**Fractals Everywhere.** *By Michael Barnsley.* Academic Press, Boston, 1988. xii + 396 pp. $39.95. ISBN 0-12-068062-9.

A mathematical fractal is a nondifferentiable set $S$, definable without using the axiom of choice, embedded in a simple space such as Euclidean space $R_n$. (The axiom of choice could be permitted but the results would then be bad models of physical reality.) Moreover, the intersections of $S$ with $n$-dimensional blocks resemble $S$ when these subsets are scaled up; in short, these subsets are also fractals. This definition obviously implies an infinite hierarchy or "tree" in the structure of a fractal. In most examples the dimensionality of $S$, say in the Hausdorff–Besicovitch sense but which I will not define, is not an integer. (For the experimenter there is a more convenient, but related, definition of dimensionality.) An example is the set of $n$-imals in which the $n$ digits have limiting frequencies $p_0, p_1, \cdots, p_{n-1}$, where these frequencies are not all equal. In this example the dimensionality is the entropic expression $-(p_0 \log_n p_0 + \cdots + p_{n-1} \log_n p_{n-1})$, and for generalized decimals there are generalizations involving cross-entropy (for a summary see Good (1989b)). Cantor's middle-third set is a special case.

A *physical fractal* is an object in the real world, whether natural or artificial, whose spatial structure can be approximately modelled by a mathematical fractal. A *natural fractal* is a physical fractal that occurs in nature; for example, a coastline, a cloud, a feather, a leaf, a vascular system, a turbulent flow, a Brownian motion, or a pattern on marble.

The dimensionality or fractal number of a set can be regarded as a measure of its *texture*. As a potential application, we might ask whether the textures of the rims of craters on the moon seem to have a unimodal distribution. If not, then we would believe that the craters were formed by more than one mechanism. Texture is just one characteristic that might not be unimodal, and this statistical approach would become multivariate if other characteristics, such as the size of the crater, were taken into account.

Although mathematical fractals have been known for seventy years, their ubiquity only fully entered the collective consciousness of the mathematical and scientific community in 1975 or 1977, with the appearance of Benoît Mandelbrot's first book on the subject, and the book's translation (see also Mandelbrot (1982)). Mandelbrot also introduced the excellent name *fractal*, which is associated with the concepts of fractional dimension and fragmentation. In view of this ubiquity, the title of Barnsley's book is appropriate enough. But it is also slightly misleading because it is not a book for the general reader: its standard of mathematical rigor is too high for that and is hardly alleviated by the occasional flippancy.

Fractal sets can readily arise from simple iterative procedures. For example, take any real number (or any set of numbers). Add either 1 or 2 at random and take the reciprocal and repeat this process many times. The points that you visit will jump about and will get arbitrarily close to a fixed set in (0, 1) (independent of the number you first chose), an "attractor" in the jargon of chaotics. Its Hausdorff–Besicovitch fractional dimension is approximately 0.531 (Good (1941), (1989a)).

Because fractal sets can arise in very simple mathematical models, the ubiquity of natural fractals should not surprise anyone who believes, with Eugene Wigner, in the "unreasonable effectiveness of mathematics."

Barnsley emphasizes what he calls the collage theorem and its use for producing computer programs to draw fractals resembling natural fractals. The theorem is attributed to "Barnsley (1985b)" (but I could not find that on the reference list). There is also emphasis on contraction mappings, iterated function systems, (strange) attractors, Julia sets, and chaotics.

Barnsley says honestly (p. 108) that typical fractals are not pretty, contrary to the impression that some may have received from the title of the book by Peitgen and Richter (1986). Even in that book, many of the pictures look like biological forms from an alien world, and they induce a feeling of uncomfortable admiration and interest, rather than an impression of beauty. As I have claimed elsewhere, *beauty arises from simplicity that emerges from complexity that emerges from simplicity*, whereas, as Barnsley (p. 43) says, a fractal is [usually] a complicated subset of a simple space. We cannot see the beauty in a fractal before we have apprehended the unity or simplicity of its structure, and this apprehension is often more mathematical than visual.

There are occasional lapses in style: for example, a too-facetious comment at the top of page 23; an over-succinct definition on page 31, where "Max" seemed to me at first to be a misprint for "Min" (I wanted immediate verbal defense of the definition to feel comfortable with it, as often happens with terse mathematical definitions); and in Definition 6 on page 23 the horrible omission of the preposition "by" following "denoted," a disease that needs to be stamped out. To compensate, there are nice touches—for example, a *simple concept* is described on page 43 as one that is easily conveyed or explained to someone else, while on page 99 the real reason for the truth of Pythagoras's theorem is given, but the reference to Euclid, Book VI, Proposition 31 is overlooked (see Heath (1956, p. 268)). The pictures on page 77 also have a history; some of it is discussed by Good (1971).

The book is based on a course of thirty lectures appropriate primarily for mathematicians, and some scientists, who want a rigorous understanding of the proliferating and fashionable field of "fractology" or "fractometry." The book would be suitable for other similar courses but an isolated reader will need both mathematical maturity and plenty of determination. As Savage (1954, p. viii) said, "Serious reading of mathematics is best done sitting bolt upright on a hard chair at a desk." A budding fractologist or fractometrician will need to take this advice, and should also study at least two of the other books on fractals, of which there are already about 10 in English. (Consult "Fractal" in the subject index of a good university library.) The general reader who wants only a quick introduction to the topic might be satisfied to look at the pictures in a few of the books, and to read short articles such as those by Steen (1977), Barcellos (1984), or Gardner (1989, Chap. 3).

## REFERENCES


A. Barcellos (1984), *The fractal geometry of Mandelbrot*, College Math. J., 15, pp. 98–113.

M. Gardner (1989), *Penrose Tiles to Trapdoor Ciphers*, W. H. Freeman, New York.

I. J. Good (1941), *The fractional dimensional theory of continued fractions*, Proc. Cambridge Philos. Soc., 37, pp. 199–228.

——— (1971), *Science in the flesh*, in Cybernetics, Art and Ideas, J. Reichardt, ed., Studio Vista, London, pp. 100–110.

——— (1989a), Errata, Proc. Cambridge Philos. Soc., 105, p. 607.

——— (1989b), Addenda, J. Statist. Comput. Simulation, 32, pp. 64–68.

Sir T. L. Heath (1956), *The Thirteen Books of Euclid's Elements*, Vol. II (Books III–IX), Dover, New York.

B. B. Mandelbrot (1975), *Les Objets fractals: forme, hasard et dimensions*, Flammarion, Paris.

——— (1977), *Fractals: Form, Chance, and Dimension*, W. H. Freeman, San Francisco.

——— (1982), *The Fractal Geometry of Nature*, W. H. Freeman, San Francisco.

H.-O. Peitgen and P. H. Richter (1986), *The Beauty of Fractals*, Springer-Verlag, Berlin, New York.

L. J. Savage (1954), *The Foundations of Statistics*, John Wiley, New York.

L. A. Steen (1977), *Fractals: a world of nonintegral dimensions*, *Science News*, 112, pp. 122–123.



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**Stochastic Processes in the Neurosciences.** *By Henry C. Tuckwell.* Society for Industrial and Applied Mathematics, Philadelphia, 1989. v + 129 pp. $24.50. ISBN 0-89871-232-7. CBMS-NSF Regional Conference Series in Applied Mathematics No. 56.

This excellent monograph by Tuckwell is a welcome addition to the literature of mathematical neurobiology. In brief, it describes the recent work of a number of analysts and statisticians concerned with the stochastic analysis of models of neural activity. In this review I will first try to summarize Tuckwell's book. Then, from the viewpoint of those of us interested in neurocomputing, I will sketch the larger picture, and what is needed.

Following a short but good introduction (Chapter 1), the author plunges (Chapter 2) directly into two fundamental issues in synaptic transmission: the amplitude and the timing of the response potentials at neurojunctions subjected to input stimulations. As the author quickly points out, in most of the