Math 219, Homework 3

Due date: 9.12.2005, Friday

1. Consider the initial value problem

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = u_4(t), \qquad y(0) = y'(0) = 0$$

(a) Solve this initial value problem using the Laplace transform.

(b) Use ODE Architect to solve the equation, and graph the solution. Also graph $\frac{dx}{dt}$ with respect to t (You can use the function $Step(t, 4)$ to create a unit step function with discontinuity at $t = 4$).

(c) Discuss how the graphs agree with the solutions in (a): in particular determine (if any) all the points where $x(t)$ and $\frac{dx}{dt}$ are discontinuous, behavior of these two functions for $t \to \infty$, their maxima and minima.

2. Write each of the following systems of differential equations in matrix form, find the eigenvalues and eigenvectors of the coefficient matrices, and using these, find all solutions of each system. Also, graph the phase portraits ($x - y$ graph) using ODE Architect. Please use a scale which includes the point $(0, 0)$, and graph several solutions in order to clearly observe the behavior around $(0, 0)$. Also, place arrows on the solution curves which indicate the direction of increasing t, and make sure that solution curves along the eigenvector directions are graphed if there are any real eigenvectors.

(a)

$$\frac{dx}{dt} = 2x - y$$
$$\frac{dy}{dt} = 3x + 3y$$

(b)

$$\frac{dx}{dt} = -x + y$$
$$\frac{dy}{dt} = 3x - 4y$$

(c)

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = 5x + 5y$$

(d)

$$\frac{dx}{dt} = -4x + 3y$$
$$\frac{dy}{dt} = -3x + 2y$$

(e)

$$\frac{dx}{dt} = -x - 3y$$
$$\frac{dy}{dt} = 2x + y$$