Fuzzy Relevant Logic: What Is It and Why Study It?

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ABSTRACT

For any correct argument in scientific reasoning as well as our everyday reasoning, the premises of the argument must be in some way relevant to the conclusion of that argument, and vice versa. On the other hand, in scientific reasoning as well as our everyday reasoning, many arguments may be correct to some degree, and therefore, a reasoning consisting of such fuzzy arguments is approximate. As a generalization of Boolean classical logic, fuzzy logic was established in order to deal with those fuzzy propositions and to underlie approximate reasoning. However, an approximate reasoning based on fuzzy logic is not necessarily relevant. In this paper, the author intends to call for attentions to such a fundamental research problem: Can we establish a formal logic system to underlie those reasoning that are both relevant and approximate? The paper presents the motivation to study fuzzy relevant logic and discusses possible research directions, problems, and difficulties to establish a formal fuzzy relevant logic system to underlie approximate and relevant reasoning.

1. INTRODUCTION

Reasoning is the process of drawing new conclusions from some premises, which are known facts or assumed hypotheses. In general, a reasoning consists of a number of arguments (or inferences). An ***argument*** (or ***inference***) is a set of declarative sentences consisting of one or more sentences as its premises, which contain the evidence, and one sentence as its conclusion. In an argument, a claim is being made that there is some sort of ***evidential relation*** between its premises and its conclusion: the conclusion is supposed to ***follow from*** the premises, or, equivalently, the premises are supposed to ***entail*** the conclusion. The correctness of an argument is a matter of the connection between its premises and its conclusion, and concerns the strength of the relation between them. Therefore, the correctness of an argument depends on the connection between its premises and its conclusion, and neither on whether the premises are true or not, nor on the conclusion is true or not. Thus, we have a fundamental question: What is the criterion by which to decide whether the conclusion really does follow from the premises or not? A ***logically valid reasoning*** is a reasoning such that its arguments are justified based on some logical criterion in order to obtain correct conclusions. Today, there are so many different logic systems established based on different philosophical motivations. As a result, a reasoning may be valid on a logical criterion but invalid on another.

Logic deals with what entails what or what follows from what, and aims at determining which are the correct conclusions of a given set of premises, i.e., to determine which arguments are valid. Therefore, the most essential and central concept in logic is the logical consequence relation that relates a given set of premises to those conclusions, which validly follow from the premises. In general, a ***formal logic system*** L consists of a formal language, denoted by F(L), which is the set of all ***well-formed formulas*** of L, and a ***logical consequence relation***, denoted by $\vdash_L$, such that for $P \subseteq F(L)$ and $t \in F(L)$, $P \vdash_L t$ means that within the framework of L, taking P as premises one can obtain t as a valid conclusion (in the sense of L). For a formal logic system (F(L), $\vdash_L$), a ***logical theorem*** t is a formula of L such that $\phi \vdash_L t$ where ϕ is the empty set. We use Th(L) to denote the set of all logical theorems of L. Th(L) is completely determined by the logical consequence relation $\vdash_L$. According to the representation of the logical consequence relation of a logic, the logic can be represented as a Hilbert style formal system, a Gentzen natural deduction system, a Gentzen sequent calculus system, or some other type of formal system. A formal logic system L is said to be ***explosive*** if and only if $\{A, \neg A\} \vdash_L B$ for any two different formulas A and B; L is said to be ***paraconsistent*** if and only if it is not explosive.

For any correct argument in scientific reasoning as well as our everyday reasoning, the premises of the argument must be in some way relevant to the conclusion of that argument, and vice versa. A well-known fact is that a reasoning based on Boolean (i.e., two-valued) classical logic is truth-preserving (i.e., the conclusion of the reasoning must be true if all premises of that reasoning are true) but not necessarily relevant. Relevant logic, which was originally established in order to find a mathematically satisfactory way of grasping the notion of conditional, can certainly underlie relevant reasoning [1, 2]. On the other hand, in scientific reasoning as well as our everyday reasoning, many arguments may be correct to some degree, and therefore, a reasoning consisting of such fuzzy arguments is approximate. As a generalization of Boolean classical logic, fuzzy logic was established in order to deal with those fuzzy propositions and to underlie approximate reasoning [17]. However, an approximate reasoning based on fuzzy logic is not necessarily relevant.

A reasoning based on relevant logic is relevant but may be not approximate, and a reasoning based on fuzzy logic is approximate but may be not relevant. Thus, in this paper, the present author intends to call for attentions to such a fundamental research problem: Can we establish a formal logic system to underlie those reasoning that are both relevant and approximate? The rest of the paper is organized as follows: Section 2 gives a brief survey of the notion of conditional and its representations in various logic systems, points out why a reasoning based on Boolean classical logic and its various classical conservative extension, various many-valued logics, and fuzzy logic is irrelevant, shows that a reasoning based on strong relevant logic is relevant and that it is the notion of conditional (entailment) that plays the most fundamental role in relevant reasoning, and Section 3 discusses possible research directions, problems, and difficulties to establish a formal fuzzy relevant logic system to underlie approximate and relevant reasoning.

2. THE NOTION OF CONDITIONAL AND RELEVANT REASONING

In various mathematical, natural, and social scientific literature, it is probably difficult, if not impossible, to find a sentence form that is more generally used to describe various definitions, propositions, theorems, and laws than the sentence form of 'if ... then ...'. On the other hand, the sentence form of 'if ... then ...' also has been used, in the form of so-called 'If-Then rules' as the most basic and key representation tool in various industrial applications. In logic, a sentence in the form of 'if ... then ...' is usually called a ***conditional proposition*** or simply ***conditional*** which states that there exists a relationship of sufficient condition between the 'if' part and the 'then' part of the sentence. Mathematical, natural, and social scientists always use conditionals in their descriptions of various definitions, propositions, theorems, and laws to connect a concept, fact, situation or conclusion and its sufficient conditions. Indeed, the major work of almost all scientists is to discover some sufficient condition relations between various phenomena, data, and laws in their research fields. On the other hand, from the abstract viewpoint of logic, every If-Then rule in various running application systems can be regarded as a conditional without exception.

In general, a conditional must concern two parts which are connected by the connective 'if ... then ... ' and called the ***antecedent*** and the ***consequent*** of that conditional, respectively. The truth-value of a conditional depends not only on the truth-values of its antecedent and consequent but also more essentially on a ***necessarily relevant and conditional relation*** between them, i.e., the notion of conditional is not truth-functional. The notion of conditional plays the most essential role in reasoning because any reasoning form must invoke it, and therefore, it is historically always the most important subject studied in logic and is regarded as the heart of logic [1, 2].

When we study and use logic, the notion of conditional may appear in both the object logic (i.e., the logic we are studying) and the meta-logic or observer's logic (i.e., the logic we are using to study the object logic). In the object logic, there usually is a connective in its formal language to represent the notion of conditional, and the notion of conditional is also usually used to represent a logical consequence relation in its proof theory or model theory. On the other hand, in the meta-logic, the notion of conditional, usually in the form of natural language, is used to define various meta-notions and describe various meta-theorems about the object logic.

From the viewpoint of object logic, there are two kinds of conditionals. One kind is empirical conditionals and the other kind is logical conditionals. For a logic, a conditional is called an ***empirical conditional*** of the logic if its truth-value, in the sense of that logic, depends on the contents of its antecedent and consequent and therefore cannot be determined only by its abstract form (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be empirical); a conditional is called a ***logical conditional*** of the logic if its truth-value, in the sense of that logic, depends only on its abstract form but not on the contents of its antecedent and consequent, and therefore, it is considered to be universally true or false (i.e., from the viewpoint of that logic, the relevant relation between the antecedent and the consequent of that conditional is regarded to be logical); moreover, a logical conditional that is considered to be universally true, in the sense of that logic, is also called an ***entailment*** of that logic. Indeed, the most intrinsic difference between various different logic systems is to regard what class of conditionals as entailments, as Diaz pointed out: "The problem in modern logic can best be put as follows: can we give an explanation of those conditionals that represent an entailment relation?" [8]

Informally, we say a reasoning is ***relevant*** if and only if in every argument of that reasoning, the premises are in some way relevant to the conclusion in the sense of meaning, and vice versa; a reasoning is irrelevant if and only if it is not relevant.

Now, let us see how the notion of conditional is represented in various logic systems and what role the notion of conditional plays in relevant reasoning.

First of all, let us see the case of Boolean classical logic (i.e., classical mathematical logic, **CML** for short). The **CML** was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them. It was based on a number of fundamental assumptions. Some of the assumptions concerning with our subject are as follows:

The ***classical abstraction***: The only properties of a proposition that matter to logic are its form and its truth-value.

The ***Fregean assumption***: The truth-value of a proposition is determined by its form and the truth-values of its constituents.

The ***classical account of validity***: An argument is valid if and only if it is impossible for all its premises to be true while its conclusion is false.

The ***principle of bivalence***: There are exactly two truth-values, TRUE and FALSE. Every declarative sentence has one or other, but not both, of these truth-values.

Taking above assumptions into account, in **CML**, the notion of conditional, which is intrinsically intensional but not truth-functional, is represented by the truth-functional extensional notion of ***material implication*** (denoted by $\rightarrow$ in this paper) that is defined as $A\rightarrow B =_{df} \neg(A\wedge\neg B)$ or $A\rightarrow B =_{df} \neg A\vee B$. This definition of material implication, with the inference rule of Modus Ponens for material implication (from A and $A\rightarrow B$ to infer B), can certainly satisfy the ***truth-preserving*** requirement of **CML,** i.e., the conclusion of a valid reasoning based on **CML** must be true if all premises of the reasoning are true. This requirement is basic and adequate for **CML** to be used as a formal description tool by mathematicians. However, the material implication is intrinsically different from the notion of conditional in meaning (semantics). It is no more than a truth-function of its antecedent and consequent but does not require the existence of a necessarily relevant and conditional relation between its antecedent and consequent, i.e., the truth-value of the formula $A\rightarrow B$ depends only on the truth-values of A and B, though there could exist no necessarily relevant and conditional relation between A and B. It is this intrinsic difference in meaning between the notion of material implication and the notion of conditional that leads to the well-known 'implicational paradox problem' in **CML**. The problem is that if one regards the material implication as the notion of conditional and regards every logical theorem of **CML** as a valid reasoning form or entailment, then a great number of logical axioms and logical theorems of **CML**, such as $A\rightarrow(B\rightarrow A)$, $B\rightarrow(\neg A\vee A)$, $\neg A\rightarrow(A\rightarrow B)$, $(\neg A\wedge A)\rightarrow B$, $(A\rightarrow B)\vee(\neg A\rightarrow B)$, $(A\rightarrow B)\vee(A\rightarrow\neg B)$, $(A\rightarrow B)\vee(B\rightarrow A)$, $((A\wedge B)\rightarrow C)\rightarrow((A\rightarrow C)\vee(B\rightarrow C))$, and so on, present some paradoxical properties and therefore they have been referred to in the literature as '***implicational paradoxes***' [1, 2, 9, 12].

Because all implicational paradoxes are logical theorems of any **CML**-theory with premises P (denoted by $T_{\mathbf{CML}}(P)$ in this paper), for a conclusion of a reasoning from a set P of premises based on **CML**, we cannot directly accept it as a true or valid conclusion in the sense of conditional, even if each of given premises is regarded to be true or valid and the conclusion can be regarded to be true or valid in the sense of material implication. For example, from any given premise A, we can infer $B\rightarrow A$, $C\rightarrow A$, ... where B, C, ... are arbitrary formulas, by using logical axiom $A\rightarrow(B\rightarrow A)$ of **CML** and Modus Ponens for material implication, i.e., $B\rightarrow A\in T_{\mathbf{CML}}(P)$, $C\rightarrow A\in T_{\mathbf{CML}}(P)$, ... for any $A\in T_{\mathbf{CML}}(P)$. However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B, C, ... and A and therefore we cannot say 'if B then A', 'if C then A', and so on. This situation means that from the viewpoint of conditional or entailment, the truth-preserving property of reasoning based on **CML** is meaningless.

On the other hand, the following proof-theoretical and model-theoretical deduction theorems hold in **CML**:

$$\Gamma\cup\{A\} \vdash_{\mathbf{CML}} B \text{ iff } \Gamma \vdash_{\mathbf{CML}} A\rightarrow B$$
$$\Gamma\cup\{A_1, ..., A_n\} \vdash_{\mathbf{CML}} B \text{ iff } \Gamma \vdash_{\mathbf{CML}} A_1\rightarrow(...\rightarrow(A_n\rightarrow B)...)$$
$$\Gamma\cup\{A_1, ..., A_n\} \vdash_{\mathbf{CML}} B \text{ iff } \Gamma \vdash_{\mathbf{CML}} (A_1\wedge...\wedge A_n)\rightarrow B$$

$$\Gamma\cup\{A\} \models_{\mathbf{CML}} B \text{ iff } \Gamma \models_{\mathbf{CML}} A\rightarrow B$$
$$\Gamma\cup\{A_1, ..., A_n\} \models_{\mathbf{CML}} B \text{ iff } \Gamma \models_{\mathbf{CML}} A_1\rightarrow(...\rightarrow(A_n\rightarrow B)...)$$
$$\Gamma\cup\{A_1, ..., A_n\} \models_{\mathbf{CML}} B \text{ iff } \Gamma \models_{\mathbf{CML}} (A_1\wedge...\wedge A_n)\rightarrow B$$

What these deduction theorems mean is that the notion of entailment in meta-logic of **CML** is "equivalent" to the notion of material implication in **CML**.

Consequently, in the framework of **CML**, even if a reasoning is valid, neither the validity of its conclusion in the sense of conditional nor the necessary relevance between its premises and its conclusion can be guaranteed necessarily. This is a direct result of the classical account of validity.

Any classical conservative extension or non-classical alternative of **CML** (including modal logic systems, intuitionistic logic, and those logic systems developed in recent years for nonmonotonic reasoning) where the notion of conditional is directly or indirectly represented by the material implication has the similar implicational paradox problem as that in **CML**. Therefore, a reasoning based on these logics also may be invalid and irrelevant in the sense of conditional.

Now, let us see how the notion of conditional is represented in many-valued logics and fuzzy logic.

Although there are many different many-valued logic systems motivated by philosophical or mathematical considerations, all the logics rejected only the principle of bivalence of **CML** and extended the case of two truth-values into the case of many truth-values, but also were based on the classical abstraction, the Fregean assumption, and the classical account of validity of **CML** [3, 11, 16]. Therefore, the notion of conditional is represented by a truth-function in various many-valued logic systems as the same as in **CML** such that "the rules for assigning values to complex formulas satisfy a generalized rule of truth functionality; the value assigned to a complex formula is a function of the values assigned to its components" [16]. As a result, those implicational paradoxes of **CML** still appear in the many-valued logic systems in the sense of many truth-values. For example, $A\rightarrow(B\rightarrow A)$, which is called 'positive paradox' and is the most typical implicational paradoxes of **CML**, is a logical axiom of Lukasiewicz's both three-valued and infinite-valued logic systems [3, 11, 16]. Lukasiewicz's infinite-valued logic system also has $(A\rightarrow B)\vee(B\rightarrow A)$, which is another typical implicational paradoxes of **CML**, as a logical axiom [3, 11, 16]. Therefore, a reasoning based on these logics also may be invalid and irrelevant in the sense of conditional. For example, in the framework of Lukasiewicz's many-valued logics, from any given premise A whose truth-value is the designated truth-value, we can infer B→A, C→A, ... where B, C, ... are arbitrary formulas, by using logical axiom A→(B→A) and Modus Ponens for many-valued material implication. B→A, C→A, ... should have the designated truth-value as their truth-values because they are conclusions of a truth-preserving reasoning based on the logic, respectively. However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded to be valid in the sense of conditional because there may be no necessarily relevant and conditional relation between B, C, ... and A and therefore we cannot say 'if B then A', 'if C then A', and so on, even though B→A, C→A, ... are "true" in the sense of logic.

Consequently, in the framework of many-valued logics, similar to the case of **CML**, even if a reasoning is valid, neither the validity of its conclusion in the sense of conditional nor the necessary relevance between its premises and its conclusion can be guaranteed necessarily.

Fuzzy logic can be regarded as a generalization of many-valued logics as well as **CML** [3, 10, 11, 13, 16]. In the development of fuzzy logic, those approaches to define the notion of fuzzy implication relation (fuzzy conditional statement) were borrowed from various many-valued logics, and they were based on various straightforward generalizations of the notion of material implication of **CML**. Therefore, similar to the case of many-valued logics, although there are many different approaches to define the notion of fuzzy implication relation, they rejected only the principle of bivalence of **CML** and extended the case of two truth-values into the case of infinite truth-values, but also were based on the classical abstraction, the Fregean assumption, and the classical account of validity of **CML** [3, 10, 11, 13, 16]. Consequently, the notion of conditional is represented by a truth-functional style in various fuzzy logic formalisms as the same as in various many-valued logic systems. Obviously, the intrinsic difference in meaning between the notion of conditional and the notion of truth-functional fuzzy implication must lead to a 'implicational paradox problem' in fuzzy logic. Because there is no standard axiomatization of fuzzy logic (in a certain sense of algebraic semantics, no fuzzy propositional logic is axiomatizable! [3]), it is difficult to say how many implicational paradoxes of **CML** appear in fuzzy logic as logical theorems or tautologies. However, for those fuzzy logic formalisms which require to have **CML** as a special case, they may have all implicational paradoxes of **CML**. For example, in an axiomatization of fuzzy logic from the viewpoint to regard fuzzy logic as a 'many-valued logic with special properties', the positive implicational paradox A→(B→A) is a logical axiom and fuzzy tautology [13].

Consequently, in the framework of these fuzzy logic formalisms, similar to the case of **CML** and many-valued logics, even if a reasoning is valid, neither the validity of its conclusion in the sense of conditional nor the necessary relevance between its premises and its conclusion can be guaranteed necessarily.

On the other hand, the notion of fuzzy implication relation was often studied in various fuzzy logic formalisms and used in various approximate reasoning applications at mata-level rather than object level, i.e., studied and used as an If-Then rule rather than a logical connective, as Dubois and Prade said: "if-then rules have been advocated as a key tool for expressing pieces of knowledge in fuzzy logic" [15].

Now, let us see some typical approximate inferences in fuzzy logic to investigate whether they are relevant or not.

Zadeh's ***compositional rule of inference*** (CRI for short) is the basic general mechanism of fuzzy reasoning [15, 17]. Let x and y be two variables running over their domain X and Y, respectively, R be a fuzzy relation on the Cartesian product of X and Y such that R: X×Y → [0, 1], A be a fuzzy set such that A: X → [0, 1] . Then, CRI states that the image of A under R, denoted by B which is a fuzzy set such that B: Y → [0, 1], is given by the composition of R and A as follows:

$$B(y) = \sup_{x\in X} \{ \min (A(x), R(x, y)) \}.$$

If we regard fuzzy sets A and B to be use to represent two fuzzy concepts, then CRI can be imaged as follows:

From 'x is A', and 'x and y has relationship R',
to infer 'y is B'.

Thus, if we use a notion of fuzzy implication in some fuzzy logic formalism or a notion of many-valued implication in many-valued logics as the fuzzy relation R in CRI`, we can obtain a special case of CRI, called the ***generalized modus ponens***, which is widely used in fuzzy reasoning as follows:

From x is A', and if x is A then y is B, to infer y is B'

$$B'(y) = \sup_{x\in X} \{ \min (A'(x), I(A(x), B(y))) \}$$

where I(A(x), B(y)) is a fuzzy implication relation or a many-valued implication relation.

Because I(A(x), B(y)) is a truth-function, without regard to how it extended the truth-function of material implication in the sense of many truth-values, even if the values of I(A(x), B(y)) is 1, it is not necessarily regarded that there is a necessarily relevant and conditional relation between A(x) and B(y). Consequently, from the viewpoint of relevant reasoning, we can say that a fuzzy reasoning is not better than a reasoning based on many-valued logics.

Traditional (weak) relevant logics were established during the 1950s~1970s in order to find a mathematically satisfactory way of grasping the notion of conditional. Some major traditional relevant logic systems are 'system **E** of entailment', 'system **R** of relevant implication', and 'system **T** of ticket entailment' [1, 2, 9, 11]. A fundamental characteristic of the logics is that they have a primitive intensional connective to represent the notion of conditional and their logical theorems include no implicational paradoxes [1, 2, 9, 11]. The underlying principle of the relevant logics is the ***relevance principle***, i.e., for any entailment provable in **T**, **E**, or **R**, its antecedent and consequent must share a sentential variable [1, 2, 9, 11]. Therefore, in the framework of relevant logic, if a reasoning is valid, then both the validity of its conclusion in the sense of conditional and the necessarily relevant relation between its premises and its conclusion can be guaranteed in a certain sense of weak relevance.

However, although the traditional relevant logics have rejected those implicational paradoxes, there still exist some logical axioms or theorems in the logics, which are not so natural in the sense of conditional. Such logical axioms or theorems, for instance, are $(A\wedge B)\Rightarrow A$, $(A\wedge B)\Rightarrow B$, $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$, $A\Rightarrow(A\vee B)$, $B\Rightarrow(A\vee B)$, $(A\Rightarrow B)\Rightarrow(A\Rightarrow(B\vee C))$ and so on, where $\Rightarrow$ denotes the primitive intensional connective in the logics to represent the notion of conditional. The present author named these logical axioms or theorems '***conjunction-implicational paradoxes***' and '***disjunction-implicational paradoxes***' [4-7]. For example, from any given premise $A\Rightarrow B$, we can infer $(A\wedge C)\Rightarrow B$, $(A\wedge C\wedge D)\Rightarrow B$, and so on by using logical theorem $(A\Rightarrow B)\Rightarrow((A\wedge C)\Rightarrow B)$ of **T**, **E**, and **R** and Modus Ponens for conditional. However, from the viewpoint of scientific reasoning as well as our everyday reasoning, these inferences cannot be regarded as valid in the sense of conditional because there may be no necessarily relevant and conditional relation between C, D, ... and B and therefore we cannot say 'if A and C then B', 'if A and C and D then B', and so on. In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed some strong relevant logics, named **Rc**, **Ec**, and **Tc** [4-7]. The logics require that the premises of an argument represented by a conditional include no unnecessary and needless conjuncts and the conclusion of that argument includes no unnecessary and needless disjuncts. As a modification of traditional relevant logics **R**, **E**, and **T**, strong relevant logics **Rc**, **Ec**, and **Tc** rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in **R**, **E**, and **T**, respectively. What underlies the strong relevant logics is the ***strong relevance principle***: If A is a theorem of **Rc**, **Ec**, or **Tc**, then every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part [6, 7]. Since the strong relevant logics are free of not only implicational paradoxes but also conjunction-implicational and disjunction-implicational paradoxes, in the framework of strong relevant logic, if a reasoning is valid, then both the validity of its conclusion in the sense of conditional and the necessarily relevant relation between its premises and its conclusion can be guaranteed in a certain sense of strong relevance.

Now, we can see that it is the notion of conditional (entailment) that plays the most fundamental role in relevant reasoning.

3. FUZZY RELEVANT LOGIC: CAN WE ESTABLISH A FORMAL SYSTEM TO UNDERLIE APPROXIMATE AND RELEVANT REASONING?

Our original problem is can we establish a formal logic system to underlie approximate and relevant reasoning. We now discuss this fundamental research problem from possible research directions, problems, and difficulties.

First of all, is there certainly some need to establish such a formal logic system? To this question, the present author's answer is 'YES'. This is based on the following considerations.

First, the fuzzy logic itself needs a standard formal calculus or a family of standard formal calculi. Until now, there is no such a standard formal calculus or a family of standard formal calculi to axiomatize fuzzy logic which is widely accepted by the fuzzy logic community. A major reason leads to this situation is that there are some different answers to the question: What is fuzzy logic? Zadeh made an important distinction between two different meanings of term 'fuzzy logic' such that "In a narrow sense, fuzzy logic, FLn, is a logical system which aims at a formalization of approximate reasoning. In this sense, FLn is an extension of multivalued logic. However, the agenda of FLn is quite different from that of traditional multivalued logic. In particular, such key concepts in FLn as the concept of a linguistic variable, canonical form, fuzzy if-then rule, fuzzy quantification and defuzzification, predicate modification, truth qualification, the extension principle, the compositional rule of inference and interpolative reasoning, among others, are not addressed in traditional systems. This is the reason why FLn has a much wider range of applications than traditional systems. In its wide sense, fuzzy logic, FLw, is fuzzily synonymous with the fuzzy set theory, FST, which is the theory of classes with unsharp boundaries. FST is much broader than FLn and includes the latter as one of its branches." [10] However, until now, even in the narrow sense of fuzzy logic, there is no standard formal calculus that formalized approximate reasoning including all contents of FLn.

Second, various approximate reasoning applications, in particular, automated reasoning systems with general purposes, need a domain-independent, sound, theoretical foundation. Those fuzzy if-then rules dependent on some special domain are difficult to be deepened into nest ones and extended into general cases. It should be the formal logic calculus of fuzzy conditional that can underlie approximate reasoning in general.

Third, approximate and relevant reasoning, which is the subject of this paper, need a sound logical criterion. To be a 'logic', fuzzy relevant logic has to be axiomatized as a formal logic system. Also, those key terms, such as Fuzziness (vagueness) and relevance, approximate reasoning and relevant reasoning, need some explicit, unambiguous, formal definitions such that various application systems invoking approximate and relevant reasoning can be designed and developed based on a sound conceptual foundation.

Now, how can we establish a formal logic system to underlie approximate and relevant reasoning? There are at least two possible directions to do this work. One direction is to modify the axiomatization of fuzzy logic which is regarded as a 'many-valued logic with special properties' such that all implicational, conjunction-implicational, and disjunction-implicational paradoxes are rejected from the logical theorems at first, and then extend this 'core' logic into a more complete family of fuzzy relevant logics such that those key concepts pointed out by Zadeh are introduced and formalized in the logics. Another direction is to introduce the notion of fuzziness (vagueness) and those key concepts pointed out by Zadeh into strong relevant logics such that they can underlie approximate reasoning as well as relevant reasoning.

It is should be pointed out that in any approach to establish fuzzy relevant logic, the notion of conditional has to be represented in the logic as a primitive intensional connective but not a truth function. Of course, the truth-value of a conditional can be many-valued even infinite-valued. Therefore, a fundamental characteristic of the fuzzy relevant logic is that the truth-value of a conditional cannot be determined simply by the truth-values of its antecedent and consequent. For those logical conditionals, their truth-values and validity are characterized by the semantic theory of the fuzzy relevant logic. On the other hand, the truth-values of all empirical conditionals to be used as premises in a reasoning must be set up by a truth assignment in the same way as that of truth-functional atomic propositions. To assign the truth-values to those empirical conditionals that are used in reasoning as premises at first may be a difficult task to some application, but it is this requirement that allows relevant logics to guarantee that a relevant reasoning is not only truth-preserving but also relevance-preserving in the sense of conditional.

Probably, the major difficulty on establishing fuzzy relevant logic is how to find some elegant and satisfactory semantics for the logic. In the framework of fuzzy relevant logic, we have to deal with conditionals such that not only truth-values of the antecedent and consequent of a conditional but also the relevant relation between the antecedent and consequent are many-valued and fuzzy.

4. CONCLUDING REMARKS

In this paper, we have proposed a fundamental research problem: can we establish a formal logic system to underlie those reasoning that are both relevant and approximate, presented the motivation to study fuzzy relevant logic, and discussed possible research directions, problems, and difficulties to establish a formal fuzzy relevant logic system to underlie approximate and relevant reasoning. Although there is a long way to go to get a satisfactory solution of the problem, this paper has made a starting point on this novel research direction. The present author believes that approximate and relevant reasoning underlain by fuzzy relevant logic must play an important role in scientific reasoning in epistemic processes of scientific discovery as well as in our everyday reasoning.

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