How to build your own paraconsistent logic:

an introduction to the Logics of Formal (In)Consistency

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> Two monks were arguing about the flag waving in the wind
> One said, "The flag moves." The other said, "The wind moves."
> Hui Neng said, "It is neither the flag, nor the wind
> It's your mind that moves."

Abstract

*The logics of formal inconsistency (***LFI***s) are logics that allow to explicitly formalize the concepts of consistency and inconsistency by means of formulas of their language. Contradictoriness, on the other hand, can always be expressed in any logic, provided its language includes a symbol for negation. Besides being able to represent the distinction between contradiction and inconsistency,* **LFI***s are non-explosive logics, in the sense that a contradiction does not entail arbitrary statements, but yet are gently explosive, in the sense that, adjoining the additional requirement of consistency, then contradictoriness does cause explosion. Several logics can be seen as* **LFI***s, among them the great majority of paraconsistent logics developed under the Brazilian tradition, as well as the sytems developed under the Polish tradition. We present here their semantical interpretations by way of possible-translations semantics, stressing their significance and applications to human reasoning and machine reasoning. We also give tableaux systems for some important* **LFI***s:* **bC***,* **Ci** *and* **LFI1***.*

KEYWORDS: contradiction, inconsistency, consistency, paraconsistency, tableaux.

1. Internalizing consistency and inconsistency

It is well accepted that in the activities of knowledge engineering and managing databases inconsistent information is the rule rather than the exception; contradictions are presumably unavoidable in our formal theories. However, this does not constitute necessarily a drawback; inconsistent theories can be quite *informative*, and it is desirable that we learn to *reason* from them in a sensible way. The first striking example are the usual proofs by *reductio ad absurdum*, where you start from a dubious premise P and are quite happy to arrive at a contradiction: this contradiction informs you that P was not correct, and a further step settles the question allowing you to conclude $\neg P$. What are you really doing in such use of *reductio*? You are starting by assuming P, and when you find a contradiction, you proceed to discharge the assumption, taking your whole reasoning as a proof of $\neg P$. There is no contradiction between P and $\neg P$ because you have concluded $\neg P$ after discharging P, but there was a moment when you had a contradiction on hands, and you decided to use it strategically to discharge the dubious hypothesis P, instead of concluding anything! This shows in a clear way that the "logic of finding proofs" cannot be classical logic, otherwise you should conclude anything from a successful proof by *reductio ad absurdum*.

Contradictions are also quite informative outside logic and mathematics. Consider, for instance, a situation in which you ask two persons, in the course of an investigation, an 'yes-no' question such as 'Were both of you out of town when the crime occurred?', so that what will result will be exactly one of the three following different possible outcomes: they might both say 'yes', they might both say 'no', or else one of them might say 'yes' while the other says 'no'. Now, it happens that only in the *last scenario*, where an inconsistency appears, *you are sure* to have received wrong information from one of your sources!

The trouble is that classical logic (and extensions of classical logic like modal logics, and even fragments of it, like intuitionistic logic) as it is well known, cannot survive contradictions, because any classical theory explodes in the presence of contradictions, in the sense that all sentences turn out then to be deducible. In more formal terms, given a logic whose language includes a symbol $\neg$ for negation let's call a theory in this logic *contradictory* if it derives A and its negation $\neg A$ for some formula A, call a theory *trivial* if any formula B can be derived from it (in the underlying logic), and finally call a theory *explosive* if adding to it any pair of contradictory sentences A and $\neg A$ is sufficient to make it trivial. The underlying logic will be called *contradictory*, *trivial*, or *explosive* if all their theories are, respectively, contradictory, trivial or explosive. It is clear that any trivial theory (or logic) will also be contradictory (if anything is derivable from a theory, in particular all pairs of formulas of the form A and $\neg A$ will be). Inside classical or intuitionistic logic, and, in a general way, inside of what we call explosive logics the contradictory and the trivial theories simply coincide, by way of their explosive character, if the logic satisfies some specific requirements (the Tarskian conditions), as explained below.

Paraconsistent logics were then proposed to be the logics to underlie those contradictory but non-trivial theories, by way of weakening or annulling the explosive character of these theories. So, paraconsistency provides a sharp distinction among the logical notions of *contradictoriness*, *explosiveness*, and *triviality*. This is the first key to paraconsistency: if there are no contradictions around, then everything is under control (once we are inside of a consistent environment); but if contradictions appear, what we must control is the *explosive* character of our underlying logic.

But paraconsistency also permits to distinguish between the logical notions of *inconsistency* and of *contradictoriness* in a purely abstract way. Distinctions have been proposed, in the literature, among the notions of *paradoxical* and of *antinomical* theories, the paradoxical ones being identified with those theories in which inconsistencies could occur without necessarily leading to trivialization, and the antinomical ones identified with those in which any occurring contradiction turns out to be fatal, as in the case of Russell's antinomy in naive set theory.

The first paraconsistent calculi were independently proposed by N. da Costa (cf. [14]) and Jaśkowski (cf. [20]), and are also related to D. Nelson's ideas on constructible falsity (as a symmetrization or dualization of intuitionistic negation) in [23]. The formal apparatus of da Costa's systems is based on the idea that 'consistency' (or what he called 'well-behavior') of a given formula is a sufficient requisite to guarantee its explosive character, and that this could be represented by another formula of the underlying logic (he chose, for his first calculus, C_1, to represent the consistency of a formula A by the formula $\neg(A\wedge\neg A)$, and referred to this last formula as intuitively reading a 'it is not the case that both A and $\neg A$ are true'. Our proposal, inspired by da Costa's idea, is exactly that of introducing consistency as a *primitive notion* of our logics: the paraconsistent logics which internalize the notion of consistency (thus introducing consistency at the object level, and not as a metamathematical concept) will be called *logics of formal inconsistency* (**LFI**s). And, given a consistent logic **L**, the **LFI**s which extend the positive basis of **L** and have connectives in their language to express consistency or inconsistency will be said to constitute **C**-

systems based on **L**. Our main contribution is to study a large class of **C**-systems based on classical logic (of which the calculi C_n, $0<n<\omega$, of da Costa will be but very particular examples).

The internalization of consistency inside our logics will be accomplished by the addition of a unary connective '∘' expressing consistency (and another connective '•' expressing inconsistency), based on the following assumption: *consistency* is exactly what makes a theory become trivial when exposed to a contradiction.[1] Consistency can be regarded as and *ideal element*, in much the same way as the (imaginary) complex numbers can be used to computing: both the formal notion of consistency and the $\boldsymbol{i} = \sqrt{-1}$ are *ideal* elements which make new calculations possible. The point is not to *validate any falsity*, but to *extend the notion of truth* (this idea is also explored in [4]). So, in a nutshell, the basic guidelines behind **LFI**s are the following:

GL1) Triviality entails contradictoriness (if we have, as usual, a symbol for negation);
GL2) contradictoriness entails inconsistency (or, to be more precise, contradictoriness entails 'non-consistency', since consistency and inconsistency are not necessarily dual, as we shall see);
GL3) contradictoriness *plus* consistency implies triviality.

We are in fact introducing, in this way, a novel and fine-grained definition of consistency that will apply for a large class of logics: here consistency has a "constructive" flavor: non-contradictoriness will be a necessary but *no longer* a sufficient requirement for us to prove consistency. In the case of explosive logics, of course, the concepts of non-contradictoriness and non-triviality will coincide, so that non-contradictoriness and consistency are also to be identified.

We can identify three basic philosophical (meta)principles governing logical systems in general: 1) The *Principle of Pseudo-Scotus* (**PPS**), also known as *ex contradictio sequitur quodlibet* (and also called *Principle of Explosion* by some contemporary logicians), stating that any theory exposed to a pair of contradictory statements A and $\neg A$ derives any other arbitrary sentence B, so that it turns out to be *trivial*; 2) The *Principle of Non-Contradiction* (**PNC**) stating that there are theories from which no such contradictions are derivable, and 3) the *Principle of Non-Triviality* (**PNT**) stating that there is at least one theory and one sentence B such that B is not derivable from this theory.

A theory Γ is said to be *gently explosive* when there is always a way of expressing the consistency of a given formula A by means of formulas which depend at most on A. A gently explosive logic is exactly a logic having only gently explosive theories, and we can now formulate, a 'gentle version' of **PPS** for a given logic **L**, asserting that this logic must be gently explosive. Gently explosive paraconsistent logics are precisely those logics that we have above dubbed **LFI**s, the logics of formal inconsistency. In the logics we will be studying along this paper, we will in general assume that the consistency of each formula A can be expressed by operators already at their object (i.e., linguistic) level, written as $\circ A$, where '∘' is the 'consistency connective'. The **C**-systems (here always having classical logic at the background), will be particular **LFI**s.

There are several other forms of explosion, as: 1) the *partial explosion,* which does not trivialize the whole logic, but just part of it (for instance, when a contradiction does not prove every formula, but does prove every *negated* formula, as it occurs in the case of Kolmogorov & Johánsson's Minimal Intuitionistic Logic, **MIL** (see [21]) where, for every Γ, A and B, we have $\Gamma, A, \neg A \Vdash \neg B$ although it is not the case that, in general, $\Gamma, A, \neg A \Vdash B$; 2) the *ex falso explosion*, which asserts that there must exist one element in the logics so that everything follows from it (a kind of *falsum*, or *bottom particle*); 3) *controllable explosion*, which states that, if not all, at least some of our formulas should lead to trivialization when taken together with their negations, and 4) *supplementing explosion*, which states that the logics should possess, or be able to define, a *supplementing*, or *strong* negation, to the effect that a pair of contradictory propositions A, $\sim A$ should explode when $\sim A$ is a strong negation of A. All of

[1] Our assumption is compatible with Jaśkowski's intuition, as expressed in [20], p.144 (our italics): 'in some cases we have to do with a system of hypotheses which, *if subjected to a too consistent analysis*, would result in a contradiction between themselves or with a certain accepted law, but which we use in a way that is restricted so as not to yield a self-evident falsehood'.

these alternative forms of explosion can be turned into logical (meta)principles alternative to 'full' Pseudo-Scotus (see [11]).

The paraconsistent logics studied in this paper will, of course, disrespect Pseudo-Scotus, and in addition to that they will also disrespect the principle regarding partial explosion while respecting the principles regarding gentle explosion, *ex falso*, supplementing explosion, and, often, controllable explosion as well.

Let ***For*** be a collection of formulas of a certain language having a unary symbol $\neg$ for negation, and call a *theory* any subset of ***For***. Let a *consequence relation* $\vdash$ over ***For*** be a relation between theories and formulas of ***For***, that is, $\vdash \subseteq (\wp(\boldsymbol{For}) \times \boldsymbol{For})$, where $\wp(\boldsymbol{For})$ denotes the power set of ***For***. We define a *logic* ***L*** to be a structure $\boldsymbol{L} = \langle \boldsymbol{For}, \vdash \rangle$. The consequence relation of some given logic is often defined by its axioms and rules, or else from some semantical interpretation associated to this logic.

The relation $\vdash$ is usually required to follow certain specific requirements, known as the Tarskian conditions, which are the following:

1) reflexivity: $A \in \Gamma \Rightarrow \Gamma \vdash A$;
2) monotonicity: $(\Gamma \vdash A$ and $\Gamma \subseteq \Delta) \Rightarrow \Delta \vdash A$;
3) transitivity: $(\Gamma \vdash A$ and $\Delta, A \vdash B) \Rightarrow \Gamma, \Delta \vdash B$.

Fix some logic ***L*** for the discussion below. In formal terms, a theory Γ of ***L*** is said to be:

1) *contradictory with respect to* $\neg$, or simply *contradictory*, if there exists a formula A such that $\Gamma \vdash A$ and $\Gamma \vdash \neg A$;
2) *trivial* if, for every A, we have $\Gamma \vdash A$.
3) *explosive* if, for every A, we have $\Gamma, A, \neg A \vdash B$, for every B.

A logic ***L*** on its turn is *contradictory*, *trivial*, or *explosive* if, respectively, all of its theories are contradictory, trivial, or explosive.

We can now restate **PNC, PPS** and **PNC** in more formal terms, for a given logic ***L***:

1. **Principle of Non-Contradiction** (**PNC**) for a logic ***L***:

L should have non-contradictory theories, that is, there should be some theory Γ such that for no A it holds that $\Gamma \vdash A$ and $\Gamma \vdash \neg A$.

2. **Principle of Pseudo-Scotus** (**PPS,** also called Principle of Explosion) for a logic ***L***:

L should have only explosive theories, that is, for every theory Γ, the theory $\Gamma \cup \{A, \neg A\}$ is trivial.

3. **Principle of Non-Triviality** (**PNT**) for a logic ***L***:

L should have a non-trivial theory, that is, there must exist a theory Γ and some formula A such that $\Gamma \nvdash A$.

Those principles hold for several logics, and it can be easily shown that, assuming **PPS**, and taking into account the transitivity of the relation $\vdash$, **PNC** and **PNT** are equivalent. So, for several logics, including classical logics, it is sufficient to assume either **PNC** or **PNT**. It seems intuitively acceptable that **PNT** should be taken as the most important of those three principles —after all, if **PNT** does not hold for a certain logic, then every Γ would deduce every A and the relation $\vdash$ would be total, that is, $\vdash = (\wp(\boldsymbol{For}) \times \boldsymbol{For})$. In this case, $\vdash$ would not be a very interesting *deductive* relation, for it would stop 'making the difference', failing to purport, we argue, any special meaning to the notion of *derivability*. We shall, accordingly, stick to **PNT** throughout this study, avoiding the consideration of trivial logics.

The logics of formal inconsistency, will define a large class of paraconsistent logics where the notions of consistency and inconsistency can be expressed inside the logic by a schema of formulas $\Delta(A)$ depending at most on A such that: i) $\Delta(A), A$ is in general non-trivial; i) $\Delta(A), \neg A$ is in general non-trivial, but: iii) $\Delta(A), A, \neg A$ is trivial for every theory Γ, i.e., $\Gamma, \Delta(A), A, \neg A \vdash B$, for every B.

A logic which follows this principle will be said to follow the Principle of Gentle Explosion; an **LFI** is thus a paraconsistent logic that satisfies a more tolerant formulation of **PPS** (expressed by the Principle of Gentle Explosion) while taking for granted **PNT** and also **PNC** (for, although we want our logics to be able to *support* contradictory theories, we may not *want* that our logics derive contradictions). Paraconsistent logics are often misunderstood as logics that derive contradictions. This is a blatant mistake, at least on our formulation of the matter: the great majority of the paraconsistent logics found in the literature, and all the ones studied here, bring no built-in contradiction in their axioms, and their inference rules do not generate contradictions from the axioms. Even so, because of suitable constraints on the power of explosiveness they can be used as underlying logics that permit to reason under contradictoriness without slippering into triviality. Although there exist some logics (the so-called *dialectical logics*, or *logics of impossible objects*, cf. [24] for example) that disrespect both **PPS** and **PNC**, and have theses which are not classical theses, this particular case of paraconsistent logics will *not* be studied here. The logics surveyed in this study just support contradictions and permit reasoning with them, but neither engender contradictions nor validate any unexpected form of reasoning. To the contrary: as we shall see, the logics of inconsistency are, in a sense, 'more conservative' than classical logic.

2. How to build your own C-system

The main idea is to conceive the concept of inconsistency in such a way that, while contradictory theories are certainly inconsistent, the converse might not necessarily be true. In a similar way as to the concept of point, which is taken in geometry as a primitive notion only describable through its relationship to other concepts, inconsistency could be taken in logic to be a primitive notion as well. Under this view, inconsistent and contradictory theories do not coincide, as our logics of formal inconsistency **bC** and **Ci** will make clear.

It is time now to give a more precise definition for the logics of formal inconsistency (**LFI**s): an **LFI** is any logic, as explained above, where a syntactic notion of *formal consistency* can be defined in such a way that this new notion of formal consistency and the notion of contradiction can be related in the light of the guideline (**GL3**). In particular, as we discuss below, in many cases this can be done by endowing the language with a new connective $\circ$ and considering new appropriate axioms.

Among the **LFI**s it is possible to identify a subclass of the so-called **C**-*systems*, as systems which preserve the positive fragment of some other consistent logic and in which consistency or inconsistency are expressible by means of connectives. As a subclass of the **C**-systems, we define the **dC**-*systems* as the ones where the notion of formal consistency can be defined in terms of other connectives of the language. The **dC**-systems comprise several classes of paraconsistent systems, including the ones in the hierarchy C_n, $0<n<\omega$, of da Costa (cf. [15]) and the logic **D2** of Jaśkowski (cf. [20]).

To be sure, not all paraconsistent logics are **C**-systems: Consider for instance the logic

∧	**1**	**½**	**0**
1	1	½	0
½	½	½	0
0	0	0	0

∨	**1**	**½**	**0**
1	1	1	1
½	1	½	½
0	1	½	0

→	**1**	**½**	**0**
1	1	½	0
½	1	½	0
0	1	1	1

	¬
1	0
½	½
0	1

Pac, given by the above matrices, where both 1 and ½ are distinguished values. (cf. Avron's [1], under the name $RM_3^{\supset}$, and, in Batens's [2] under the name PI^s). It is easy to see that in this logic, for no formula *A* it can be the case that $A, \neg A \vDash_{Pac} B$, for all *B*. So, *Pac* is a non-explosive, thus paraconsistent, logic. Although conjunction, disjunction and implication in *Pac* are fairly classical (in fact, the whole positive classical logic is validated by its matrices) the negation in *Pac* is too strongly non-classical. Indeed, no negation having all classical properties can be definable in *Pac*, since any truth-function of this logic having only ½'s as input will also have ½ as output. As a consequence, *Pac* provides a very weak interpretation for negation, once in this logic all contradictions are admissible. Other paraconsistent logics like Priest's logic *LP* defined in [24] (which is just the implicationless fragment of *Pac*) are simply immune to any contradictions. Those are below the line of **C**-systems, which attempt to express some classical forms of re asoning.

However, *Pac* can be easily extended to a **C**-system by adding a strong negation or a *falsum* constant (a bottom particle) what will result in the logic called $\mathbf{J}_3$ (studied by D'Ottaviano and da Costa in 1970, cf. [19] but already introduced for purely proof-theoretical purposes in [25], and also called **CLuNs,** cf. [3]). The extension so obtained is still paraconsistent and has all those special explosive theories absent in *Pac*. In [13] we have explored the possibility of applying (a variant of) this logic to the study of inconsistent databases, introducing the connective $\circ$ as primitive. As a result, this logic (renamed **LFI1**, one of the main 'logics of formal inconsistency') has been shown to be adequate, among other options, for the task of formalizing the notion of (in)consistency in a flexible and sensible way. Its matrices are given in Section 5.

We want to show a wide class of **C**-systems based on classical logic, defining several systems by axiomatic extensions. Call C_{min} the logic characterized by all positive axioms of classical logic, taking $\wedge$, $\vee$, $\to$ and $\neg$ as primitive connectives, plus the axioms $(\neg\neg A\to A)$ and $(A\vee\neg A)$ and closed under the rules of Modus Ponens and Substitution, that is

(Min1) $\vdash_{min} (A\to(B\to A))$;
(Min2) $\vdash_{min} ((A\to B)\to((A\to(B\to C))\to(A\to C)))$;
(Min3) $\vdash_{min} (A\to(B\to(A\wedge B)))$;
(Min4) $\vdash_{min} ((A\wedge B)\to A)$;
(Min5) $\vdash_{min} ((A\wedge B)\to B)$;
(Min6) $\vdash_{min} (A\to(A\vee B))$;
(Min7) $\vdash_{min} (B\to(A\vee B))$;
(Min8) $\vdash_{min} ((A\to C)\to((B\to C)\to((A\vee B)\to C)))$;
(Min9) $\vdash_{min} (A\vee(A\to B))$;
(Min10) $\vdash_{min} (A\vee\neg A)$;
(Min11) $\vdash_{min} (\neg\neg A\to A)$.

While C_{min} is positive preserving relative to classical logic, the logic obtained by deleting (Min9) from C_{min} coincides with the logic C_ω proposed by da Costa (cf. [15]). C_{min} is only positively preserving relative to intuitionistic logic. The main properties of C_{min} are summarized below (all proofs, unless otherwise indicated, are to be found in [8] or [11]):

THEOREM 1.

(i) The *Deduction Metatheorem* holds, i.e., $\Gamma, A \vdash_{min} B \Rightarrow \Gamma \vdash_{min} (A\to B)$.
(ii) The theorem of Pseudo Scotus (tPS) $(A\to(\neg A\to B))$ is not provable in C_{min}.
(iii) Adding (tPS) to C_{min} provides a sound and complete axiomatization for Classical Propositional Logic, **CPL**.
(iv) A form of *proof-by-cases* holds in C_{min}, i.e, $(\Gamma, A \vdash_{min} B)$ and $(\Delta, \neg A \vdash_{min} B) \Rightarrow (\Gamma, \Delta \vdash_{min} B)$.
(v) C_{min} does not have neither a strong negation nor a bottom particle, and is not finitely trivializable (that is, every finite theory in C_{min} is non-trivial);
(vi) C_{min} does not have any negated theorem, i.e. $(\nvdash_{min} \neg A)$.
(vii) No two different negated formulas of C_{min} are provably equivalent. ■

Theorem 1.(v) shows that C_{min}, (and C_ω as well) cannot be a **C**-system based on classical logic, or intuitionistic logic, once it cannot be gently explosive, and thus cannot formalize 'consistency', in the precise sense formulated here. We need for this purpose a deductively stronger calculus.

The *basic logic of formal inconsistency,* **bC**, is then defined as C_{min} plus the following axiom, called the *Gentle Principle of Explosion* (where $\circ$ is a new unary operator):

$$\circ A, A, \neg A \vdash B$$

reading $\circ A$ as 'A is consistent'. The reader will notice that this axiom is coherent with the (**GL3**) above: a contradictory theory (the one containing A and $\neg A$) is not necessarily trivial —it becomes trivial if, besides being contradictory, the contradictory formula is consistent. In such a case, its very consistency becomes contradictory, and this situation leads thus to triviality.

It can be proven that **bC** does have negated theorems, and equivalent negated formulas (but, on the other hand, it has no consistent theorems, that is, theorems of the form $\circ A$). Examples of theorems relating contradiction and consistency in **bC** are:

THEOREM 2

i. $A, \neg A \vdash_{\mathbf{bC}} \neg\circ A$ ('if *A* is contradictory, then *A* is not consistent');

ii. $\circ A \vdash_{\mathbf{bC}} \neg(A\wedge\neg A)$ ('if *A* is consistent then *A* is non-contradictory', first form);

iii. $\circ A \vdash_{\mathbf{bC}} \neg(\neg A\wedge A)$ ('if *A* is consistent then *A* is non-contradictory', second form).[2] ■

The converses of the above rules do *not* hold in **bC**. Also, interdefinability of connectives is usually not valid:

THEOREM 3. The following rule holds in **bC**:

$(\neg A\rightarrow B) \vdash_{\mathbf{bC}} (A\vee B)$, but none of the other usual De Morgan rules holds in **bC**. ■

The property of *intersubstitutivity of provable equivalents* (IpE) is not valid in **bC** and in many other paraconsistent logics, and this is responsible for the difficulties of algebraization of these logics.

THEOREM 4 In **bC** we have:

(i) $(A\wedge B) \dashv\vdash_{\mathbf{bC}} (B\wedge A)$ holds, but $\neg(A\wedge B) \dashv\vdash_{\mathbf{bC}} \neg(B\wedge A)$ does not;

(ii) $(A\vee B) \dashv\vdash_{\mathbf{bC}} (B\vee A)$ holds, but $\neg(A\vee B) \dashv\vdash_{\mathbf{bC}} \neg(B\vee A)$ does not;

(iii) $(A\wedge\neg A) \dashv\vdash_{\mathbf{bC}} (\neg A\wedge A)$ holds, but $\neg(A\wedge\neg A) \dashv\vdash_{\mathbf{bC}} \neg(\neg A\wedge A)$ does not.

(iv) $(A\vee\neg A) \dashv\vdash_{\mathbf{bC}} (B\vee\neg B)$ holds, but $\neg(A\vee\neg A) \dashv\vdash \neg(B\vee\neg B)$ does not. ■

An important observation is that in **bC** the notions of 'not consistent' and 'inconsistent' do *not* necessarily coincide. Indeed, even if we introduce the concept of 'consistent' as internal to the language, through a new connective $\bullet$ added to the language (reading $\bullet A$ as '*A* is inconsistent') $\circ A$ and $\neg\bullet A$, as well as $\neg\circ A$ and $\bullet A$, will not necessarily be interdefinable, contrary to what one could hastily conclude -the notions of 'not consistent' and 'inconsistent' will or will not be interdefinable depending upon which new axioms would govern the connectives $\circ$ and $\bullet$.

THEOREM 5

(i) $\neg\bullet A \nvdash_{\mathbf{bC}} \circ A$ ('If *A* is not inconsistent, then it is consistent' is not provable in **bC**);

(ii) $\neg\circ A \nvdash_{\mathbf{bC}} \bullet A$ ('If *A* is not consistent, then it is inconsistent' is not provable in **bC**);

(iii) $\bullet A \nvdash_{\mathbf{bC}} \neg\circ A$ ('If *A* is inconsistent, then it is not consistent' is not provable in **bC**);

(iv) $\circ A \nvdash_{\mathbf{bC}} \neg\bullet A$ ('If *A* is consistent, then it is not inconsistent' is not provable in **bC**). ■

In **bC** a new negation, called *strong negation*, can be defined as: $\sim A =_{\text{def}} \neg A\wedge\circ A$, which recovers several features of classical negation, though not all. We have, for instance, that $A, \sim A \vdash_{\mathbf{bC}} B$ for all *A* and *B*, and thus **PPS** holds relative to this strong negation (that is, this negation is explosive) but $\nvdash_{\mathbf{bC}}$ $(A\vee\sim A)$ and $\nvdash_{\mathbf{bC}}$ $(A\rightarrow\sim\sim A)$. Consequently, the strong negation $\sim A$ is not classical, even though it is explosive (in intuitive terms, $\sim A$ is somehow analogous to the intuitionistic negation). But another strong negation, this one having all properties of classical negation, is definable in **bC** by letting $\dot{\sim} A =_{\text{def}}$ $A\rightarrow(A\wedge\sim A)$

Now, a stronger logic of formal inconsistency, **Ci**, is defined by adjoining to **bC** the axiom:

$$\bullet A \vdash (A\wedge\neg A),$$

and defining $\bullet A =_{\text{def}} \neg\circ A$.

In the system **Ci**, inconsistency and contradictions are equivalent to each other, due to the fact that $(A\wedge\neg A) \vdash_{\mathbf{bC}} \neg\circ A$ (as noted before) and to the axiom just introduced, plus the definitions. Omitting the definitions, however, we obtain intermediate logics between **bC** and **Ci**.

[2] In several systems of paraconsistent logics, although $(A\wedge\neg A)$ and $(\neg A\wedge A)$ are equivalent, $\neg(A\wedge\neg A)$ and $\neg(\neg A\wedge A)$ cannot be proven equivalent.

Moreover, in **Ci** both strong negations mentioned above are equivalent and acquire all the properties of classical negation —but it is still possible to define non-classical strong negations in **Ci**, via for example $\neg\neg{\sim}A$ and $\neg\neg\dot{-}A$. Some properties of this logic are:

THEOREM 6 The following rules hold in **Ci**:

(i) $\circ A,\ \bullet A \vdash_{\mathbf{Ci}} B$;
(ii) $\circ A,\ \neg\circ A \vdash_{\mathbf{Ci}} B$;
(iii) $\bullet A,\ \neg\bullet A \vdash_{\mathbf{Ci}} B$;
(iv) $\vdash_{\mathbf{Ci}} \circ\circ A$;
(v) $\vdash_{\mathbf{Ci}} \neg\bullet\circ A$;
(vi) $\vdash_{\mathbf{Ci}} \circ\bullet A$;
(vii) $\vdash_{\mathbf{Ci}} \neg\bullet\bullet A$.;
(viii) $\neg\circ A \vdash_{\mathbf{Ci}} (A\wedge\neg A)$

but the following do not hold: $\neg(A\wedge\neg A) \vdash_{\mathbf{Ci}} \circ A$; $\neg(\neg A\wedge A) \vdash_{\mathbf{Ci}} \circ A$. ■

What does not hold in this logic? Still, **PPS** does not hold, that is, $A,\ \neg A \nvdash_{\mathbf{Ci}} B$, for some A and B. De Morgan laws and the rule of contraposition only hold in restricted forms, for instance $(A\to B) \nvdash_{\mathbf{Ci}} (\neg B\to\neg A)$, but $(A\to\circ B) \vdash_{\mathbf{Ci}} (\neg\circ B\to\neg A)$. Also, **Ci** does not prove any formulas to be consistent, unless they already refer to consistency or inconsistency, that is, $\circ A$ is provable in **Ci** if, and only if, A is itself of the form $\circ\neg B$, $\circ B$, $\bullet B$ or $\bullet\neg B$ for some B.

On the light of Theorem 6.(vii) we may consider the addition of some other axioms to **Ci**, as for example:

Levo-based axiom for contradictoriness (**cl**):[3] $\neg(A\wedge\neg A) \vdash \circ A$
Dextro-based axiom for contradictoriness (**cd**): $\neg(\neg A\wedge A) \vdash \circ A$
Bi-directional axiom for contradictoriness (**cb**): $\neg(A\wedge\neg A) \vee\neg(\neg A\wedge A) \vdash \circ A$
Global axiom for contradictoriness (**cg**): $(B\leftrightarrow(A\wedge\neg A))\to(\neg B\leftrightarrow\neg(A\wedge\neg A))$
Expansion of double negations: (**ce**) $A \vdash \neg\neg A$.

A **dC**-*system* is any **C**-system where $\circ$ and $\bullet$ can be defined in terms of other connectives in the language. In the case of da Costa's C_1, the stronger calculus of his hierarchy of paraconsistent logics in [15], $\circ A$ is defined as $\neg(A\wedge\neg A)$ and $\bullet A$ as $\neg\circ A$, so that this logic is an extension of **Cil**, the logic obtained by the addition of the axiom (**cl**) to **Ci**.

Several other distinct **dC**-systems can be defined by choosing adequate axioms of 'propagation', for consistency and inconsistency, and many different choices are possible —depending on the particular ones, we may obtain finite many-valued paraconsistent logics or infinite-valued paraconsistent logics (and , in an extreme case, even classical logic).

Some choices which have been tried for the axioms of propagation are the following:

First Choice: (**ca**) $(\circ A\wedge\circ B) \to \circ(A\#B)$, for all binary connectives #;
Second Choice: (**ca**) $(\circ A\vee\circ B) \to \circ(A\#B)$, for all binary connectives #;
Third Choice: (**cv**) $\circ(A\#B)$ for all binary connectives #;
Fourth Choice: (**cr**) $\circ(A\#B) \vdash (\circ A\vee\circ B)$ for all binary connectives #;
Fifth Choice: (**cj1**) $\bullet(A\wedge B) \leftrightarrow (\bullet A\wedge B) \vee (\bullet B\wedge A)$
(**cj1**) $\bullet(A\vee B) \leftrightarrow (\bullet A\wedge\neg B) \vee (\bullet B\wedge\neg A)$
(**cj1**) $\bullet(A\to B) \leftrightarrow (\bullet B\wedge A)$
Sixth Choice: (**cw**) $\circ(\neg A)$ for every formula A;
Seventh Choice: $\circ(A)$ for every formula A.

Now several **C**-systems can be defined by the particular modes of propagating consistency (or inconsistency), adjoining to **Ci** the following axioms:

1) (**ca**) plus (**cl**) above defines the calculus **Cila**, which coincides with C_1 of the hierarchy of C_n;

[3] This axiom will hold for the system C_1 of da Costa, for example, as discussed in [11].

2) (**co**) plus (**cl**) defines the calculus **Cilo**, equivalent to a calculus called C_1^+ (cf. [17]);
3) (**cv**) and (**cw**) plus (**cb**) define the three-valued maximal logic **Cibvw** equivalent to $\mathbf{P}^1$, introduced in [26] (here (**cb**) is provable, so **Cibvw = Civw**);
4) (**cv**) plus (**cb**) and (**ce**) define the three-valued maximal logic **Cibve** equivalent to $\mathbf{P}^2$, (cf. [22]) (here (**cb**) is provable, so **Cibve = Cive**);
5) (**co**), (**cr**) and (**cw**) plus (**cb**) define the three-valued maximal logic **Ciborw** also called $\mathbf{P}^3$, (here (**cb**) is provable, so **Ciborw = Ciorw**);
6) similarly, **Cije** and **Ciore** define, respectively, the three-valued maximal logics **LFI1** and **LFI2** studied in [13];
7) finally, the seventh choice defines classical propositional logic.

Many other combinations are possible: the logics **Cido**, **Cibo**, **Cigo**, **Ciloe**, **Cidoe**, **Ciboe** and **Cigoe** for example are all extended by the three-valued paraconsistent logic $\mathbf{P}^2$. The logic **LFI1**, in the sixth choice, gives yet another axiomatization for the three-valued paraconsistent calculus $\mathbf{J}_3$. As mentioned before, this logic has reappeared in the literature under different formulations.

Though many extensions of the logics of formal inconsistency are relatively simple, semantical interpretations are a complicated issue for paraconsistency in general. The initial da Costa's **C**-systems have been introduced only in proof-theoretical terms and only some years later semi-truth-functional bivalued semantics (cf. [16]) were proposed as their interpretation. These semantics, however, offer a weak meaning to paraconsistent logics, as they do not provide a reductive explanation for the presence of contradictory information; in particular, this makes difficult to apply and to implement them. In next section we describe an attractive alternative semantics called *possible-translations semantics*.

4.The semantics of possible-translations

The Rosetta Stone, found by Napoleon troops in 1799, contained inscriptions that were the key to deciphering Egyptian hieroglyphic writing. The deciphering was only possible due to the fact that the inscriptions appear in three forms: hieroglyphic, Demotic and Greek. By comparing the hieroglyphic and Demotic scripts with the Greek version, and starting from the fact that they contained the same text (this information was actually written in the readable Greek part of the stone) Thomas Young and Jean François Champollion were able to decipher the hieroglyphic and Demotic versions in 1822. The *possible-translations semantics* as a general tool for providing interpretation for non-standard logics (and applied to paraconsistent logics in [6] and [22]) are in many aspects similar to the deciphering of the Rosetta Stone (cf. also [9] and [12]).

A *translation* from a logic ***L*** into a logic ***L'*** is just a mapping between their sets of formulas which preserves derivability, that is, if A is provable in ***L*** from premises Γ and * is a translation from ***L*** into ***L'***, then A^* should be provable in ***L'*** from premises $\Gamma^* = \{B^*: B\in \Gamma\}$, i.e., if $\Gamma \vdash A$ then $\Gamma^* \vdash A^*$. In intuitive terms, the idea is to project a given 'hieroglyphic' logic by means of translations of it into simpler (usually many-valued) systems, and combine their respective forcing relations in order to obtain a sound and complete semantical interpretation for the initial complicated system. The simpler systems would thus play the role of the known languages for the Rosetta Stone analogy. We may think of this process as working in two distinct directions when analyzing a complicated logic in terms of simpler components, we call the process *splitting logics*; but it is also possible to conceive this process in the direction of synthesis, by defining a complex logic starting from simpler ones, and in this case we call *splicing logics*.

We give here an example of how this kind of semantics helps to give meaning to contradictions. For **Ci** the logics playing the role of 'simple known languages' in the Rosetta Stone analogy will be copies of the three-valued logic pictured in the following, with truth-values T, t and F, of which T and F are absolute 'true' and 'false', while t can be seen as hypothetically true. The meaning of $\wedge$, $\vee$, $\rightarrow$ is fixed for **Ci**,, but the meaning of $\neg$ and $\circ$ varies: each of them will be assigned two distinct interpretations, namely a weak and a strong one. For negation $\neg$, the weak interpretation $\neg_w$ regards the value t as

$\wedge$	T	t	F
T	t	t	F
t	t	t	F
F	F	F	F

$\vee$	T	t	F
T	t	t	t
t	t	t	t
F	t	t	F

$\to$	T	t	F
T	t	t	F
t	t	t	F
F	t	t	t

	$\neg_w$	$\neg_s$
T	F	F
t	t	F
F	T	T

	$\circ_w$	$\circ_s$
T	T	T
t	T	F
F	T	T

really true, and assigns to the negation of t also the value t. On the other hand, the strong interpretation $\neg_s$ makes no distinction between t and T, and assigns F to the negation of t. For $\circ$ the weak interpretation $\circ_w$ forgets the distinction between t and T, while the strong interpretation $\circ_s$ recognizes the value t as 'provisionary true', and thus potentially inconsistent.

For the system **Ci** the set of all recursive possible translations to be considered is definable by the following clauses, to be obeyed by any translation * in this set:

Tr 1. for atomic p, $p^* = p$, $(\neg p)^* = \neg_w p$;
Tr 2. $(\neg A)^* = \neg_s A^*$ or $(\neg A)^* = \neg_w A^*$, for non-atomic A;
Tr 3. $(A \# B)^* = A^* \# B^*$, for every $\# \in \{\wedge, \vee, \to\}$;
Tr 4. $(\circ A)^* = \circ_s A^*$ if $(\neg A)^* = \neg_w A^*$, and $(\circ A)^* = \circ_w A^*$ if $(\neg A)^* = \neg_s A^*$.

As an example, a formula of the form $\neg\circ\neg A$, will have in principle eight possible distinct translations, according to the above clauses: in case $(\neg\neg A)^* = \neg_w(\neg A)^*$, then $(\neg\circ\neg A)^*$ will be either $\neg_w\circ_s\neg_w A^*$, $\neg_s\circ_s\neg_w A^*$, $\neg_w\circ_s\neg_s A^*$, or $\neg_s\circ_s\neg_s A^*$; if $(\neg\neg A)^* = \neg_s(\neg A)^*$, then $(\neg\circ\neg A)^*$ will be either $\neg_w\circ_w\neg_w A^*$, $\neg_s\circ_w\neg_w A^*$, $\neg_w\circ_w\neg_s A^*$, or $\neg_s\circ_w\neg_s A^*$.

In other words, the syntax will be interpreted in different semantic scenarios, and here of course we have infinitely many distinct translations interpreting the formulas of **Ci** into distinct fragments of the above three-valued logics, according to the choices for the interpretation of the connectives $\wedge$, $\vee$, $\to$, $\neg$ and $\circ$.

Possible-translations semantics are a powerful tool for combining logics, complementary to other methods as fibring (cf. [5]). Possible-translations semantics have already been given to the calculi in the hierarchy C_n (cf. [22]), for a slightly stronger version of C_n (cf. [6]) and can be given to many other logics, offering thus a solution to the difficult problem of finding good semantics not only for paraconsistent logics, but for several other non-standard logics. It is also possible to give such semantics to some many-valued logics and other truth-functional logics as well, by means of a particular case called *society semantics*.

5. Modeling human and computer reasoning

Paraconsistent logics can be applied in the domain of automated reasoning and knowledge-based systems and databases. This first example refers to designing an implementation of deductive databases which are robust enough to work under contradictions (see [13] and [18]). Different users having equal access to some given database may introduce new facts, and even new rules or constraints, which, despite being possibly consistent from the point of view of each user, can still be globally contradictory. Traditional databases try to detect contradictory information and then start a complicated and expensive procedure of 'restoring consistency'. It is thus very natural to embed databases in logical environments which permit *reasoning* with contradictory information, while maintaining all other desirable features of traditional logic, such as reasoning under the law of excluded middle, reasoning by cases, and reasoning by means of quantifiers.

Information stored in databases must be checked to verify predefined conditions called integrity constraints in order to be safely integrated in the database. Integrity constraints are expressed by (fixed) first-order sentences; for example, a database storing information about books may contain the requirement that no book have more than one title, expressed by the following first-order formula:

$$\forall x \forall y \forall z((\mathrm{Title}(x, y) \wedge \mathrm{Title}(x, z)) \rightarrow (y = z));$$

where Title(x, y) means "y is the title of x". Updates in traditional databases are only performed if the new database would satisfy the integrity constraints; if not, the database maintains its previous state. While in traditional databases the integrity constraints must be fixed, using a paraconsistent logic in the background we can allow integrity constraints to be changed in time, permitting what we call *evolutionary databases*, which seem very much interesting for the domain of artificial reasoning. In [13] we show that **LFI1** and **LFI2**, as well as for their first-order extensions, **LFI1*** and **LFI2***, are very appropriate logics to treat databases that are robust with respect to inconsistency (see [18]).

Several **C**-systems can also be treated from the *analytical tableaux* perspective (see [10]). Tableaux are very important for real applications, as they provide an operative meaning for the connectives in the logic. (Indeed, a logic given in purely Hilbertian terms, if no tableau or sequent systems are provided, is practically useless from the perspective of automatic theorem proving, which is basic for applications).

1) Tableau rules for bC :

Tableaux are dyadic trees, and the well-known "α-β notation" means the following: a rule is of type α (an α–rule) if its consequences go in a same branch of the tree, and is of type β (a β-rule) if its consequences go into distinct branches.

α–rules for **bC**

α	α_1	α_2
1 $T(A \wedge B)$	$T(A)$	$T(B)$
2 $F(A \vee B)$	$F(A)$	$F(B)$
3 $F(A \rightarrow B)$	$T(A)$	$F(B)$
4 $T(\neg\neg A)$	$T(A)$	$T(A)$
5 $F(\neg A)$	$T(A)$	$T(A)$

β-rules for **bC**

β	β_1	β_2
6 $F(A \wedge B)$	$F(A)$	$F(B)$
7 $T(A \vee B)$	$T(A)$	$T(B)$
8 $T(A \rightarrow B)$	$F(A)$	$T(B)$
9 $T(\circ A)$	$F(A)$	$F(\neg A)$

The following is a derived rule of type β:

10. $T(\neg A)$	$F(A)$	$F(\circ A)$

To show that this is a derived rule, it is sufficient to show the existence of a closed tableau for $T(\neg A)$, $T(A)$, $T(\circ A)$ (use rule 9).

2) Tableau rules for Ci.

α-rules for **Ci**: are the same as for **bC**, plus:

α	α_1	α_2
11. $F(\circ A)$	$T(A)$	$T(\neg A)$
12. $T(\neg\circ A)$	$F(\circ A)$	$F(\circ A)$

β-rules for **Ci** are the same as for **bC**.

A tableau for **bC** or **Ci** is *closed* if all branches contain formulas $T(A)$ and $F(A)$ for some A.

3) Tableau rules for LFI1:

The logic **LFI1**, as mentioned in Section 2, is a three-valued paraconsistent logic with truth-values 1 and ½ (for "true" and "partially true") and 0 (for "false"); the matrices are the following:

$\wedge$	**1**	**½**	**0**
1	1	½	0
½	½	½	0
0	0	0	0

$\vee$	**1**	**½**	**0**
1	1	1	1
½	1	½	½
0	1	½	0

$\rightarrow$	**1**	**½**	**0**
1	1	½	0
½	1	½	0
0	1	1	1

	$\neg$	$\bullet$
1	0	0
½	½	1
0	1	0

where 1 and ½ are the designated values.

The α-rules for **LFI1** are the same as for **Ci** (i.e., α-rules 1, 2, 3, 4 for **bC,** 11, 12 for **Ci)** plus 5* (a modification of rule 5) and new rules 14 to 17 (we can here assume $\circ A$ as being defined by $\bullet A$):

α	α_1	α_2
5*. $F(\neg A)$	$T(A)$	$F(\bullet A)$
13. $F(\neg\neg A)$	$F(A)$	$F(A)$
14. $F(\bullet(A\wedge B))$	$F(\bullet A\wedge B)$	$F(\bullet B\wedge A)$
15. $F(\bullet(A\vee B))$	$F(\bullet A\wedge\neg B)$	$F(\bullet B\wedge\neg A)$
16. $F(\bullet(A\rightarrow B))$	$F(A\wedge\bullet B)$	$F(A\wedge\bullet B)$
17. $T(\bullet(A\rightarrow B))$	$T(A\wedge\bullet B)$	$T(A\wedge\bullet B)$

The β-rules for **LFI1** are the β-rules for **Ci (**i.e., β-rules 6, 7, 8, 9 for **bC**) plus:

β	β_1	β_2
18. $T(\bullet(A\wedge B))$	$T(\bullet A\wedge B)$	$T(\bullet B\wedge A)$
19. $T(\bullet(A\vee B))$	$T(\bullet A\wedge\neg B)$	$T(\bullet B\wedge\neg A)$

The following is a derived rule of type $\beta$for **LFI1**:

20. $F(\bullet A)$	$F(A)$	$F(\neg A)$

A tableau for **LFI1** is *closed* if all branches contain formulas $T(A)$ and $F(A)$ for some A, or contain a formula of the form $T(\bullet\bullet A)$ or of the form $T(\circ\circ A)$. A good exercise is to show that rule 5* permits to prove that $\neg(A\wedge\neg A)$ is a theorem of **LFI1.**

It is important to notice that rule 13 can also be added to**bC** and **Ci,** simply by adding an extra axiom $(A\rightarrow\neg\neg A)$.to these logics. Completeness and other results can be easily adapted, and we get slightly stronger logics.

Completeness of similar rules for the propositional calculus C_1 (and for its first-order version, C_1^*) was proven in [7] and other examples for automated reasoning were given in [10], [13] and [18].

Some simple yet illustrative examples of applications are given below.

Example 1: Revising an ill-defined concept

The so-called 'Nixon Diamond' is a problem frequently considered the literature. Suppose we have the following statements:

1) Nixon is a Quaker ($N \rightarrow Q$)
2) Quakers are pacifists ($Q \rightarrow P$)
3) Nixon is not a pacifist ($N \rightarrow \neg P$)

These statements are obviously contradictory assuming that there is some person having all such properties. At this point, a human reasoner would suspect that one of them should be disqualified, but this maneuver would be blocked if all of them had equal confidence status. If all of them are to be taken as true, there is no other rational possibility besides some concept being inexact, or vague, or subject to contradictions. Supposing it can be always clarified who is a Quaker and who is not, the only candidate for inexactness or possible contradictoriness is 'pacifist'. This is exactly the conclusion given by our system in [7] running a tableau for the set of propositions S (adding $\mathrm{T}(N)$ to guarantee the existence of such creature):

$$S = \{\mathrm{T}(N), \mathrm{T}(N \rightarrow Q), \mathrm{T}(Q \rightarrow P), \mathrm{T}(N \rightarrow \neg P)\}$$

the system concludes, instead of becoming blocked, that $\mathrm{F}(\circ P)$, that is, P is not consistent. This means that the concept of 'pacifist' is at least not well-defined in this context, and the system gives the user the opportunity to revise its definition.

Example 2: Belief revision: diamonds again

A diamond was stolen, and only two persons were present: Adam and Bob. Since there are no proofs (but only evidences) against them, the judge initially considers that they are not guilty, that is, his belief basis contains $\Delta_o = \{\neg A, \neg B\}$,where the sentences A and B stand, respectively, for "Adam is guilty" and "Bob is guilty". Later on, the diamond was found in their car, so the judge's belief basis must be enlarged to $\Delta_1 = \{\neg A, \neg B, A \vee B\}$. In this situation, if the judge used Classical Logic, he would derive $\Delta_1 \vdash_{\mathbf{CPC}} A \wedge B$, since Δ_1 is contradictory and causes explosion. Fortunately, however, judges (and by the way, everybody else) do not take classical logic so seriously: they cautiously wait for more information. The judge's attitude can be formalized by means of an **LFI**: using either **bC**, **Ci** or **LFI1**, for example, he does not conclude this. Let's choose the logic **LFI1** to express his rationale.

It is clear (just run a **LFI1**-tableau) that $\Delta_1 \nvdash_{\mathbf{LFI1}} A$ and $\Delta_1 \nvdash_{\mathbf{LFI1}} B$. A new investigation discovers that only Adam used the car since the diamond was stolen, and thus the judge's basis is again update to $\Delta_2 = \{\neg A, \neg B, A \vee B, \circ(\neg B)\}$ where $\circ(\neg B)$ means that the initial supposition about Bob's innocence is indeed consistent. Now it can be easily checked by means of **LFI1**-tableaux that:

$\Delta_2 \vdash_{\mathbf{LFI1}} A$ and $\Delta_2 \vdash_{\mathbf{LFI1}} \bullet(\neg A)$

that is, Adam is now proven to be guilty, and the initial supposition about his innocence was inconsistent.

Example 3: Inventing and solving logic puzzles

A popular kind of logic puzzle asks you to deduce some facts from a theory, giving some minimum of information. For example: three fellows A, B and C (a logician, a philosopher and a mathematician) live in Rome, Paris and São Paulo. From the following information, deduce who is what, and who lives where:
1) The logician lives in Paris;
2) A is not a philosopher;
3) B is a logician;
4) C is not a mathematician. and does not live in Rome;
5) The philosopher does not live in São Paulo.
considering that each person leaves in a different city and has a distinct profession.

The information is designed to guarantee a unique solution: if the puzzle is underdetermined (that is, information is insufficient) there is more than one solution, and if the game is overdetermined (that is, the information is contradictory) there is no solution. Moreover, the puzzle can be expressed in first-order logic (or, in fact, because the universe is finite, in propositional logic) and can be solved automatically (by means of logic programming, or equivalently, by means of tableaux).

If you try to invent a puzzle like this, you may of course be considering inconsistent information, and you certainly do not want to deduce everything from this temporary inconsistency; is there way to automatically check for the inconsistencies, and to revise the information?

Yes, there is: just run your samples using a **Ci**-tableau (or an **LFI1**-tableau), and the system will solve the puzzle, or automatically point to which information should be revised!
(Is this particular puzzle contradictory?)

6. Closure

These notes intend to defend the idea that reasoning with contradictions is not only useful but perfectly sound from the logico-mathematical standpoint. We have also discussed some underlying questions of paraconsistency, indicating some real applications of it to human and automated reasoning and to database models. We have shown how several axiomatic systems of paraconsistent logics can be formulated, and how possible-translations semantics and automated proof systems can be assigned to some of them. We also intended to show how intuitively and philosophically appealing are those semantics.

By taking seriously the task of constructing logics for formal inconsistency we may also get a better grip on certain philosophical questions, and understanding the role of contradictory procedures in conveying information (as in the case of proofs by *reductio ad absurdum*), besides the immediate applications in databases and belief revision.

7. References

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