BEILAGE

zu

№. 168. DER ASTRONOMISCHEN NACHRICHTEN.

Disquisitiones circa theoriam perturbationum quae motum corporum coelestium afficiunt,

auctore

P. A. Hansen,

Observatori Seebergensis Directore.

(Beschlufs.)

Hinc colligitur, ipsam $\Omega_{\prime\prime}$ et quotientes ejus differentiales quibus utimur, ex terminis compositas esse, quorum quivis productis constat binorum integralium horum

$$\begin{array}{ll}
\int r \cos w \cos ig\, dg & \int r \cos w \sin ig\, dg \\
\int r \sin w \sin ig\, dg & \int r \sin w \cos ig\, dg \\
\int \frac{1}{r'^2} \cos w' \cos i'g'\, dg' & \int \frac{1}{r'^2} \cos w' \sin i'g'\, dg' \\
\int \frac{1}{r'^2} \sin w' \sin i'g'\, dg' & \int \frac{1}{r'^2} \sin w' \cos i'g'\, dg'
\end{array}$$

quae omnia a valore ipsius g aut $g' = 0$ usque ad valorem $= 2\pi$ extendi debent. Quantitas itaque $\Omega'_{\prime\prime}$ et quotientes ejus differentiales quae in motu planetae m' occurrunt, terminis compositae sunt, qui productis constant binorum integralium horum

$$\begin{array}{ll}
\int r' \cos w' \cos i'g'\, dg' & \int r' \cos w' \sin i'g'\, dg' \\
\int r' \sin w' \sin i'g'\, dg' & \int r' \sin w' \cos i'g\, dg' \\
\int \frac{1}{r^2} \cos w \cos ig\, dg & \int \frac{1}{r^2} \cos w \sin ig\, dg \\
\int \frac{1}{r^2} \sin w \sin ig\, dg & \int \frac{1}{r^2} \sin w \cos ig\, dg
\end{array}$$

quorum limites iisdem sunt ut praecedentium. Quum vero evolutio quantitatis $r^h \cos w$ aut $r'^h \cos w'$ in seriem sinus multiplicium anomaliae mediae, et evolutio quantitatis $r^h \sin w$ aut $r'^h \sin w'$ in seriem cosinus multiplicium anomaliae mediae non contineat, facile concluditur, integralia columnae posterioris omnia cifram aequare debere. His positis jam demonstrabimus esse identicas aequationes has

$$\left.\begin{aligned}
\frac{i^2}{a^3}\int r \cos w \cos ig\, dg &= \int \frac{1}{r^2} \cos w \cos ig\, dg \\
\frac{i^2}{a^3}\int r \sin w \sin ig\, dg &= \int \frac{1}{r^2} \sin w \sin ig\, dg \\
\frac{i'^2}{a'^3}\int r' \cos w' \cos i'g'\, dg' &= \int \frac{1}{r'^2} \cos w' \cos i'g'\, dg' \\
\frac{i'^2}{a'^3}\int r' \sin w' \sin i'g'\, dg' &= \int \frac{1}{r'^2} \sin w' \sin i'g'\, dg'
\end{aligned}\right\} \quad (38)$$

intra limites 0 et 2π. Quarum aequationum priores transeunt in has

$$\left.\begin{aligned}
0 &= \int \left\{ i^2 \frac{r}{a} \cos w \cos ig\, dg - \frac{a^2}{r^2} \cos w \cos ig\, dg \right\} \\
0 &= \int \left\{ i^2 \frac{r}{a} \sin w \sin ig\, dg - \frac{a^2}{r^2} \sin w \sin ig\, dg \right\}
\end{aligned}\right\} \quad (39)$$

sed, designante z anomaliam excentricam, habetur

$$\begin{aligned}
r \cos w &= a \cos z - ae \\
r \sin w &= a\sqrt{(1-e^2)} \,.\, \sin z \\
r &= a - ae \cos z \\
g &= z - e \sin z \\
dg &= (1 - e \cos z)\, dz
\end{aligned}$$

quibus substitutis, ex priore aequationum (39) sequitur, esse confirmandam aequationem hanc

$$0 = \int \left\{ i \cos z \cos ig\, d.ig - \frac{\cos z - e}{(1 - e\cos z)^2} \cos ig\, dz - ie \cos ig\, d.ig \right\}$$

Multiplicato e per quantitatem $\cos^2 z + \sin^2 z$, quo valor expressionis non mutatur, quantitas $\frac{\cos z - e}{(1 - e\cos z)^2}$ abit in hanc

$$\frac{\cos z - e\sin^2 z - e\cos^2 z}{(1 - e\cos z)^2} = -\frac{e\sin^2 z}{(1 - e\cos z)^2} + \frac{\cos z}{1 - e\cos z} = \frac{d.\dfrac{\sin z}{1 - e\cos z}}{dz}$$

aequatio igitur praecedens erit

$$0 = \int \left\{ i \cos z\, d.\sin i g - \cos i g\, d.\frac{\sin z}{1 - e\cos z} - i e\, d.\sin i g \right\}$$

Hac per partes integrata, quum $dg = (1 - e\cos z)\,dz$, statim prodit indefinite

$$i \cos z \sin i g - \cos i g \frac{\sin z}{1 - e\cos z} - i e \sin i g$$

quae expressio, si ad limites designatos restringitur, certe evanescit.

Introducta z, aequatio altera (39) praebet confirmandam aequationem hanc

$$0 = \int \left\{ i^2 \sin z \sin i g\, dg - \frac{\sin z \sin i g}{(1 - e\cos z)^2}\, dz \right\}$$

Cui aequationi addatur quantitas haec

$$\int \left\{ \frac{i}{e} \frac{\cos i g}{1 - \cos z}\, dg - \frac{i}{e} \cos i g\, dg - i \cos z \cos i g\, dz \right\}$$

quam cifrae aequari patebit, si in termino ultimo pro dz valorem suum per dg expressum substitueris, hac quantitate addita, erit aequatio confirmanda

$$0 = \int \left\{ i^2 \sin z \sin i g\, dg - i \cos z \cos i g\, dg - \frac{i}{e} \cos i g\, dg + \frac{i}{e} \frac{\cos i g}{1 - e\cos z}\, dg - \frac{\sin z \sin i g}{(1 - e\cos z)^2}\, dz \right\}$$

quae per se integrabilis esse reperitur. Nam

$$i^2 \sin z \sin i g\, dg - i \cos z \cos i g\, dz = - i d.\sin z \cos i g$$

$$\frac{i}{e} \frac{\cos i g}{1 - e\cos z}\, dg - \frac{\sin z \sin i g}{(1 - e\cos z)^2}\, dz = \frac{1}{e}\, d.\frac{\sin i g}{1 - e\cos z},$$

igitur aequatio praecedens erit

$$0 = \int d.\left\{ \frac{1}{e} \frac{\sin i g}{1 - e\cos z} - \frac{1}{e} \sin i g - i \sin z \cos i g \right\}$$

quae intra limites designatos integrata, haud minus quam illa prior evanescit. Mutatis mutandis, posteriores duae ipsarum (38) quoque demonstratae sunt. Ex aequationibus vero (38) sequitur, si terminus quicunque illorum, qui in expressione (36) continentur, per

$$\frac{i'^2}{a'^3} M$$

repraesentatur, esse terminum correspondentem in (37) $=$

$$\frac{i^2}{a^3} M$$

Hinc colligitur, si ipsius $\Omega_{\prime\prime}$ vel quotientium ejus differentialium quibus in motu planetae m utimur, evolutiones resp. repraesentantur per

$$\frac{m'}{\mu} \overset{i,i'}{C_c} \cos(ig + i'g') + \frac{m'}{\mu} \overset{i,i'}{C_s} \sin(ig + i'g')$$

et ipsius $\Omega'_{\prime\prime}$ vel quotientium ejus differentialium quibus in motu planetae m' utimur, evolutiones resp. per

$$\frac{m}{\mu'} \overset{i,i'}{D_c} \cos(ig + i'g') + \frac{m}{\mu'} \overset{i,i'}{D_s} \sin(ig + i'g')$$

haberi

$$\overset{i,i'}{D_c} = \frac{a'^3\, i^2}{a^3\, i'^2} \overset{i,i'}{C_c}$$

$$\overset{i,i'}{D_s} = \frac{a'^3\, i^2}{a^3\, i'^2} \overset{i,i'}{C_s}$$

Quae aequationes tamen ad terminos in quibus $i' = 0$ applicari non possunt, quoniam infinitae fiunt, sed, pro quantitatibus $\Omega_{\prime\prime}$ et $\Omega'_{\prime\prime}$ illâ de qua in fine art. 24 locuti sumus, scilicet

$$rr' \left\{ \cos^2 \tfrac{1}{2} I \cos(u - u') + \sin^2 \tfrac{1}{2} I \cos(u + u') \right\}$$

atque quotientibus eius differentialibus evolutis, et resp. per

$$\overset{i,i'}{B_c} \cos(ig + i'g') + \overset{i,i'}{B_s} \sin(ig + i'g')$$

repraesentatis, emergunt

$$\overset{i,i'}{C_c} = \frac{i'^2}{a'^3} \overset{i,i'}{B_c}$$

$$\overset{i,i'}{C_s} = \frac{i'^2}{a'^3} \overset{i,i'}{B_s}$$

$$\overset{i,i'}{D_c} = \frac{i^2}{a^3} \overset{i,i'}{B_c}$$

$$\overset{i,i'}{D_s} = \frac{i^2}{a^3} \overset{i,i'}{B_s}$$

quas semper tuto adhibere licet.

Quod artificium tamen de approximatione prima praesertim valet, quum in approximatione secunda atque in posterioribus evolutionem quantitatis $\Omega_{\prime\prime}$ per integrationes singulas exsequi non liceat.

Denique, quamquam nomine planetae ubique usi sumus, formulas tamen nostras (13) (14) (29) etc. apte evalutas ad satellites etiam, imo ad cometas in ellipsibus Solem ambientes transferri licere, per se quidem intelligitur, sed tamen disertis verbis monendum esse, nobis visum est.

Hansen.