∞ -calculus and sums of closed operators, *Math. Ann.* **321** (2001), 319–345.
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- N. Kalton, P. Kunstmann, and L. Weis, Perturbation and interpolation theorems for the H^∞ -calculus with applications to differential operators, *Math. Ann.* **336** (2006), 747–801.
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- N. J. Kalton and L. Weis, The H^∞ -functional calculus and square function estimates, pp. 716–771.