umes only as a four-page note. It appeared on both heuristic and empirical grounds that the prime counting function $\pi(x)$ was smaller than its asymptotic approximant li x for all $x \geqslant 2$. It came as a great surprise when Littlewood proved in 1914 that $\pi(x) > \text{li } x$ held for a sequence of values of x tending to infinity. Littlewood's proof (whose details were given in a joint paper with Hardy) did not provide a method of estimating when such a value of x occurred. This stirred a philosophical debate on constructivity in number theory. Further interest was raised when S. Skewes, a student of Littlewood, showed that the inequality is achieved for a number x not exceeding the gargantuan "Skewes number" exp exp exp exp (7.705). Skewes's bound has been reduced greatly, but it is still not known where $\pi(x)$ first overtakes li x.

Littlewood's human side and his pungent writing style are revealed in such articles as his account of his mathematical education and his views upon how mathematicians work. We find in the latter piece such items of good sense as the advice to "either work all out or rest completely" (p. 1629) along with the curious pronouncement that a vacation should be of exactly twenty-one days duration—nineteen days is probably too short.

These volumes have considerable scientific value, and they provide a fitting memorial to one of the mathematical giants of the first half of the twentieth century.

HAROLD G. DIAMOND

Gregory H. Moore. *Zermelo's Axiom of Choice: Its Origins, Development, and Influence.* (Studies in the History of Mathematics and Physical Sciences, 8.) xiv + 410 pp., illus., bibl., index. New York/Heidelberg/Berlin: Springer-Verlag, 1982. $38 (cloth).

Euclid's parallel postulate is the classic example of the fruitfulness of studying basic mathematical assumptions. Its study led J. Bolyai and N. I. Lobachevsky to construct non-Euclidean (hyperbolic) geometry, thus demonstrating the independence of the parallel postulate from the other axioms of Euclid's system. Another famous example is Georg Cantor's continuum hypothesis, a form of which states that every subset of the real line $\mathbf{R}$ can be placed into one-to-one correspondence either with $\mathbf{R}$ itself, or with a set of integers. In 1900 Hilbert made its proof the first of his famous list of twenty-three problems. Cantor, K. Weierstrass, and D. Hilbert (briefly, in 1927) were all under the illusion that they had established its truth.

The subject of Moore's book is a third famous example. Ernest Zermelo's axiom of choice asserts that for every set S there is a function f that selects from each nonempty subset A of S a unique element $f(A)$ in A. This axiom plays a basic role in twentieth-century mathematics and logic, and Moore's book traces in considerable detail its origin, development, and influence. The story is as follows.

In 1883, while studying the implications of his theory of arbitrary sets (*Mengenlehre*), Georg Cantor assumed as a valid "law of thought" that every set can be well ordered. (A set is *well ordered* by an ordering imposed on its members under which each of its nonempty subsets has a least element.) However, many of Cantor's contemporaries rejected this "well-ordering principle."

In his 1904 attempt to prove the well-ordering principle, Zermelo introduced the (logically equivalent) axiom of choice. Both Zermelo's proof and his axiom came under attack on mathematical and philosophical grounds. In a 1908 attempt to dispel this criticism, and probably also in order to avoid the paradoxes of Cantorian set theory that had surfaced in the 1890s, Zermelo listed explicitly the axioms he was assuming for the theory of sets. This axiomatization, and the axiom of choice in particular, occasioned another round of attacks. In particular, E. Borel pointed out in 1914 that the axiom of choice led to a basic paradox of measure theory: the "existence" of a subset A of the sphere congruent to its complement (hence of measure one-half), and also to two other disjoint sets having no point in common with it (whence its measure must be one-third). In Borel's opinion, A should be regarded as *undefined*.

In 1918 W. Sierpinski published the first thorough study of the role of the axiom of choice in set theory and analysis. In 1922 Abraham Fraenkel incorporated the axiom into a formal axiomatization of set theory, revising Zermelo's 1908 list. Throughout the 1920s a lively debate continued concerning the validity of Cantorian set theory, the deepest results being obtained by the Moscow (Lusin, Suslin, Alexandroff) and Warsaw (Sierpinski, Kuratowski, Tarski) schools. In 1934 Sierpinski published his famous monograph, *Hypothèse du continu*, in which he proved the equivalence of the continuum hypothesis to eleven other basic propositions of set theory.

K. Gödel, having shown the inadequacy of the Russell-Whitehead-Hilbert axiomatization of logic in 1931, continued this research on models of set theory to show in 1938 that Zermelo-Fraenkel set theory including the choice axiom is consistent provided that the theory is consistent without it. Gödel's result convinced most early critics of the axiom that there was no danger in utilizing it in mathematical research.

In the meantime, the axiom of choice had been found in many new equivalent (or weaker) forms such as Zorn's lemma, Tychonoff's compactness theorem, the existence of a basis for any vector space, and the representation of Boolean algebras by sets. As a result, logicians had become accustomed to think of those parts of mathematics which do not depend on choice as constituting a model somewhat analogous to absolute geometry, in which only the theorems of Euclidean geometry hold that are not dependent on the parallel postulate or its denial.

Moore's book provides an encyclopedic account of all these developments, thus continuing the historical documentation of modern logic and set theory begun in Jean Van Heijenoort's *From Frege to Gödel* and Joseph Dauben's *Georg Cantor*. Moore includes thorough citations and an unusually complete forty-page bibliography. Useful appendixes chart the deductive relations of other axioms to the axiom of choice and present the first English translation of the famous letters of the French empiricists discussing the axiom. Moore's bibliography and analysis show the exhaustive study he has made over more than a decade of both the published and unpublished literature. These sources have been handled with historical sensitivity and mastery of technical detail. This careful scholarship has enabled him to straighten out several historical myths appearing throughout the published literature, for example as to the roles played by Beppo Levi and Guiseppe Peano in introducing the axiom of choice, or in the actual origins of the Burali-Forti paradox.

The book does have some imperfections. At times Moore's consistently exhaustive scholarship makes it hard for the reader to distinguish which are major accomplishments. Too much space is devoted to the Polish school and too little to the roles of major figures, especially Poincaré and Hilbert, in shaping standards and areas of mathematical research. Also, Moore is perhaps overzealous in claiming Zermelo's axiomatization resulted not from efforts to resolve the set-theoretic paradoxes, but rather from a more selfish desire to justify his proof of the well-ordering principle. Moore's argument rests on the plausible claim that two papers written sixteen days apart in 1908 should be regarded as one unified piece of work. However, as Moore himself admits, the latter paper, which contains the axiomatization, also contains three references to the paradoxes. These and other facts described by Moore suggest that he should temper his claim.

Moore's account of "the four decades that have elapsed since Gödel established the relative consistency of the Axiom of Choice and the Generalized Continuum Hypothesis" is limited to a fifteen-page epilogue. Perhaps it is too early to analyze the change in the attitudes of logicians and mathematicians toward axiomatic set theory that has taken place since 1963, when Paul Cohen introduced the technique of "forcing" to prove the independence of the choice axiom from the other axioms of set theory, without resorting to the *Urelementen* Fraenkel had introduced in 1922 to prove a similar, but weaker, independence result.

In any case, Moore's book seems likely to stand for a long time as *the* definitive study of the axiom of choice, from its first glimmerings around 1871 to World War II.

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Constance Reid. *Neyman: From Life*. 292 pp., illus., index. New York/Heidelberg/Berlin: Springer-Verlag, 1982.

Mathematical statistician Jerzy Neyman was part of the intellectual migration from Europe to America which took place in the 1930s. He not only had a major role in the development of modern statistical ideas and procedures, but, as founder in 1939 and director for several decades of the Statistical Laboratory at the University of