Math 219, Homework 2

Due date: 23.11.2005, Wednesday

This homework concerns two (fictitious) design problems about the solar car "MEŞ-e" of the METU Robotics Society, which won the Formula-G trophy in September 2005. Just for the purposes of this homework, assume that they want to modify the car, and they are asking for your help on two issues.

1. The first problem is about the shock absorbing system of the car. We may model the shock absorber as a single linear spring. This question concerns how to adjust the damping coefficient in order to meet certain requirements.

(a) It is known that when the pilot, weighing $80kg$, gets into the car seat, the shock absorber is compressed by $5cm$. From this data, compute the spring constant k (in kg/sec^2).

(b) The car (without the pilot) weighs $240kg$. Write a differential equation which governs the vertical motion of the car (this could for instance describe the vertical

displacement when the car goes over a speed bump). (Hint: Check "Damped free vibrations" from Boyce,Di Prima, section 3.8).

(c) It is required that, when the car goes over a speed bump of $5cm$ high, the vertical displacement $x(t)$ should approach the equilibrium point 0 in a way that for $t \geq 1sec$, $|x(t)| \leq 1cm$. For several values of the damping coefficient and for this initial condition, sketch the graph of the solution curve for $0 \leq t \leq 2sec$. Finally, decide which values of the damping coefficient are allowable in order to meet this requirement.

2. The second problem is about the power supply of the motor. The panels convert solar energy to electrical energy and store it in the accumulator. Assume that the accumulator provides a voltage of $E(t) = 48\cos(\omega t)$ Volts to the system. The frequency ω is adjusted by the gas pedal. The circuit can be modeled as a series $L - C$ circuit. Take $L = 0.05$Henry and $C = 10^{-6}$Farad.

(a) Write a differential equation for the current $I(t)$ through the voltage source (Hint: Check "Electric Circuits" from Boyce,Di Prima, section 3.8. We are assuming $R = 0$).

(b) Solve this differential equation for $I(t)$ in terms of ω using the method of undetermined coefficients.

(c) Currents over 20 Amperes may harm the accumulator. Using your result from part (b), find out which frequencies (in Hertz) should not be allowed. For several values of ω graph $I(t)$ for $0 \leq t \leq 0.2sec$ using ODE Architect, and denote which of these are potentially harmful and which are not.