Instructions. Put your answers on the blank paper provided. Show all your work.

1. (28 points) Consider the matrices $A = \begin{pmatrix} 2 & -6 & 3 \\ -1 & 3 & 1 \\ 0 & 6 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, vector $\vec{v} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$.

a. Compute the product AB.

b. Find, if it exists, B^{-1}.

c. Compute $\det(B)$.

d. If we consider A, B to represent isomorphisms f, g from $\mathbb{R}^3$ to $\mathbb{R}^3$ with respect to the standard basis, find $f(g(\vec{v}))$.

2. (16 points) Consider the basis $B = \langle \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \rangle$ for $\mathbb{R}^2$.

a. Use the Gram Schmidt process to find an orthogonal basis retaining the same first vector. Call the new basis K.

b. Find the matrix H that represents the change of basis from B to K.

3. (40 points) For each of the following situations, decide if the statement is true or false. If true, provide a short proof, if false provide a counterexmple.

a. If A and B are matrix equivalent then they must be rwo equivalent.

b. If G is a singular $n \times n$ matrix then GH must also be singular for any $n \times n$ matrix H.

c. For the set of 2×2 matrices under multiplication, "commutes with" is an equivalence relation. *HINT.* Equivalence relation means reflexive, symmetric, and transitive.

d. Let $\vec{s}$ be any vector in $\mathbb{R}^3$. if $\vec{u}$ is orthogonal to $\vec{s}$ then $\text{proj}_{\vec{s}}(\vec{u}) = \vec{0}$.

e. Let P be an $n \times n$ permutation matrix, i.e., an identity matrix in which the rows may have been interchanged, then $\det(P) = \pm 1$.

4. (16 points) Consider the following two bases for $\mathcal{P}_2$: $D = \langle 2x, 1+x, 1-x^2 \rangle$, $B = \langle 1, x, x^2 \rangle$.

a. Find the matrix G that represents the change of basis matrix from D to B.

b. Multiply G by the following matrix

$$H = \begin{pmatrix} -.5 & .5 & -.5 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

and explain in a sentence what the answer means.