

Math 219, Homework 4

Due date: 30.12.2005, Wednesday

Suppose that $K > 0$, and $f(t)$ is defined as

$$f(t) = \begin{cases} 1 & \text{if } 4n \leq t < 4n + 1 \\ 0 & \text{otherwise} \end{cases}$$

where n runs through the set of integers.

(a) Determine the Fourier series for $f(t)$.

(b) Consider the differential equation

$$\frac{d^2x}{dt^2} + x = f(t).$$

By using ODE Architect, solve (and graph the solutions of) this equation for as many values of K as possible between 0.5 and 10 for $0 \leq t \leq 100$. (You can enter $f(t)$ in ODE Architect using the built in command $SqWave(t, L, K)$ and then set $L = 4 * K$, and assign K definite values on the lines below). Record the maximum values of $x(t)$ for each of these K 's, and plot a K vs. $max(x(t))$ graph by hand.

(c) Which values of K result in a resonance in the system? (Hint: you should find 5 such values). Can you relate these values to the Fourier series terms? Please print the resonance graphs.

(d) What do the Fourier coefficients correspond to on the resonance graphs?

We wish you a happy new year and success on your finals!