Fuzziness and the Sorites Paradox.
From Degrees to Contradictions.

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Brief description of the proposed dissertation research:

The dissertation has two parts, each dealing with a problem, namely: 1) What is the most adequate account of fuzziness -the so-called phenomenon of vagueness?, and 2) what is the most plausible solution to the sorites, or heap paradox?

I will try to show that fuzzy properties are those which are gradual, amenable to be possessed in a greater or smaller extent. Acknowledgement of degrees in the instantiation of a property allows for a gradual transition from one opposite to the other, each intermediate stage constituting an overlap in certain proportion of both contraries. Hence, degrees in the possession of a property give rise to simple contradictions.

The reason why I have chosen those two questions is that they provide the main philosophical motivation for a particular brand of an infinite valued and paraconsistent logic. I will claim that Classical logic (CL) is not adequate to handle fuzzy situations, and, being deficient, is in need of being expanded to make room for degrees of truth and weak contradictions.

One can hardly deny the importance of the debate, since what is ultimately at stake is what the limits of truth, rationality, intelligibility and possibility are.

The main disciplines within which the research moves are the philosophy of language, philosophy of logic, and ontology.

Detailed presentation of the proposal

PART I.- I will consider an inventory of analyses of fuzzy expressions according to possible ways of classifying a sentence, S, like "Frank is bald", in regard to its truth value, assuming that Frank is a borderline case of bald. I enumerate the alternatives with an indication of their leading advocates.

1a) *Truth-functional bivalent classical logic* (CL): S must have one of the truth values: it is either True or False, but not both. The principle of bivalence holds. The meaning of a fuzzy expression can somehow be legitimately precisified, or stipulated. No epistemic considerations involved. (Quine [1981]).

1b) *Agnosticism* (wrongly called *epistemicism*): S has a classical truth value, but it is impossible in principle to discover which. Its truth value is unknowable. Fuzziness is a special kind of ignorance. (Williamson [1994], [1996], [1999]; and Sorensen [1988], [2001]).

2a) *Indeterminism*: S is neither true nor false. The principle of excluded middle fails. (Frege -for whose position see Heijenoort [1985], and Russell [1923]). There is no fact of the matter to be known as to whether Frank is bald or not. Fuzziness is this *ontic* indeterminacy. (Tye [1990], [1994a], [1994b]; Horgan [1994]; Burgess [1998]). The indeterminacy may also be merely *linguistic*: a fuzzy word partially or indeterminately refers to each of a range of precise things. (Field [1998]).

Supervaluationism (Fine [1975]; Pinkal [1995]; Keefe [2000]) and *intuitionism* (Putnam [1983]; Wright [1975], [1987], [2001]) are special cases of this theory of fuzziness as indefiniteness.

2b) *Many-valued logic* and *fuzzy set theory*: S has none of the standard truth values: it is neither (totally) True nor (completely) False, but has an intermediate truth value. The extension of a fuzzy predicate is a fuzzy set mapping objects into the unit interval [0, 1]. Membership in a set as well as truth itself are gradual. A central feature of fuzziness is its graduality. The *more* hairs Frank has, the *less* bald he is. Even one hair makes a difference to the truth of "Frank is bald". (Goguen [1969]; Lakoff [1973], [1987]; Machina [1976]; Sanford [1976], [1995]).

3a) *Nihilism*: Were fuzziness real, S would be both True and False. But, since this is impossible, fuzzy expressions are empty or non-referential. Properties such as bald, red, etc. do not exist; and there are no fuzzy objects, like heaps, mountains, children, tables, etc. (Wheeler [1975]; Unger [1979]; Heller [1996]).

3b) *Pragmatism*: S is both True and False, but each in a different context. Fuzziness is context- dependent. The role of subjects involved in a discussion is essential. An analysis in terms of ambiguity may be in order here. (Burns [1991]; Walton [1992]; Raffman [1996]; Graff [2000]).

3c) *Paraconsistent logic*: S is both true and false (a glut), at the same time and in the same respect. Fuzziness results in an over-determination. It is a source of benign inconsistency, which is logically under control. In a fuzzy situation, there is an overlap of opposites. The principle of non contradiction is not (perfectly) true. (Hyde [1997], *Relevantist* Sylvan [1993], and Read [1995]; *Dialetheist* Priest [1998]).

The vexed question which I will tackle in the dissertation is how to choose among these conflicting views. What are the criteria of adequacy for theories of fuzziness? This will require a method to adjudicate between them. My claim is that the selection must ultimately be dictated by a general conception of reality. Fuzziness, as a characteristic of linguistic expressions, ought to be explained in terms of certain features of the mind- independent world. It is hard to imagine how a fuzzy language successfully operates within a non-fuzzy reality. We cannot truly describe the way objects, properties and relations are in themselves without fuzzy words. I will argue for a particular combination of 2b) and 3c).

PART II.- The structure of the sorites argument is the following. Imagine a sequence of adult human beings ordered by the relation of *having one more hair than*, such that it begins with a completely bald person, a_0, and ends with the hairiest person in the world, say, $a_{100,000}$. Suppose further that the predicate "bald" definitely applies to a_0 (first premiss), and definitely does not apply to $a_{100,000}$. These are the extremes of the series. Finally, assume that the difference in the hairy situation between any pair of adjacent individuals is so minute that it is beyond the powers of discrimination of the naked eye. Thus any pair of contiguous members should receive the same treatment: either both are bald, or none is. It cannot be that only a_i is bald, while a_{i+1} fails to be bald. Equivalently, either a_i is not bald, or a_{i+1} is bald. This is the major premise. Now reason repeatedly by disjunctive syllogism, and you will conclude that the hairiest person in the world is bald too, which is absurd.

How should we judge the reasoning? I will exhaustively examine the range of conceivable answers.

First of all, there are some authors (Frege, Russell) who consider the deduction not to be amenable to logical evaluation: an assessment in terms of validity is beside the point. Fuzziness lies outside the realm of logic. But, if we have reasons to discard this very radical stance, which reduces the scope of logic to ideal objects, there are still some substantial questions we have to ask.

Does the argument have true premises and is valid? Nihilists say yea: the sound inference is taken to be a *reductio* of the first premise. The predicate "bald" is meaningless: it does not apply to anything. But this is a desperate attitude, which annihilates all objects of ordinary experience.

If we reject the conclusion, then there are mainly two ways out: either at least one of the major premises is not true, or the rule of inference employed is invalid. And this is the dilemma. In other words, is the transition from the property *F*, to its opposite, not *F*, abrupt or gradual? Is the extension of a fuzzy predicate sharp or blurred? The dismissal of the major premiss is adopted by CL, agnosticism, indeterminism, supervaluationism, intuitionism and pragmatism, while the restriction or rejection of the validity of the argument is taken by many valued and paraconsistent logics. I will examine these two routes.

The objections facing the abandonment of the major premise is that to dismiss it classically entails to affirm its negation: $\sim\forall i(\sim Fa_i \vee Fa_{i+1})$, which in turn entails $\exists i(Fa_i \wedge \sim Fa_{i+1})$. And this seems to entail the existence of a unique *sharp cutoff* point somewhere along the series. Although a_i and a_{i+1} are indiscrimin- able, one would be compelled to treat them dissimilarly, thus threatening the observationality of phenomenal properties. But it seems to be unjustified to postulate a difference where there is no distinction. Besides, an explanation is owed of why the major premises appeared *prima facie* plausible. Supporters of this solution react differently according to which conception of fuzziness they adhere to. Partisans of position (1b) reply that the sharp divide is compatible with fuzziness. Those favouring (2a) retort that their posture does not imply the existence of a clear-cut boundary, by rejecting at least one of the three entailments just mentioned. In contrast to these proposals, many valued and paraconsistent logics respect the second premise -though it may not be altogether true-, thus allowing for a *smooth transition*, a *continuum* from one pole to the other of the sequence, due to tiny gradual steps. All the intermediary objects in the stretch between the extremes share in the qualities of the opposites in different proportions. To dissolve the paradox, the rule of inference is invalidated. Relevantists, dialetheists, many valued logicians deny the validity of *modus ponens*. The main problems with these radical systems is that they depart largely from CL in postulating degrees of truth or accepting true contradictions and dropping a fundamental rule of reasoning. The debate turns around these contentious issues in the philosophy of logic.

In my dissertation, I will develop the program advanced by Lorenzo Peña, and I will rationally defend it against objections found in the literature. It comprises the following points. Fuzzy properties are those which can be had in greater or smaller extent. Now, degrees of possessing a property (or degrees of membership in a set) are identical to degrees of truth. Therefore, truth itself is gradual. If this is so, then a fuzzy situation arises when an object neither fully possesses a property nor totally lacks it, but has it *up to a point*, and also *in some extent* fails to have it. Thus fuzzy situations are indeed *weakly* inconsistent. This acceptable contradiction is arrived at by means of the rule allowing us to pass from the premiss that an object *x* has *F to some degree or other* to the conclusion that *x* is *F*, *tout court*. This is a *minimalist* demand. Not to accept this rule is to commit us to the opposite *maximalist* requirement, to wit, for *x* to be *F*, it is necessary that *x* be wholly *F*!

But in order to avoid *absolute* inconsistency (all sentences being true in the system), a crucial distinction is introduced between two kinds of negation: the strong "¬" or classical, and the weak "~", obeying the principle that /~p/ = 1-/p/. "¬" is read as "it is *wholly* false", "not (true) *at all*", while "~", simply as "not". Accordingly, two kinds of contradictions result, depending on which negation is involved. Simple contradictions, "p and not p", are allowed, and may be partially true, but never totally true, whereas over- contradictions, "p and not p at all", are completely false, absurd, unintelligible. The principle of Cornubia (p, ¬p ⊢ q) is valid only for strong negation, not for the simple one, ~.

With the help of these distinctions, we can say that a fuzzy situation involves the weak denial of the principles of excluded middle, and of non-contradiction, but such denial does not prevent those principles from being also partly true.

As for the sorites, all the major premises are considered true, but not totally true, since they are also partly false. Consequently, fuzzy predicates are bounded, and have many boundaries. The conclusion is averted by renouncing disjunctive syllogism for ~ (but keeping it for ¬). Hence, the argument is a sophism. The advantages of the present solution are that, by upholding the major premisses, a direct account is offered of the little by little transition from *F* to not *F*. A small difference in the input cannot produce a great difference in the output. If the input is multiply valued, the output should not be bivalent. Furthermore, graduality is not eliminated, and the incoherence is explained as supervening on degrees. Additionally, since like cases are treated alike, observational predicates are not threatened. And finally, absolutely all tautologies and rules of inference of CL are preserved. Even a version of the principle of bivalence survives, to wit, any sentence has a designated (true) value or an antidesignated (false) one. But of course, more truths are added. We have everything that is wanted.

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