Errata List for the Second Printing

The Second Revised Edition has had two printings, and it also exists on the web. The second printing can be distinguished from the first printing by the fact that on the title page in the second printing, the affiliations of the authors are given. Note that the phrase 'Second Revised Edition' is on the cover of both printings of the second edition. Some errors that were present in the first printing were corrected in the second printing. The only errors in the list below are those that have been found in the second printing. All of these errors have been corrected in the current web version. We also list items that should be added to the index.

page 38, line -2: Replace the sentence "Explain why this is a fallacy" with the sentences "Is this phenomenon a fallacy? If so, why?"

page 45, line -2: The number 0.001 should be 0.01.

page 49, line 9: The interval [-1, 1] should be [0, 1].

page 76, line -2: Replace the word "measure" by the word "distribution." (At a few other places in the text, the word measure should be replaced by the word distribution in exactly the same way.)

page 82, line -11: Insert the phrase "has no fixed points" after the set $\{1, 2, \ldots, n\}$.

page 86, line -13: Delete the first "he."

page 139, lines -5 to -1: We have changed the definition of independent events to the following. Let E and F be two events. We say that they are independent if either 1) both events have positive probability and

$$P(E|F) = P(E) \text{ and } P(F|E) = P(F) \ ,$$

or 2) at least one of the events has probability 0.

page 142, line 16: After the inequality $a_1 \leq a_2 \leq a_3$, we added a sentence containing another restriction, namely that for $i = 1, 2$, we have $a_{i+1} - a_i \leq 1$.

page 142, line -6. The second R_1 in the displayed expression should be R_2.

page 145, line -12: After the phrase "such that" add the phrase "the sample space Ω satisfies the equation"

page 146, line -1: The number 3125 should be 3215.

page 147, line 1: Some of the entries in the matrix in Table 4.4 are incorrect and should be changed. The actual values are:

$$\begin{pmatrix} .700 & .131 & .169 \\ .075 & .033 & .892 \\ .358 & .604 & .038 \\ .098 & .403 & .499 \end{pmatrix}$$

page 152, Exercise 16. This exercise was changed to: Prove that for any three events A, B, C, each having positive probability, and with the property that $P(A \cap B) > 0$,

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)\ .$$

page 153, Exercise 27. Part b) should be changed to "the probability that he brings his umbrella, given that it doesn't rain."

page 181: Delete the paragraph after Exercise 1 that starts "In the next two problems....".

page 181: The last four lines of problem 2 should be replaced by the following text: "A bridge hand has been dealt, i. e. thirteen cards are dealt to each player. Given that your partner has at least one ace, what is the probability that he has at least two aces? Given that your partner has the ace of hearts, what is the probability that he has at least two aces? Answer these questions for a version of bridge in which there are eight cards, namely four aces and four kings, and each player is dealt two cards. (The reader may wish to solve the problem with a 52-card deck.)"

page 181: In problem 3, the sentence "(Again, assume the deck consists of eight cards.)" should be added after the sentence that ends "...we might get this information."

page 185: The plots in Figure 5.1 have mass at 0, but the geometric distribution has no mass at 0. The spikes in the plots should all be shifted to the right by one unit.

page 185, line -8: Delete the sentence that begins "Let X be a geometrically ... ." Delete the word "Now" and the comma from the beginning of the next sentence.

page 186, lines 3 and 6: Replace the floor functions by ceiling functions. In addition, at the end of the second sentence on this page, add the phrase "...

where the notation $\lceil x \rceil$ means the least integer which is greater than or equal to x."

page 187, line 17: Replace the floor function in the summation by a ceiling function.

page 196, line 11: The word "Negrini" should be changed to "Nigrini."

page 197, Exercise 2. Add the phrase 'at random' after the second occurrence of the symbol S.

page 204, line -6: The reference "Example 5.10" should be changed to "Example 5.6."

page 206, line -1: The denominator of the right-hand expression should be $1 - F(r)$, not $1 - F(s)$.

page 235, line 6. Add 'A more accurate approximation to $E(S_n)$ is given by the expression

$$\log n + \gamma + \frac{1}{2n} \ ,$$

where γ denotes Euler's constant, and is approximately equal to .5772.' Then, delete 'log 10', change 2.3 to 2.9298, and change 2.9 to 2.9290. Then delete the last sentence.

page 241, line -6. Change the word 'we' to 'way.'

page 242, line -8. After the word "recall," add the phrase ", from the Historical Remarks section of Chapter 1, Section 2," .

page 248, Exercise 12. Change this exercise to: Recall that in the martingale doubling system (see Exercise 1.1.10), the player doubles his bet each time he loses. Suppose that you are playing roulette in a fair casino where there are no 0's, and you bet on red each time. You then win with probability 1/2 each time. Assume that you enter the casino with 100 dollars, start with a 1-dollar bet and employ the martingale system. You stop as soon as you have won one bet, or in the unlikely event that black turns up six times in a row so that you are down 63 dollars and cannot make the required 64-dollar bet. Find your expected winnings under this system of play.

page 248, Exercise 13: Add the word "expected" before the word "final."

page 254, Exercise 30, part (d): Change the first sentence to: In Example 6.11

we stated that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \sim \log n + .5772 + \frac{1}{2n} \ .$$

page 259, line 13. Replace both occurrences of ω by x.

page 260, lines 11-15. Add the following two lines to the statement of the theorem:

$$\sigma(S_n) = \sigma\sqrt{n}$$
$$\sigma(A_n) = \frac{\sigma}{\sqrt{n}} \ .$$

page 260, line -9: Change the word "statement" to the word "equation" and change the word "proof" to the word "theorem."

page 267, Exercise 25: part (b) should be part (c) and part (b) should be

> (b) We draw a ball from the urn, examine its color, replace it, and then draw another. Under what conditions, if any, are the results of the two drawings independent; that is,
>
> $$Pr(\text{white}, \text{white}) = Pr(\text{white})^2?$$

page 267: Exercise 26 should be deleted.

page 279, Exercise 6(b): The density function should be $f_T(t) = 9te^{-3t}$.

page 296, line 4: The expression $\frac{1}{4\pi}$ to the left of the integral should be changed to $\frac{1}{2\pi}$. The term $r-s$ in the numerator of the integrand should have parentheses around it. The two 2's in the denominator of the integrand should be deleted. The endpoints of integration should be 0 and r, not $-\infty$ and ∞.

page 296, line 5: The expression $\frac{1}{2}e^{-r^2/2}$ should be changed to $\frac{1}{2}e^{-r/2}$.

page 313, Exercise 10: Add the word "positive" before the word "integer."

page 314 Exercise 16 is incorrect as stated. A correct replacement exercise, sent to us by David Maslen, is as follows:

In this exercise, we shall construct an example of a sequence of random variables that satisfies the weak law of large numbers, but not the strong law.

The distribution of X_i will have to depend on i, because otherwise both laws would be satisfied. As a preliminary, we need to prove a lemma, which is one of the Borel-Cantelli lemmas.

Suppose we have an infinite sequence of mutually independent events $A_1, A_2, \ldots$. Let $a_i = \text{Prob}(A_i)$, and let r be a positive integer.

a. Find an expression of the probability that none of the A_i with $i > r$ occur.

b. Use the fact that $x - 1 \leq e^{-x}$ to show that

$$\text{Prob}(\text{No } A_i \text{ with } i > r \text{ occurs}) \leq e^{-\sum_{i=r}^{\infty} a_i}$$

c. Prove that if $\sum_{i=1}^{\infty} a_i$ diverges, then

$$\text{Prob}(\text{infinitely many } A_i \text{ occur}) = 1.$$

Now, let X_i be a sequence of mutually independent random variables such that for each positive integer $i \geq 2$,

$$\text{Prob}(X_i = i) = \frac{1}{2i \log i}, \quad \text{Prob}(X_i = -i) = \frac{1}{2i \log i}, \quad \text{Prob}(X_i = 0) = 1 - \frac{1}{i \log i}.$$

When $i = 1$ we let $X_i = 0$ with probability 1. As usual we let $S_n = X_1 + \cdots + X_n$. Note that the mean of each X_i is 0.

d. Find the variance of S_n.

e. Show that the sequence $\langle X_i \rangle$ satisfies the weak law of large numbers. I.e. prove that for any $\epsilon > 0$

$$\text{Prob}(\left| \frac{S_n}{n} \right| \geq \epsilon) \to 0,$$

as n tends to infinity.

We now show that $\langle X_i \rangle$ does not satisfy the strong law of large numbers. Suppose that $S_n/n \to 0$. Then because

$$\frac{X_n}{n} = \frac{S_n}{n} - \frac{n-1}{n} \frac{S_{n-1}}{n-1},$$

we know that $X_n/n \to 0$. From the definition of limits, we conclude that the inequality $|X_i| \geq \frac{1}{2} i$ can only be true for finitely many i.

f. Let A_i be the event $|X_i| \geq \frac{1}{2} i$. Find $\text{Prob}(A_i)$. Show that $\sum_{i=1}^{\infty} \text{Prob}(A_i)$ diverges (think integral test).

g. Prove that A_i occurs for infinitely many i.

h. Prove that

$$\text{Prob}(\frac{S_n}{n} \to 0) = 0,$$

and hence that the strong law of large numbers fails for the sequence $\langle X_i \rangle$.

page 330, lines 10-17: The statement of the theorem should be changed to: Let S_n be the number of successes in n Bernoulli trials with probability p for success, and let a and b be two fixed real numbers. Then

$$\lim_{n\to\infty} P\left(a \le \frac{S_n - np}{\sqrt{npq}} \le b\right) = \int_a^b \phi(x)\,dx \ .$$

page 333, line -18: Change the word "to" to the word "two."

page 343, line -11: The number "240" should be changed to "420."

page 345, line 12: Add the word "independent" before the word "experiments." Change the word "standardized" to "standardize."

page 368. Delete Theorem 10.1.

page 369. Replace all but the first sentence of the proof of Theorem 10.2 by the following. Conversely, assume that $g(t)$ is known. We wish to determine the values of x_j and $p(x_j)$, for $1 \le j \le n$. We assume, without loss of generality, that $p(x_j) > 0$ for $1 \le j \le n$, and that

$$x_1 < x_2 < \ldots < x_n \ .$$

We note that $g(t)$ is differentiable for all t, since it is a finite linear combination of exponential functions. If we compute $g'(t)/g(t)$, we obtain

$$\frac{x_1 p(x_1)e^{tx_1} + \ldots + x_n p(x_n)e^{tx_n}}{p(x_1)e^{tx_1} + \ldots + p(x_n)e^{tx_n}} \ .$$

Dividing both top and bottom by e^{tx_n}, we obtain the expression

$$\frac{x_1 p(x_1)e^{t(x_1-x_n)} + \ldots + x_n p(x_n)}{p(x_1)e^{t(x_1-x_n)} + \ldots + p(x_n)} \ .$$

Since x_n is the largest of the x_j's, this expression approaches x_n as t goes to ∞. So we have shown that

$$x_n = \lim_{t\to\infty} \frac{g'(t)}{g(t)} \ .$$

To find $p(x_n)$, we simply divide $g(t)$ by e^{tx_n} and let t go to ∞. Once x_n and $p(x_n)$ have been determined, we can subtract $p(x_n)e^{tx_n}$ from $g(t)$, and repeat the above procedure with the resulting function, obtaining, in turn, $x_{n-1}, \ldots, x_1$ and $p(x_{n-1}), \ldots, p(x_1)$.

page 376, Exercise 8. Delete parts d) and e).

pages 384-385. Delete lines -6 to -3 on page 384. Then change the reference 10.6 in line -2 to 10.5. Change the next equation to

$$h'_{n+1}(z) = h'_n(h(z))h'(z).$$

Change the 'facts' in the next sentence to "$h(1) = 1,\ h'(1) = m,$ and $h'_n(1) = m_n$".

page 385, line -10. Add the phrase "$0 < c < 1$. Then we have" after the expression "$1 - c$."

page 393, line -6. Delete the reference to Example 5.7.

page 393, line -5. Change '0 or 1' to '1 or 0.'

page 409, line 11: The number .188 should be .198.

page 410, footnote: The word "Physicalishce" should be "Physicalische".

page 412, line -12. After the word "boundaries," add the sentences: "The top left-hand corner square is adjacent to three obvious neighbors, namely the squares below it, to its right, and diagonally below and to the right. It has five other neighbors, which are as follows: the other three corner squares, the square below the upper right-hand corner, and the square to the right of the bottom left-hand corner. The other three corners also have, in a similar way, eight neighbors."

page 414, Exercise 6. Change the w's to u's.

page 417, lines -12 to -9: Delete the entire paragraph that begins "In the following..."

page 418, line 1: The expression "$2n$" should be "$2m$."

page 423, line -9: The last summand on the left-hand side of the equation should be -1, not 1.

page 434, line -11: The first entry in the third line of the first matrix should be 1/2, not 1/3. The first entry in the third line of the second column vector should be 1/2, not 1/3.

page 450, line 10: The expression to the right of the equal sign should be

$$\mathbf{P}(i,k) \cdot \mathbf{P}(j,l) \ .$$

page 452. Add Exercise 9: (Suggested by Peter Doyle) In the proof of Theorem 11.14, we assumed the existence of a fixed vector $\mathbf{w}$. To avoid this assumption, beef up the coupling argument to show (without assuming the existence of a stationary distribution $\mathbf{w}$) that for appropriate constants C and $r < 1$, the distance between αP^n and βP^n is at most Cr^n for any starting distributions α and β. Apply this in the case where $\beta = \alpha P$ to conclude that the sequence αP^n is a Cauchy sequence, and that its limit is a matrix W whose rows are all equal to a probability vector w with $wP = w$. Note that the distance between αP^n and w is at most Cr^n, so in freeing ourselves from the assumption about having a fixed vector we've proved that the convergence to equilibrium takes place exponentially fast.

page 454, line -4: The term m_{jk} should be m_{kj}.

page 470, Exercise 24. This exercise should be replaced by the following exercise. In the course of a walk with Snell along Minnehaha Avenue in Minneapolis in the fall of 1983, Peter Doyle (private communication) suggested the following explanation for the constancy of *Kemeny's constant* (see Exercise 19). Choose a target state according to the fixed vector $\mathbf{w}$. Start from state i and wait until the time T that the target state occurs for the first time. Let K_i be the expected value of T. Observe that

$$K_i + w_i \cdot 1/w_i = \sum_j P_{ij} K_j + 1 \ ,$$

and hence

$$K_i = \sum_j P_{ij} K_j \ .$$

By the maximum principle, K_i is a constant. Should Peter have been given the prize?

page 478, line 13: The expression

$$\frac{n!}{j!k!(n-j-k)!}$$

should be replaced by

$$\frac{1}{3^n} \frac{n!}{j!k!(n-j-k)!} \ .$$

page 478, line -3: The number ".65" should be changed to ".34."

page 480, line -11: The plus sign should be a minus sign and in the second summation there is a missing factor of 2^{2m}.

page 481, line -4: In part (c) of Exercise 2 the phrase 'Using part (a)' should be replaced by 'Using part (b)'.

page 482, Exercise 2 (d): The phrase 'Using Exercise 2' should be changed to 'Using parts (a) and (b)'.

page 482, line -7: After the first occurrence of the word 'that,' add the phrase 'if m is odd,'.

page 483, Exercise 9: The second displayed equation of the introduction should be deleted.

Additions and Corrections to Index

2. distribution – binomial, 99, 184