China's 1989 National College Entrance Examination

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On a recent visit to the People's Republic of China I had an opportunity to examine a copy of the examination given to all high school students who wish to enter college. One can judge from the questions on the mathematics section of this exam the standard to which college students are held in the world's most populous country.

Students must choose between the "Liberal Arts" and the "Science & Engineering" exams. The former is comprised of tests on Chinese language, English, history, geography, politics, and mathematics, with an optional test on biology, while the Science & Engineering exam replaces history and geography with chemistry and physics. The two mathematics examinations are slightly different, as might be expected, with more emphasis on technical and advanced problems for Science & Engineering students. But in both cases they offer a total of 120 points, whereas the other required examinations each offer only 100 points, and biology a mere 70. When I inquired whether students in China complain about a lack of practical applications of school mathematics, I was reminded that the 120 points worth of questions on the college entrance examination are considered a most significant application!

Seriously, though, I asked students in many fields of study, professors of Chinese and English, and companions on various train trips about their attitudes toward mathematics and was surprised at the uniformly positive attitudes they expressed. Rather than being a conversation stopper, the news that I was a college mathematics teacher often stimulated fond recollections of my companions' experiences in mathematics classes. Many Chinese share the view common in Japan that mathematics is not something of interest only to an intellectual elite, but is an important ingredient in everyone's education (see [**1**]). The extra weight given to mathematics on the college entrance exam and the high standard of the mathematics questions even for students in Liberal Arts confirm this impression.

After elementary school, students in China attend middle school for three years and high school for another three. Each of these final six years they study algebra, in the first year of middle school for five hours per week, and thereafter for three hours every week. In addition to algebra, they have another three hour mathematics class each week in their final five years: plane geometry in the second and third years of middle school, trigonometry in the first year of high school, solid geometry in the second year and analytic geometry in the final year. Three years of physics are also required. The textbooks formerly were adapted from Russian schoolbooks, but more recently they have been written by mathematicians at Chinese universities.

1989 Mathematics Examination for Liberal Arts Candidates

Part I. Multiple-choice. (3 points each)

1. If $I=\{a,b,c,d,e\}$, $M=\{a,c,d\}$, $N=\{b,d,e\}$, where I is the universal set, then $\overline{M}\cap\overline{N}$ equals
   A. $\varnothing$ B. $\{d\}$ C. $\{a,c\}$ D. $\{b,e\}$

2. A function which has the same graph as that of $y=x$ is

A. $y=\sqrt{x^2}$ B. $y=\dfrac{x^2}{x}$ C. $y=a^{\log_a x}$, where $a>0$, $a\neq 1$

D. $y=\log_a a^x$, where $a>0$, $a\neq 1$.

3. If a circular cylinder has base radius $\sqrt{2}$ and height 2, then its lateral area is
A. $2\sqrt{3}\pi$ B. $2\sqrt{2}\pi$ C. $4\sqrt{3}\pi$ D. $4\sqrt{2}\pi$

4. Given that $\{a_n\}$ is a geometric sequence with $a_1+a_2=12$ and $a_2+a_3=-6$, then if $S_n=a_1+a_2+\cdots+a_n$, the value of $\lim_{n\to\infty} S_n$ is
A. 8 B. 16 C. 32 D. 48

5. If $(1-2x)^7=a_0+a_1x+a_2x^2+\cdots+a_7x^7$, then the value of $a_1+a_2+\cdots+a_7$ is
A. -2 B. -1 C. 0 D. 2

6. If $|\cos\theta|=1/5$, $5\pi/2<\theta<3\pi$, then the value of $\sin(\theta/2)$ is

A. $\dfrac{-\sqrt{10}}{5}$ B. $\dfrac{\sqrt{10}}{5}$ C. $\dfrac{-\sqrt{15}}{5}$ D. $\dfrac{\sqrt{15}}{5}$

7. The line symmetric to the line $2x+3y-6=0$ with respect to the point $(1,-1)$ has the equation
A. $3x-2y+2=0$ B. $2x+3y+7=0$ C. $3x-2y-12=0$
D. $2x+3y+8=0$

8. Two parallel planes a unit distance apart intersect a certain ball in disks whose areas are 5π and 8π respectively. Given that the two planes lie on the same side of the center of the ball, then the ball's radius is
A. 4 B. 3 C. 2 D. 5

9. Among the 5-digit numbers without repeating digits which can be formed from the digits $1,2,3,4,5$ the number of even numbers is
A. 60 B. 48 C. 36 D. 24

10. If the distance from a point P on the hyperbola $\dfrac{x^2}{64}-\dfrac{y^2}{36}=1$ to the right focal point of the hyperbola is 8, then the distance from P to the right directrix is

A. 10 B. $\dfrac{32\sqrt{7}}{7}$ C. $2\sqrt{7}$ D. $\dfrac{32}{5}$

11. The minimum value of $f(x)=\cos^2 x+\sin x$, for $|x|\leq\pi/4$, is

A. $\dfrac{\sqrt{2}-1}{2}$ B. $-\dfrac{1+\sqrt{2}}{2}$ C. -1 D. $\dfrac{1-\sqrt{2}}{2}$

12. Given $f(x)=8+2x-x^2$, if $g(x)=f(2-x^2)$ then
A. on the interval $(-2,0)$, $g(x)$ is an increasing function
B. on the interval $(0,2)$, $g(x)$ is an increasing function
C. on the interval $(-1,0)$, $g(x)$ is a decreasing function
D. on the interval $(0,1)$, $g(x)$ is a decreasing function

Part II. Write the answer in the blank space. (4 points each)

13. Given three points $A(1,0)$, $B(-1,0)$, $C(1,2)$, the line through A perpendicular to the line BC has equation ____________.

14. The solution set of the inequality $|x^2 - 3x| > 4$ is ____________.

15. The domain of definition of the inverse of the function

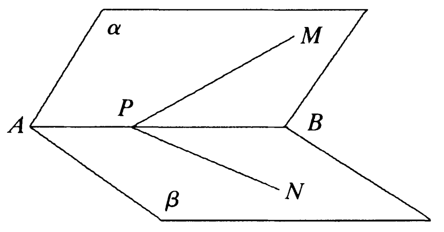
$$y = \frac{e^x - 1}{e^x + 1}$$

is ____________.

16. Given two propositions A and B, if A is a sufficient condition for B, then B is a ____________ condition for A; $\bar{A}$ is a ____________ condition for $\bar{B}$.

17. Given $0 < a < 1$, $0 < b < 1$, if $a^{\log_b(x-3)} < 1$ then the range of possible values for x is ____________.

18. As shown in the figure, P is a point on edge AB of the dihedral angle $\alpha - AB - \beta$, from which depart rays PM, PN on planes α, β respectively. If $\angle BPM = \angle BPN = 45°$ and $\angle MPN = 60°$, then the magnitude of the dihedral angle $\alpha - AB - \beta$ is ____________.



Part III. Solve, showing all work. (points as indicated)

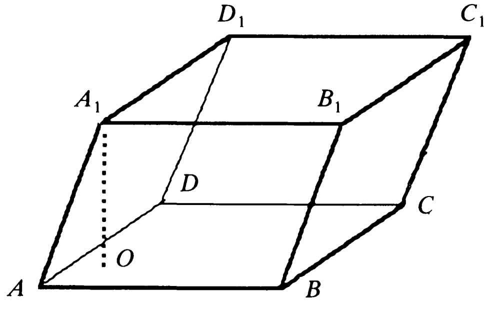
19. (8 pts.) If the complex number $z = (1 - \sqrt{3}\, i)^5$, find the modulus and principal value of the argument of z.

20. (8 pts.) Prove that $\tan\frac{3x}{2} - \tan\frac{x}{2} = \frac{2\sin x}{\cos x + \cos 2x}$.

21. (10 pts.) In the parallelepiped $ABCD - A_1B_1C_1D_1$ shown in the figure, with $AB = 5$, $AD = 4$, $AA_1 = 3$, $AB \perp AD$, and $\angle A_1AB = \angle A_1AD = \frac{\pi}{3}$,

i) prove that the projection O of the point A_1 onto the lower face $ABCD$ lies on the bisector of $\angle BAD$.

ii) Find the volume of the parallelepiped.



22. (10 pts.) Use mathematical induction to prove

$$(1 \cdot 2^2 - 2 \cdot 3^2) + (3 \cdot 4^2 - 4 \cdot 5^2) + \cdots + \left[(2n-1)(2n)^2 - 2n(2n+1)^2\right]$$
$$= -n(n+1)(4n+3).$$

23. (12 pts.) Given $a > 0$, $a \neq 1$, find the range of values of k which make the equation $\log_a(x - ak) = \log_{a^2}(x^2 - a^2)$ have solutions.

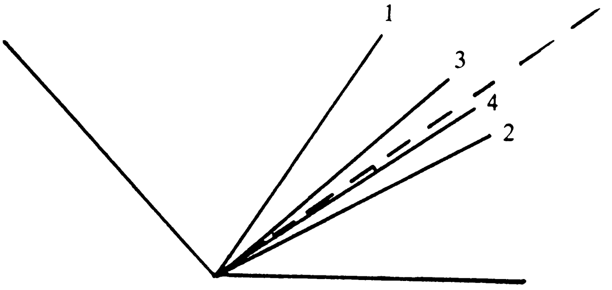
24. (12 pts.) Given the ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $a > b > 0$, find the hyperbola, with the same foci as this ellipse, which maximizes the area of the quadrilateral whose vertices are the intersection points of the two curves. Find the coordinates of the vertices of this area-maximizing quadrilateral.

Reference

1. Jerry P. Becker et al., *Mathematics Teaching in Japanese Elementary and Secondary Schools*, A Report of the 1988 ICTM Japan Mathematics Delegation, Illinois Council of Teachers of Mathematics, Southern Illinois University at Carbondale, March 1989.

Trisection of an Angle in an Infinite Number of Steps

$1/3 = 1/2 - 1/4 + 1/8 - 1/16 \cdots$



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