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Fractals Everywhere. By Michael Barnsley. Academic Press, Boston, 1988. xii + 394 pp.

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> I am very ambivalent about fractals becoming a field because my work was inspired and supported by a very strong belief that the division of science into fields has been extremely harmful.
>
> Benoit B. Mandelbrot, 1983 [**1**]

Occasionally it is possible to coax an opinion out of Benoit Mandelbrot. As proud as the "Father of Fractals" may be of his brainchild, he looks toward its domestication with trepidation. Known as a polymath, Mandelbrot has carried his fractal message anywhere that people might listen, collecting academic appointments in mathematics, applied mathematics, economics, engineering, physiology, and medicine [**2**]. His success has been singular.

Fractal images have become a commonplace of math department offices and computer centers. Fractal techniques have become acceptable in cosmology, dynamical systems, and computer graphics. Not only is fractal geometry becoming a respectable tool, it is, despite Mandelbrot's concerns, becoming a topic in its own right.

The evidence of this is everywhere (along with the fractals themselves). Since the last edition of Mandelbrot's original fractal manifesto [**6**], we have been treated to lavish picture books [**7**], best-selling popularizations [**5**], meticulous treatises with yet more pictures [**8**], and philosophical collaborations between physicists and psychologists [**4**].

Among these varied offerings we have the present book, *Fractals Everywhere*, by Michael Barnsley of the Georgia Institute of Technology and Iterated Systems, Inc. It is a textbook for a course on fractals. However, Benoit need not fear that his creation is being set up for codification and sterile abstraction. Barnsley treats fractals as the unifying thread in his presentation of metric spaces, mappings, dynamical systems, image compression [**3**], and graphical models.

Fractals Everywhere is yet another picture book. This is good, because anyone who ignores the visual side of fractals abandons not only its esthetic dimension, but also its ability to represent what we otherwise cannot see. Mandelbrot's *The Fractal Geometry of Nature* is in part a casebook of missed opportunities, where he points out how inadequate visualization of complex systems obscures their nature (see [**6**, p. 179] in particular).

Barnsley's cover illustration, a depiction of an Andes Indian girl, is a collage of 160 iterated maps. The emphasis on visuals is a fundamental part of the book and not, as in so many other cases, a sugar-coating with little relation to what's inside. Barnsley's notation is often iconic; that is, specialized symbols provide strong visual reminders of what they represent. Given the inherent complexity of the material, it's nice to see a heavy dot representing a closed disk in $\mathbf{R}^2$ and a filled square standing in for the unit square; that's easier than having to remember what D and S stand for (or whatever). Even more striking are the icon for a Sierpiński triangle and the little mummy representing a frozen cadaver (a useful $\mathbf{R}^3$ subset that Barnsley calls "Body space").

These playful elements pervade the book, and a good thing, too. Barnsley uses them as memory devices to minimize the mental baggage one must carry during his fractal trek, leaving more room for the booty to be gathered as the trip progresses.

Barnsley's fractal geometry course at Georgia Tech was the basis for his text. The prerequisite is two years of calculus, which makes the entry requirements rather low; a healthy dollop of mathematical maturity would be a good idea. Ramping up from an elementary discourse on the properties and topology of metric spaces, Barnsley moves into contraction mappings and defines the iterated function system (IFS), a tool that recurs time and again in all that follows. The probabilistic Chaos Game is contrasted with the Deterministic Algorithm for fractal generation. Barnsley introduces dynamical systems and fractal dimension.

By this time the atmosphere is getting rarefied as Barnsley tracks through fractal interpolation, Julia sets, and measures on fractals. In actual practice, Barnsley reserves the latter chapters of his text for special topics that round out the basic introductory material. The rich selection ensures that each instructor will be able to tailor his or her own course in many different ways.

Fractals Everywhere has many praiseworthy features. The prose is clear and generally straightforward, though with enough passive voice so we don't forget we're in a textbook. Exercises abound—at all levels of difficulty. Practically every page has at least one illustration, and two-page spreads of pure text are almost impossible to find. (In fact, given Barnsley's predilection for iconic notation, several of the text pages could be said to be illustrated as well.) However, some of the figures are rather crudely drawn, and they suffer by comparison with the slick computer graphics that dominate the illustrations.

The author occasionally forgets that his readers may not be as familiar with fractal terminology as he is. Thus he mentions the Chaos Game or Chaos Algorithm in a few places, but spends a lot more time talking instead about the Random Iteration Algorithm. They're one and the same, but I never saw where Barnsley expressly said so. Unfortunately, the index references for the two names are disjoint, with no cross-references to each other.

The book may claim to have 9 chapters, but it really has only 8. I regret that the author refers to his five-page introduction as Chapter 1, as it is not even a member of the same species as the other chapters. The concept of Chapter 0 was invented for chapters of measure zero like this one. This is, of course, a trifling matter, but so is Chapter 1.

We can probably expect that fractal geometry will become a fixture in the college mathematics curriculum. As a descriptive tool it has great power, providing a new model for natural processes that has already proved fruitful in many fields. *Fractals Everywhere* takes Mandelbrot's thesis of fractal ubiquity and frames it with sufficient structure to make classroom presentation possible. Barnsley's text will inspire rival books, but I think that *Fractals Everywhere* will be commonly referred to as the "classic" in its field.

Perhaps, somewhere, someone is writing (or has written) a textbook on fractals that begins with a definition in terms of Hausdorff-Besicovitch dimension and proceeds to adduce therefrom a numbered sequence of theorems and corollaries. This is not that book. Benoit would be pleased.

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