Relating a small decrease of (m_p/m_e) with cosmological time to a small cosmological constant

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Abstract

The possible small variation downward by about 10^{-5}, of the ratio of the proton mass to the electron mass, over cosmological time is related to the decrease with time of a small vacuum expectation value for a Goldstone-like pseudoscalar field. The initial vacuum expectation value controls the magnitude of a very small cosmological constant.

Recently, new data [1] has given an indication that the ratio of the proton to electron mass $\mu = (m_p/m_e)$, has decreased over a cosmological time interval. If interpreted in terms of an effective decrease in the proton mass, the data suggest a decrease by about $(10 \text{ eV}) \times \mu \sim 18 \text{ keV}$, over a period of about the past twelve billion years. Natural questions which arise are then the following.

(1) Can one obtain the direction of change, a decrease, independently of the possibility that coupling parameters such as α_{em}, depend upon time? Data on the latter possibility [2, 3], have stimulated the search for time variation of physical "constants" [4, 1]. Recent data [5, 6] have not yet confirmed a variation [2, 3].

(2) Can one obtain an estimate of the small absolute scale of the mass decrease, a few keV, by relating it to other small energy-scale effects, like a cosmological constant of about $3 \times 10^{-47} \text{ GeV}^4$, and possibly to a neutrino mass of less than 0.1 eV?

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(3) Is it possible to estimate the cosmological time scale $\bar{t}$, for significant mass decrease (on the above small energy scale), in terms of another cosmological parameter associated with the early universe?

The purpose of this note is to illustrate how there might be affirmative answers to these questions. The basis is the standard assumption that the main contributions to particle mass arise from the nonzero vacuum expectation values of spin-zero fields. It is usually assumed that such a vacuum expectation value arises at the stable minimum of some effective potential-energy density. Here, the first assumption is that a small, particular vacuum expectation value of a pseudoscalar, Goldstone-like field b (which spontaneously breaks CP invariance in a cosmological context [F1]) is a metastable maximum value (i. e. over a cosmological time interval $\bar{t}$) [7]. We note that the possibility that the standard, scalar-field inflationary dynamics in the very early universe originates at a maximum of an effective potential for a (classical) scalar (inflaton) field ϕ, has already been considered in detail [8, 9, 10]. It has been shown that radiative corrections to a $\lambda\phi^4$ potential-energy density can set up an effective energy density with a maximum, with ϕ at or just above the Planck mass $M_{\mathrm{Pl}} \cong 1.2 \times 10^{19}$ GeV, and a minimum with ϕ just below M_{Pl} [9]. Inflation occurs with the inflaton field at the maximum and during its movement to the minimum. The positive second derivative of the effective potential with respect to the field variable, at the minimum, corresponds to the large squared mass of inflaton quanta (estimated [7, 8] to be $m_\phi^2 \sim (5 \times 10^{11} \text{ GeV})^2$). If metastable (i. e. essentially decoupled from big-bang radiation) these quanta can constitute dark matter today [7, 8, 11]. The pseudoscalar field b can be connected with the scalar ϕ, in a hypothetical, idealized model of a cosmological, spontaneously-broken chiral symmetry [7, 8]. However, the pseudoscalar field b is a separate hypothesis from the scalar field ϕ whose vacuum energy density generates the hypothetical inflation near $t = 0$, and the b-field dynamics over cosmological time intervals is distinct. The hypothetical, small vacuum expectation value b_0 of the b field, contributes a vacuum energy density $\lambda b_0^4 \sim 2.7 \times 10^{-47}$ GeV^4 for $b_0 \sim 5.5$ eV [7], using the same value of the self-coupling parameter as for the ϕ field, $\lambda \sim 3 \times 10^{-14}$ [12, 13]. An attempt is made to obtain a separate estimate of b_0 by coupling the b field to ν_τ (with g_{ν_τ}) [7]. This gives rise to a (presumably largest) neutrino mass $m_{\nu_\tau} = \sqrt{\tilde{m}_{\nu_\tau}^2 + (g_{\nu_\tau} b_0)^2} \sim \sqrt{(g_{\nu_\tau} b_0)^2} \sim 0.055$ eV, for $b_0 \sim 5.5$ eV and $g_{\nu_\tau} \sim 10^{-2}$ [7], and "bare" neutrino mass $\tilde{m}_{\nu_\tau} \sim 0$. This provides a representation, including the significant role of λ, of the often-remarked similarity between the empirical energy scales relevant for a cosmological constant and for neutrino mass. As illustrated below the effective squared mass of potential b quanta is negative.

[F1]Spontaneous CP violation is a motivation for considering a nonzero vacuum expectation value for the b field. For a brief time near to $\sim 10^{-36}$ s, there can be a CP-violating effect such as an antineutrino-neutrino asymmetry from a primary, radiation-producing decay process [7, 11].

The second assumption here is that quanta with negative squared mass (i. e. superluminal tachyons [14, 15]) are not present. This effectively prohibits strong long-range forces due to exchange of b quanta. Thus, we assume that the main effect of a hypothetical coupling to quarks of the b field is to give a mass contribution to primordial quarks. A coupling g of the b field to primordial ordinary quarks gives a quark mass contribution of gb; subsequently for three confined valence quarks, a nucleon mass contribution of order $3gb$. We have assumed that primordial quarks have zero bare mass, and we do not consider possible thermal effects in this paper. We assume that electroweak symmetry-breaking generates the standard-model MeV mass contribution for light quarks at a later time, and we assume that this contribution then simply adds to the mass contribution estimated here, gb. This is the assumption that the electroweak mass term arises from the Higgs vacuum expectation value times a tiny coupling to the quark field which has acquired a small mass term gb.

We consider a very small perturbation which causes b to change with time. The essential physical assumption is that $b(t)$, coupled to quarks, moves from a largest value at $t \sim 0$, toward zero, over cosmological time intervals parameterized by $\bar{t}$, but comes only so far as $t \to \infty$, such that the effective squared mass of potential b quanta approaches zero through negative values. (The vacuum energy density λb_0^4 is assumed as a cosmological constant.) Here, we briefly indicate the consistency of a hypothetical time-dependence for $b(t)$ with an equation of motion, using a particular, hypothetical, generalized force. An ansatz for the time dependence is

$$b^2 = \epsilon b_0^2 + \frac{(1-\epsilon)b_0^2}{(1+(t/\bar{t})^2)^n}; \quad 0 < \epsilon < 1, \quad \dot{b} = \frac{d(b^2)/dt}{2b} \tag{1}$$

The time t can be viewed as parameterized by $b(t)$.

$$\frac{t}{\bar{t}} = \left\{ \left(\frac{(1-\epsilon)b_0^2}{b^2 - \epsilon b_0^2} \right)^{1/n} - 1 \right\}^{1/2} \tag{2}$$

In the following, we use the Hubble parameter in cosmological time as approximately given by

$$H(t) = \frac{2}{3t} \tag{3}$$

A usual equation of motion (with a term in squared mass m^2) is

$$\ddot{b} + m^2 b + 3H\dot{b} = F = -\frac{dV(b)}{db} = -V'(b) \tag{4}$$

F is a generalized force, usually assumed to be derivable from a potential, via $V'(b)$. With neglect of $\ddot{b}$, and with m^2 taken as zero, a usual equation of motion (over relatively brief time intervals in which H changes little), $3H\dot{b} = -V'(b)$,

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simply represents a variation of b with time, from some initial value at which V' is assumed to be nonzero, down to a final value at which $V' = 0$, presumably a potential minimum, stable in time. More generally, $F = F(b, \dot{b}, H(t))$. Then, the analogue of V'' is $-F'$, with $\dot{b}, H(t)$ inserted as explicit functions of b. An effective, b-dependent (i. e. time-dependent) squared mass in the equation of motion can be viewed as $(m^2 - F')$. With neglect of terms from $\ddot{b}$, the equation of motion gives

$$m^2 = \frac{(-3H\dot{b} + F)}{b} \qquad (5)$$

Thus, at a maximum value of b for which $F = 0$, $\dot{b}$ is nonzero because of the term m^2, which can here be viewed as a phenomenological device for causing motion down from a (metastable) maximum. (Inclusion of terms from $\ddot{b}$, which must be proportional to $1/\bar{t}^2$, does not change the essence of the following argument.) Using (1-3) and (5), for a hypothetical $-F = (\mu_0^2 b + 3H\dot{b})$, with $\mu_0^2 = 2n(1 - \epsilon)/\bar{t}^2$ fixed by the initial (i. e. at $t = 0, b = b_0$) condition $F(b = b_0) = 0$, one obtains the following b-dependent functions

$$-F \;=\; \mu_0^2 \left[b - \frac{b_0^2}{b} \left\{ \frac{(b^2 - \epsilon b_0^2)}{(1 - \epsilon)b_0^2} \right\}^{\frac{n+1}{n}} \right] \qquad (6)$$

$$-F' \;=\; \mu_0^2 \left[1 + \left(\frac{b_0^2}{b^2} \right) \left\{ \frac{(b^2 - \epsilon b_0^2)}{1 - \epsilon)b_0^2} \right\}^{\frac{n+1}{n}} \right.$$
$$\left. - \frac{2(n+1)b_0^2}{n} \left\{ \frac{(b^2 - \epsilon b_0^2)^{1/n}}{((1 - \epsilon)b_0^2)^{(n+1)/n}} \right\} \right] \qquad (7)$$

$$m^2 \;=\; \mu_0^2 \left[-1 + 2\frac{b_0^2}{b^2} \left\{ \frac{(b^2 - \epsilon b_0^2)}{(1 - \epsilon)b_0^2} \right\}^{\frac{n+1}{n}} \right] \qquad (8)$$

For illustration, consider $n = 1$. Then, the effective, squared mass is

$$(m^2 - F') = \mu_0^2 \left[3 \left(\frac{b_0^2}{b^2} \right) \left\{ \frac{(b^2 - \epsilon b_0^2)}{(1 - \epsilon)b_0^2} \right\}^2 - \frac{4(b^2 - \epsilon b_0^2)}{(1 - \epsilon)^2 b_0^2} \right] \qquad (9)$$

This quantity is negative at $b^2 = b_0^2$ (at $t = 0$), $\mu_0^2 [3 - (4/(1 - \epsilon))] = -(2/\bar{t}^2)(1 + 3\epsilon)$. It goes to zero through negative values, as $b^2 \to \epsilon b_0^2$ (as $t \to \infty$).

With eq. (1), we have for time-dependent, approximate contributions to m_p.

$$\begin{aligned}
&\text{at } t \sim 0 && 3gb_0 \\
&\text{at } t \sim \bar{t} && 3g\sqrt{(1 + \epsilon)/2}\, b_0 \qquad (10) \\
&\text{at } t \to \infty && 3g\sqrt{\epsilon}b_0
\end{aligned}$$

The direction of mass change is downward in the model. There is a definite decrease as $t \to \infty$, the magnitude is related to b_0. Even with a sizable effective

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"magnification" factor [F2], $g \sim 10^3$, one obtains a small scale of mass change, $\sim$ keV. Conceptually, this is related through b_0 to a very small cosmological constant, and possibly to a very small neutrino mass. The electron mass can change, but the leptonic b coupling may be like that estimated for neutrinos, $g_l \sim 10^{-2}$. Thus, the hypothetical downward change in (m_p/m_e) is probably controlled by the downward change in m_p. The parameter $\bar{t}$ might be related to other dynamical quantities in the early universe. It can be connected with the ratio of vacuum expectation values, which ratio is numerically closely given in terms of the very small parameter λ [11], that scales the primordial, vacuum energy densities: $\lambda^2 \sim b_0/\phi_0 \sim 5.5$ eV/10^{18} GeV $\sim 5.5 \times 10^{-27}$. (Here $\phi_0 \sim 10^{18}$ GeV, at an assumed minimum of zero for the inflaton effective potential.) Numerically, $\bar{t} \sim 3 \times 10^{16}$ s $\sim (10^{-36}$ s$) \times (1/\lambda^4)$, where 10^{-36} s is the time near the end of inflation generated by the ϕ field [11] [F3]. With this representation for $\bar{t}$, the expansion scale factor is given approximately as $a(\bar{t}) \sim (\bar{t}/10^{-36}$ s$)^{1/2} \sim (1/\lambda^2)$ [F4].

To recapitulate, the unusual assumption is that the b field can move from a maximum value toward the value zero, over cosmological time intervals, but comes only a part of the way as $t \to \infty$. The second essential assumption is that quanta of the b field with negative squared mass are not present to induce strong long-range forces. In the numerical estimates, an essential number is the empirically very small self-coupling parameter λ for the inflaton (ϕ) field; this parameter is common to the b field self coupling in the model.

We conclude with the remark that a general idea seems to receive support from the possible small decrease of (m_p/m_e) with cosmological time [1]. This is that there is a small energy scale b_0, associated with the early universe [7, 11], in addition to the usual very high energy scales, i. e. inflaton mass and radiation temperature. Effects of the b field and of the ϕ field are related, when depending upon the single parameter λ [11]. The above smallness of λ from the ratio of vacuum expectation values, can give the relatively slow evolution of the field and energy density on a short time scale for ϕ, and possibly on a long time scale for b from a relationship of the very small parameter λ to a cosmological time parameter $\bar{t}$. Clearly, if λ were to approach zero, then the vacuum energy density in the b field would approach zero, and $\bar{t}$ would approach infinity. The small energy scale is capable of relating a very small cosmological constant today and

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[F2]With reference to possible "magnification" of the effective g, it might be useful to note that the confinement of quarks at $\sim 10^{-6}$ s, does involve electroweak mass $\sim$ MeV, being substantially increased to constituent quark mass. The QCD energy-scale parameter is $\Lambda \sim 220$ MeV.

[F3]As counted from the time of ϕ leaving its value $\gtrsim M_{\rm Pl}$ at the effective potential maximum.

[F4]It is interesting to note that the necessary minimal value of the expansion scale factor for an initial inflation over Δt is a similar number. That is $a_{\rm infl}(\Delta t) = e^{H_{\rm in}\Delta t} \sim 1/\lambda^2 \sim 2 \times 10^{26}$, for $\Delta t \sim 1/H_{\rm in} \times \ln(1/\lambda^2)$, where $H_{\rm in}$ is the Hubble parameter as fixed by the initial vacuum energy density of the inflaton field, which is proportional to λ.

a small decrease of mass over cosmological time.

Appendix I

At the end of inflation, there are two very high, energy-scale parameters, $\phi_0 \cong 10^{18}$ GeV and the initial radiation temperature $T_0 \cong 10^{15.5}$ GeV. But there is also a very small, dimensionless parameter λ, of the order of 10^{-14}. [F5] A hypothetical small energy scale $b_0 \cong 5.5$ eV is obtained from a relation $\lambda^2 = b_0/\phi_0 = 5.5 \times 10^{-27}$ (for $\lambda = 7.4 \times 10^{-14}$). There are then two time-scale parameters: $1/\sqrt{\lambda}\phi_0 = 2.4 \times 10^{-36}$ s at the end of inflation; and $1/\sqrt{\lambda}b_0 = 4.4 \times 10^{-10}$ s. Over the latter time scale, the radiation energy scale evolves downwards to a relatively low scale

$$T_0 \left(\frac{2.4 \times 10^{-36} \text{ s}}{4.4 \times 10^{-10} \text{ s}} \right)^{1/2} = h = 234 \text{ GeV} \tag{11}$$

This scale is almost the empirical Higgs energy scale (the assumed nonzero vacuum expectation value v) of the standard model. The coupling g_e, of $v/\sqrt{2}$ to electrons, is empirically another small number: $g_e^2 \cong 9 \times 10^{-12}$. Again, a ratio of energy scales almost gives this very small number $g^2 = b_0/h = 23.5 \times 10^{-12}$.[F6] In summary, the above emphasis on the role of λ allows for simply

$$\begin{aligned} b_0 &= \lambda^2 \phi_0 \propto \lambda^2 \\ h &= \lambda \phi_0 (T_0/\phi_0) \propto \lambda \\ g^2 &= \lambda(\phi_0/T_0) \propto \lambda, \text{ so } g \propto \sqrt{\lambda} \end{aligned} \tag{12}$$

The essential parameters are ϕ_0, T_0 and λ^2 (or b_0). Here, it is the extreme smallness of λ which relates the high energy scale ϕ_0 (or T_0) to the much lower energy scale h. Also, the smallness of a minimal Higgs coupling is related to the smallness of $\sqrt{\lambda}$.[F7]

[F5]Heuristically, this can be seen by equating the maximum energy density in the inflaton field, to the initial (maximum) radiation energy density, which appears just after inflation. Then, $\sim \lambda M_{\text{Pl}}^4 \sim T_0^4$ gives $\lambda \sim 10^{62}$ GeV$^4/10^{76}$ GeV$^4 = 10^{-14}$.

[F6]Another ratio of energy scales with b_0 possibly gives a dimensionless measure of electromagnetic interaction strength, that is $\alpha^2 = 5.3 \times 10^{-5} \cong b_0/m_e \cong 5.5$ eV$/0.5$ MeV $= 1.1 \times 10^{-5}$; $\alpha^2 \propto \sqrt{\lambda}$. The ratio of a nuclear (QCD) energy scale to an atomic (binding) energy scale is $1/\sqrt{\lambda}$, giving a nuclear scale of order 40 MeV for an atomic scale of order 10 eV, with $\lambda \cong 7 \times 10^{-14}$ (clearly, λ must be much less than unity).

[F7]The contribution of zero-point energies in massless quanta to the vacuum energy density, may be initially limited by a specific large dimension of the inflation region, $(1/\sqrt{\lambda}\phi_0) \times (1/\lambda^2) = 1/\sqrt{\lambda}b_0$, that is the very rapid expansion may result in a density of order $(\sqrt{\lambda}b_0)^4$. (The empirical, apparent cosmological constant is $\sim \lambda b_0^4$.)

Appendix II - "Cosmic coincidence"

The present ratio of vacuum energy density ρ_Λ, to approximate dark-matter energy density $\rho_{\mathrm{d.m.}}$, empirically $\rho_\Lambda/\rho_{\mathrm{d.m.}} \sim 3$, appears to be a curious accident, in particular when these densities are assumed to originate in completely different dynamics, and the matter density falls with the expansion of the universe. Here, we note that the ratio of order unity is not particularly accidental, when the two densities are considered together as functions of only two quantities: the very large energy scale ϕ_0, and the very small dimensionless parameter λ, using relations in the text. We have [F8]

$$\rho_\Lambda = \lambda b_0^4 = \lambda(\lambda^2\phi_0)^4 = \lambda^9\phi_0 \tag{13}$$

$$\rho_{\mathrm{d.m.}} = (\rho_{\mathrm{d.m.}})_0 \left\{ \left(\frac{10^{-36}\ \mathrm{s}}{\bar{t}}\right)^{3/2} \left(\frac{\bar{t}}{10^{11}\ \mathrm{s}}\right)^{3/2} \left(\frac{10^{11}\ \mathrm{s}}{4\times 10^{17}\ \mathrm{s}}\right)^2 \right\}$$

$$= \left(\frac{1}{2}f(\lambda)m_\phi^4\right)\left\{(\lambda^4)^{3/2} \times (\sim \lambda^{0.39})\right\} \tag{14}$$

$$= (\lambda^{2.5}\phi_0^4)\left\{(\lambda^6) \times (\sim \lambda^{0.39})\right\}$$

$$\cong \lambda^{8.89}\phi_0^4$$

In (14), $(\rho_{\mathrm{d.m.}})_0$ is the initial energy density for inflatons created at $\sim 10^{-36}$ s. This density is written in terms of the inflaton mass $m_\phi \cong \sqrt{\lambda}\phi_0$, as $m_\phi^2/2 \times (f(\lambda)m_\phi^2)$ with an assumption for the fluctuation scale [8], $\lambda < f(\lambda) < 1$. [F9] As a definite example, we used in (14) the geometric mean $f(\lambda) = \sqrt{\lambda}$. The terms bracketed as $\{\dots\}$ give the time evolution to the present age of the universe, $\sim 4\times 10^{17}$ s. The first term explicitly uses the text's hypothetical λ dependence of $\bar{t}$, for evolution to the time $\bar{t} = 3\times 10^{16}$ s, which is about a characteristic time for galaxy-halo formation from dark matter. The next two terms give the evolution from $\bar{t}$ to the present, with $\sim 10^{11}$ s taken as an approximate time of matter dominance. These two factors are expressed as an effective, small power of λ. Eqs. (13,14) give[F8]

$$\rho_\Lambda/\rho_{\mathrm{d.m.}} \sim 1/30 \tag{15}$$

This differs from the approximate empirical value by only a factor of about 90. From this point of view, a ratio of order of unity may be explainable in terms of only two primary, dynamical quantities, ϕ_0 and λ (or b_0).

[F8]For $\lambda \sim 7.4 \times 10^{-14}$, Eq. (13) gives ρ_Λ above the empirical value by only a factor of about 2.5.

[F9]This is consistent with an estimate of the size of f from production of massive dark matter by a time-varying gravitational field, when H is of order m_ϕ.[11]

References

[1] E. Reinhold et al., Phys. Rev. Lett. **96**, 151101 (2006)
A. Ivanchik et al., Astron. Astrophys. **440**, 45 (2005)

[2] J. K. Webb et al., Phys. Rev. Lett. **82**, 884 (1999)

[3] J. K. Webb et al., Phys. Rev. Lett. **87**, 091301 (2001)

[4] J. D. Bekenstein, Phys. Rev. **D66**, 123514 (2002)

[5] R. Srianand et al., Phys. Rev. Lett. **92**, 121302 (2004)

[6] R. Quast et al., Astron. Astrophys. **415**, L7 (2004)

[7] S. Barshay and G. Kreyerhoff, Mod. Phys. Lett. **A19** 2899 (2004). In Eq. (2) correct a misprint to read $b \rightarrow b + \beta\phi$.

[8] S. Barshay and G. Kreyerhoff, Eur. Phys. J. **C5**, 369 (1998)

[9] S. Barshay and G. Kreyerhoff, Z. Phys. **C75**, 167 (1997); Erratum: ibid. **C76**, 577 (1997)

[10] S. W. Hawking and T. Hertog, hep-ph/0204212

[11] S. Barshay and G. Kreyerhoff, Mod. Phys. Lett. **A21**, 1183 (2006)

[12] H. Kurki-Suonio and G. J. Mathews, Phys. Rev. **D50**, 5431 (1994)

[13] P. J. E. Peebles and A. Vilenkin, Phys. Rev. **D59** 063505 (1999)

[14] G. Feinberg, Phys. Rev. **159**, 1089 (1967)
G. Feinberg, Phys. Rev. **D17**, 1651 (1978)

[15] M. E. Arons and E. C. G. Sudarshan, Phys. Rev. **173**, 1622 (1968)
J. Dhar and E. C. G. Sudarshan, Phys. Rev. **174**, 1808 (1968)