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COMPLEX POLYNOMIALS
(Cambridge Studies in Advanced Mathematics 75)

By TERRY SHEIL-SMALL: 428 pp., £65.00 (US\$95.00), ISBN 0-521-40068-6 (Cambridge University Press, 2002).

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This is a book about polynomials in the complex plane, and in particular about the geometric theory of such polynomials. It touches on a breadth of topics in classical complex analysis (Blaschke products, univalent functions, the fundamental theorem of algebra, the Grace–Szegö theorem). It also treats some far less standard parts of the theory, like the ABC conjecture for polynomials and the Ilieff–Sendoff conjecture.

The Ilieff–Sendoff conjecture is that any polynomial with all its zeros in the unit disc must have each zero of the derivative within distance one of a zero of the polynomial. This is a frustratingly elementary problem, that ought to be easy and is probably true, but it has resisted solution for forty years. It illustrates a central issue in this general area. The relationship between zeros of polynomials and their derivatives is subtle; likewise, the relationship between the location of zeros of polynomials (and analytic functions) and their coefficients.

On some level, this subtle relationship between zeros and coefficients is the intrinsic problem with the Riemann hypothesis – which is, after all, just a statement about the zeros of one very explicit analytic function (how hard can that be?). One suspects (and in some cases one knows) that a good deal of this theory developed around a wish to solve the Riemann hypothesis (certainly, this motivated Pólya). Number theorists tend to believe that a solution of the Riemann hypothesis will come from above, and not through hands-on study of the zeta-function but maybe, just maybe, the proof is elementary. (Even if the first proof found doesn't prove to be elementary, it would be an unjust mathematical universe that did not contain an elementary proof.)

Chapter 2 of *Complex polynomials* serves as a 'mini course' in plane topology. It is based on the degree principle (the continuous change in the argument of a function around a loop), and is used as a non-standard foundation for the classical theory of complex analysis. Included in the chapter is a proof of Brouwer's fixed-point theorem. Also included is a discussion of harmonic polynomials.

Chapter 3 is on the 'Jacobian problem': when is it possible to solve uniquely a set of algebraic equations? This is an attractive introduction to a beautiful and central problem concerning the non-vanishing of the Jacobian, and the property of being one-to-one. Pinchuk's example of a degree-25 polynomial mapping that has positive Jacobian but is not globally one-to-one is presented.

This is only a taste of the material presented in this eleven-chapter book. Scattered throughout are a wealth of diverse and interesting material and a number of open problems and conjectures. This includes the Craven–Csordas–Smith conjecture and the Smale conjecture, two innocent-looking, but apparently difficult, unsolved problems.

The Craven–Csordas–Smith conjecture states that, for a real polynomial P, the number of real critical points of P'/P does not exceed the number of non-real zeros of P. The Smale conjecture states that a polynomial of degree at least 2 must, for each z, have a critical point ζ where

$$|P(z) - P(\zeta)| < |(z - \zeta)P'(z)|.$$

I like this book for a variety of reasons. I found myself reading large pieces for pleasure, before I knew I would be asked to review it. It is carefully and attractively presented. Shiel-Small has written an original book; there is no book like it currently available. It does not particularly compare to anything extant, and it will not soon be duplicated. I suspect that it will be in my collection for many years.

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SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS IN HILBERT SPACES

(London Mathematical Society Lecture Note Series 293)

By GIUSEPPE DA PRATO and JERZY ZABCZYK: 379 pp., £29.95 (LMS members' price £22.46), ISBN 0-521-77729-1 (Cambridge University Press, 2002).

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What do we mean by 'analysis in infinite-dimensional spaces'? First, we have in mind functional analysis; more precisely, most mathematicians will think of linear operators acting between infinite-dimensional topological vector spaces, mostly Banach or Hilbert spaces. The 'infinite-dimensional character' stems from these spaces, but often in (elementary) linear functional analysis we do not identify such spaces to be a substitute for $\mathbb{R}^n$ or a (finite-dimensional) manifold.

This changes when we turn our attention to nonlinear operators. Of course, now we search for a linear approximation, and we start to consider differentiation of functionals $F: V \to \mathbb{R}$, V being, for example, a Banach space, or operators $A: V \to V$; that is, the mapping $x \mapsto A(x)$. Now we identify elements in V much more as substitutes for points in $\mathbb{R}^n$, and the rise of topological methods in the theory of nonlinear (Banach space) operators is a very convincing argument for doing analysis and topology on infinite-dimensional manifolds. Often, however, V is a space of functions $\varphi: \mathbb{R}^n \to \mathbb{R}$ (for example), and we still feel that 'true analysis' (in the sense of calculus) is related to these functions.

There are two topics that lead us to a different kind of thinking right from the start. The first comes from mathematical physics: in statistical mechanics, or in parts of solid-state physics, we have to handle systems with infinitely many degrees of freedom, and a macroscopic description needs a formulation based on functions that depend on infinitely many variables and the operators acting thereon.