African Journal of Mathematics and Computer Science Research Vol. 5(6), pp. 112-119, 15 March, 2012
Available online at http://www.academicjournals.org/AJMCSR
DOI: 10.5897/AJMCSR12.011
ISSN 2006-9731©2012 Academic Journals

Full Length Research Paper

Logistic preference function for preference ranking organization method for enrichment evaluation (PROMETHEE) decision analysis

S. K. Amponsah, K. F. Darkwah* and A. Inusah

Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

Accepted 20 February, 2012

Decision problems relate to problems of ranking, choice and detection with regards to whether a decision alternative efficient parameter satisfies some given conditions. The preference ranking organization method for enrichment evaluation (PROMETHEE) methods of decision analysis are recognized as being efficient in solving problems involving ranking. Various preference functions have been established in the literature as being useful in the PROMETHEE methodology. The Gaussian preference function is preferred when the performance data is continuous. This paper presents a new logistic preference function, which can be used for continuous performance data. The proposed logistic preference function was used on telecommunications operators performance data of the National Communication Authority of Ghana. When used in the PROMETHEE methodology, the proposed logistic preference function and the Gaussian preference function produced the same order of ranking. However, the proposed logistic preference function performed more efficiently than the Gaussian preference function.

Key words: Preference ranking organization method for enrichment evaluation (PROMETHEE), multicriteria, preference function, telecommunication.

INTRODUCTION

The most fundamental challenge faced by managers in both public and private sectors is the making of optimal decisions on problems that are multicriteria in nature. In recent times, the giant development in computer technology coupled with advance in theory has made decision analysis an indispensable tool in both government and in business as far as the making of multicriteria decision is concerned (Covaliu, 2001). It is worthwhile to note that the solution of a multicriteria problem does not only depend on the fundamental data employed in the evaluation table, but also on the decision maker (Brans et al., 1986). There exists only a compromise solution, which partly depends on the preferences of each decision maker and as a result additional information representing these preferences is required to provide the decision maker with useful decision aid.

The preference ranking organization method for enrichment evaluation (PROMETHEE) methodology is a family of six outranking methods, which are the PROMETHEE I to VI (Villota, 2009). PROMETHEE I and II were first proposed by Brans (1982). Other multicriteria decision aids (MCDA), such as the PROMETHEE group decision support system (GDSS) for group decision-making (Brans and Mareschal, 2010) and the visual interactive module, geometrical analysis for interactive aid (GAIA), for pictorial representation to complement the algebraic methodology were developed to facilitate the analysis of more complex decision-making problems (Brans and Mareschal, 2010). Two extensions of PROMETHEE have recently been proposed as PROMETHEE TRI for multicriteria decision-making problems involving sorting and the PROMETHEE CLUSTER for problems dealing with nominal

*Corresponding author. E-mail: fordarkk@gmail.com.

classification (Figueira et al., 2004).

RELATED WORKS

PROMETHEE II method was used to solve a facility location problem in which there were eight criteria against four alternative locations solutions (Athawale and Chakraborty, 2010). At the end, the most cost-effective and highest yielding location alternative was identified and selected. Maragoudaki and Tsakiris (2005) identified PROMETHEE methodology as one of the most efficient MCDA outranking techniques that could be used to arrive at the optimal flood mitigation plan for a river basin. Four alternative irrigation projects for the East Macedonia-Thrace district were evaluated using analytic hierarchy process (AHP) and PROMETHEE multicriteria methods (Anagnostopoulos et al., 2005). The project goal was the rational water resources management of Nestos River in relation to the operation of two recently constructed dams. A preventive maintenance decision model based on integrating PROMETHEE method and the Bayesian approach was developed to help decision makers establish replacement intervals (Ferreira et al., 2007). A numerical application example was given to illustrate the proposed decision model and showed the effectiveness of the model in terms of the decision maker's preferences.

Albadvi (2004) formulated national information technology strategies: a preference ranking model using PROMETHEE method. The sole purpose of the research was to define a national strategy model for Information Technology (IT) development in developing countries and to apply the model in a real case of Iran. The model was a multicriteria decision making and in order to solve it and select a set of IT application flagships in different budgeting levels, they used the PROMCALC and GAIA decision support system.

Zhou et al. (2010) developed a fuzzy based pipe condition assessment model using PROMETHEE II. This method was used to calculate pipe breakage risk to reflect the condition assessment in order to enable them rehabilitate the deteriorated pipes in a planned and proactive way. The numerous influential factors they identified as responsible for pipe breakage included ground load, pipe material, soil corrosion, pipe age, construction quality, pipe length, soil condition, breakage history, etc. The authors argued that the proposed model was different from the previous model, being used in that it only required readily usually available data, and that it gave an insight into the uncertainty and preference opinion of the expert as shown in the relation between expert opinion's uncertainty and preference that had a pipe breakage risk signification and in each criterion.

A PROMETHEE based uncertainty analysis of United Kingdom (UK) police force performance rank improvement was designed for a periodic comparison of the units of the police force in the UK with each other in terms of performance by both government and non-government bodies (Barton and Beynon, 2009). The study demonstrated the employment of PROMETHEE in an investigation of the targeted performance rank improvement of individual units of the UK police forces. The graphical representations presented offered an insight into the implications of series of such a PROMETHEE based series of perceived improvement analysis. The goals of their study were two folds: firstly, namely to exposit PROMETHEE based uncertainty analysis in rank improvement and secondly, to show how the subsequent results could form part of the evidence to that aided in their performance strategies.

A new sorting method (flow sort) based on the ranking methodology of PROMETHEE for assigning actions to completely ordered categories, defined either by limiting profiles or by central profiles was established by Nemery and Lamboray (2007). The flow sort assignment rules were based on the relative position of an action with respect to the reference profiles in terms of the incoming, leaving and/or net flows.

Manzano et al. (2011) conducted an economic evaluation of the Spanish port system using the PROMETHEE multicriteria decision method. The work established an ordering relationship among twenty-seven Spanish port authorities at different strategically considered time points.

Aburas et al. (2010) conducted call quality measurement for telecommunication network and proposition of tariff rates research. The idea of their research was basically the measurement of call quality from the end users perspective and could be used by both end user and operator to benchmark the network. The call quality was measured based on certain call parameters as average signal strength, the successful call rate, drop rate, handover success rate, handover failure rate and location area code (LAC). The quality parameters were derived from active calls and the results were analyzed and plotted for detailed analysis and benchmarking as well as used as a base for charging the customer by the operators. The authors suggested charging rates based on the signal quality and the call statistics recorded.

Michailidis and Chatzitheodoridis (2006) proposed a model based on PROMETHEE, a multicriteria decision aid, to be used to evaluate and rank three tourism destinations, located in the Northern and Central Greece. Additionally, innovatory innovative elements were the incorporation of differing levels of socioeconomic data (destination image and destination personality) within the decision frame work and the direct determination of the PROMETHEE II preference thresholds. According to them, the developed methodology provides a user-friendly approach, promotes the synergy between different stakeholders and could pave a way towards consensus. The authors identified the act of describing

Table 1. Relations between alternatives in PROMETHEE partial preorder ranking.

Preference relation	Conditions	Graphical representation
$A_K P A_l$	$\phi^+(A_K) > \phi^+(A_l)$ and $\phi^-(A_K) < \phi^-(A_l)$ $\phi^+(A_K) > \phi^+(A_l)$ and $\phi^-(A_K) = \phi^-(A_l)$ $\phi^+(A_K) = \phi^+(A_l)$ and $\phi^-(A_K) < \phi^-(A_l)$	$A_K \longrightarrow A_l$
$A_K I A_l$	$\phi^+(A_K) = \phi^+(A_l)$ and $\phi^-(A_K) = \phi^-(A_l)$	-
$A_k R A_l$	$\phi^+(A_K) > \phi^+(A_l)$ and $\phi^-(A_K) > \phi^-(A_l)$ $\phi^+(A_l) > \phi^+(A_k)$ and $\phi^-(A_l) > \phi^-(A_k)$	-

the design implementation and use of a decision support system (DSS), which applied new methodological approaches for the evaluation and ranking of several tourism destinations as the main focus of their study.

Due to its reach and capability to share information, the World Wide Web has become an important tool for business (Villota, 2009). According to the author, there were some so-called usability criteria, which should be respected by web designers in order to make websites useful. As a result, using a multicriteria decision making approach, they evaluated the performances, based on seven usability criteria, of five websites from which one could buy books online. Considering usability as a subjective matter, they used two well-known methodologies that deal with this issue: AHP and PROMETHEE. Through PROMETHEE, they related the preference of a decision maker with specially defined criterion functions.

MODEL FORMULATION AND PROMETHEE ALGORITHM

Decision problem statement is stated as follows: given a finite set of alternatives $A = \{A_j\}$, $j = 1, \ldots, m$ against a set of criteria, $C = \{C_j\}$ and weights w_i, $i = 1,\ldots, n$ what alternative A_j is the best alternative?

The PROMETHEE algorithm for ranking the alternatives (Villota, 2009) is given as follows:

Step 1: Input data of performance table and table of weights: The performance data shows in quantitative terms the performance value x_{ij} of each A_j on each criterion, C_i.

Step 2: Calculate deviations of various criteria i :

$$d_i(A_k, A_l) = \begin{cases} x_{ik} - x_{il} & \text{for maximization criteria } i \\ -(x_{ik} - x_{il}) & \text{for minimization criteria } i \end{cases}$$

Step 3a: Select a generalized preference function $P(d_i) = P(d_i(A_k, A_l))$: There are currently eight generalized preference functions from which to choose to reflect the priorities of the decision maker (Podvezko and Podviezko, 2010).

Step 3b: Calculate preference (criterion) value using $P(d_i) = P(d_i(A_k, A_l))$. This measures the intensity of the decision maker's preference for the alternative A_k over A_l on the same criterion C_i.

Step 4: Calculate the aggregate preference index of alternative A_k over A_l for all criteria C_i by using the relation: $\pi(A_k, A_l) = \sum_{i=1}^{n} w_i P_i(A_k, A_l)$, where w_i is the weight of criterion i.

Step 5: Perform partial ranking (PROMETHEE I):

i. Calculate the positive outranking flow of alternative, A_j, : over all other alternatives A_k with $A_k \neq A_j$ and using,

$$\phi^+(A_j) = \frac{1}{m-1}\sum_{k=1}^{m} \pi(A_j, A_k) \quad j = 1, 2, 3...m$$

ii. Calculate the negative outranking flow of all alternatives, A_k over alternatives A_j with $A_k \neq A_j$ and using, $\phi^-(A_j) = \frac{1}{m-1}\sum_{k=1}^{m} \pi(A_k, A_j) \quad j = 1, 2, 3..m$.

iii. Determine the outranking relation existing between various alternatives by using Table 1 where $A_k P A_l$ signifies the preference of the alternative A_k over A_l, $A_k I A_l$ signifies the indifference between alternatives A_k and A_l and $A_k R A_l$ indicates the incomparability of the two alternatives A_k and A_l over all criteria.

If the resulting incidence table I of the resulting directed graph from column 3 of Table 1, satisfy the condition that:

Table 2. Relations between alternatives in complete ranking.

Preference relation	Cases	Graphical representation
$A_k\ P\ A_l$	$\phi(A_k) > \phi(A_l)$	$A_k \longrightarrow A_l$
$A_k\ I\ A_l$	$\phi(A_k) = \phi(A_l)$	_

Table 3. Performance table of five telecom networks in the Greater Accra region as of June, 2010.

Criteria	Type of criteria	Alternative				
		A_1	A_2	A_3	A_4	A_5
C_1	Min	15.12	12.09	11.67	13.86	15.28
C_2	Max	80	96	41	81	88
C_3	Min	17	3	27	12	10
C_4	Min	3	1	32	8	2

$$\sum_j I_{ij} + \sum_j I'_{ij} = m-1 \quad \text{for all } i = 1,2,3...m$$

with *I'* being the transpose of *I.* Then, the alternatives are completely ranked, stop. The alternative represented by the row with the highest sum of entries is the decision. Otherwise the alternatives are partially ranked, go to step 6.

Step 6: Perform complete ranking (PROMETHEE II): Compute the net outranking flow $\phi(A_k)$ for each alternative A_k such that $\phi(A_k) = \phi^+(A_k) - \phi^-(A_k)$:

i. The alternative A_k is preferable to A_l if and only if $\phi(A_k) > \phi(A_l)$

ii. The alternative A_k is indifferent to A_l if and only if $\phi(A_k) = \phi(A_l)$

This is illustrated as shown in Table 2.

PREFERENCE FUNCTION

Podvezko and Podviezko (2010) categorized eight generalized preference functions found in the literature. These include multistage, c-shape and Gaussian preference functions. Villota (2009) suggested that for continuous performance data, the Gaussian preference function is preferred. The Gaussian preference function is given by:

$$P(d_i) = \begin{cases} 0 & d_i \le 0 \\ 1 - e^{\left(-\frac{d_i^2}{2\sigma_i^2}\right)} & d_i > 0 \end{cases}$$

where d_i is deviation of over criterion *i* over respective pairs of alternatives, A_k, A_i. σ_i^2 is the variance of the data for criterion *i*.

We introduce a new preference function which we call the logistic preference function. It performs better than the Gaussian preference function. The logistic preference function *P(x)* is the difference of logistic probabilities for success *p(x)* and failure *q(x)*.

$$\begin{aligned} P\ x &= p - q = p - (1-p) = 2p - 1 \\ &= 2\left(\frac{1}{1+e^{-x}}\right) - 1 = \frac{2-1-e^{-x}}{1+e^{-x}} = \left(\frac{1-e^{-x}}{1+e^{-x}}\right) \end{aligned}$$

$$\text{where} \quad p = \frac{1}{1+e^{-x}}$$

Put $x = \dfrac{d_i^2}{\sigma_i^2}$ to obtain:

$$P\ d_i = \begin{cases} 0 & d_i \le 0 \\ \dfrac{1 - e^{\left(-\frac{d_i^2}{\sigma_i^2}\right)}}{1 + e^{\left(-\frac{d_i^2}{\sigma_i^2}\right)}} & d_i > 0 \end{cases}$$

COMPUTATIONAL EXPERIENCE AND RESULTS

Our preference function was tested on data of performance measure of the National Communications Authority (NCA) of Ghana. Five mobile telecommunications network operators in Greater Accra were selected. Table 3 displays the data and indicates whether a criterion is minimizing or maximizing criterion.

Table 4. The mean and standard deviation of the four criteria.

Criteria	Mean (μ)	Standard deviation (σ)
C_1	13.60	1.67
C_2	77.20	21.23
C_3	13.80	8.93
C_4	10.20	15.22

Table 5. Aggregated preference indices $\pi(A_k, A_l)$.

	A_1	A_2	A_3	A_4	A_5
A_1	0.00	0.00	0.61	0.01	0.00
A_2	0.52	0.00	0.74	0.33	0.33
A_3	0.24	0.01	0.00	0.12	0.25
A_4	0.11	0.00	0.67	0.00	0.09
A_5	0.09	0.00	0.73	0.04	0.00

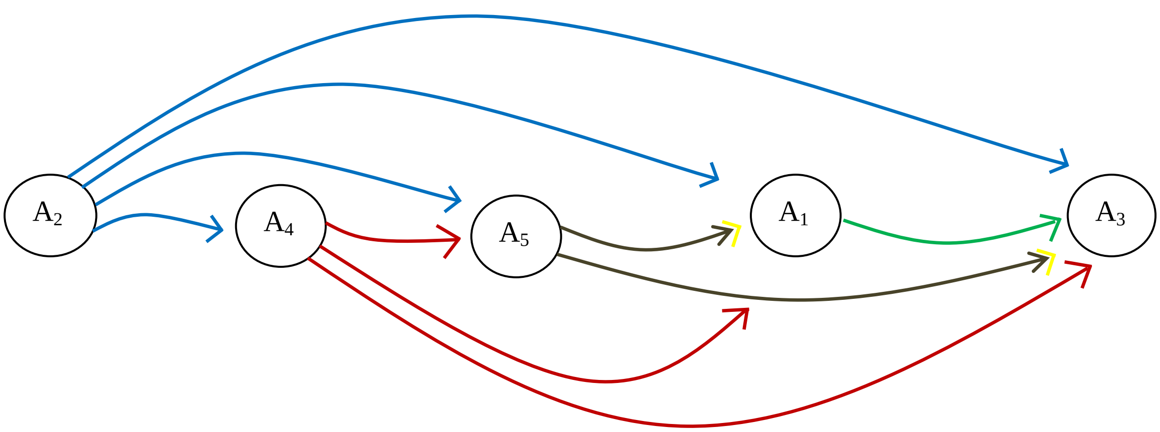

Figure 1. Graph of ordered complete ranking of network operators at partial ranking step of the algorithm when the logistic function is used.

A = {A_j} for *j* = 1, …,5 are the set of alternatives and *C* = {C_j} for *i* = 1,…,4 are the set of criteria. Table 4 presents the mean (μ_i) and the standard deviation (σ_i) for each of the four criterion C_i.

Going through steps 1 to 4 of the PROMETHEE algorithm, Table 5 shows the aggregate preference indices of pairs of alternatives A_k and A_l. From step 5 of partial ranking, we obtain the directed graph as shown in Figure 1 with the network operators represented as nodes. The resulting incidence matrix of the directed graph satisfies the condition that:

$$\sum_j I_{ij} + \sum_j I'_{ij} = m-1 \quad \text{for all } i, j = 1, 2 \ldots 5 \quad \text{and} \quad m = 5$$

Thus, the alternatives are completely ranked. From Figure 1, the network ranking order is A_2, A_4, A_5, A_1 and A_3 and the ranking is done based on the number of directed arcs that is recorded by each alternative, such that the best alternative (A_2) is the one with the highest number of directed arcs and the alternative A_3 with no directed arc becomes the worst one.

DISCUSSION

The new logistics preference function is the difference between the success and failure probabilities of the logistic function. Applying the proposed preference function in the PROMETHEE algorithm, we had a complete ordered ranking at the partial ranking step 5 of the algorithm.

The same data was used to rank the five network

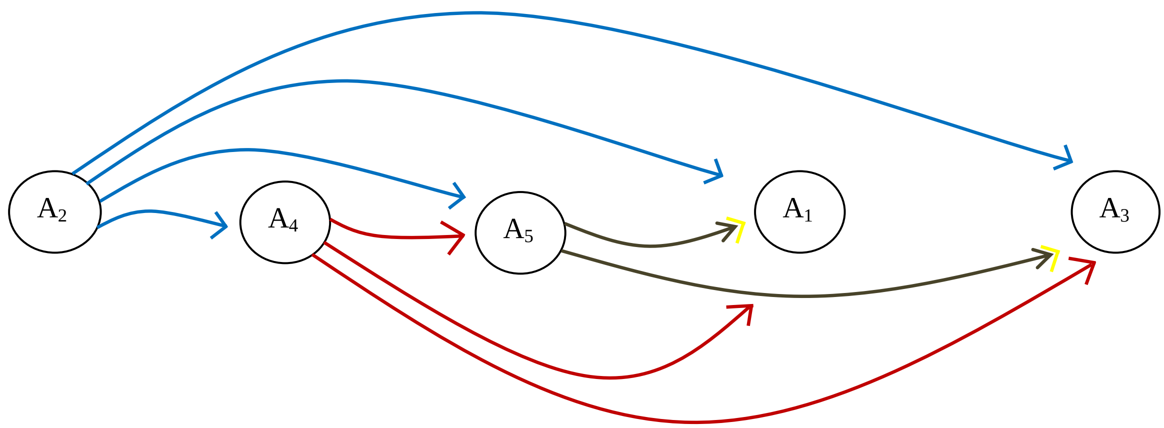

Figure 2. Graph of partial ranking of network operators when Gaussian function is used.

operators using the Gaussian preference function. In this case, step 5 of the algorithm produced partial ranking of the network operators. The directed graph of the partial ranking is as shown in Figure 2, which shows that, the condition,

$$\sum_j I_{ij} + \sum_j I'_{ij} = m - 1 \quad \text{for all } i, j = 1, 2...5 \quad \text{and } m = 5$$

is not satisfied for nodes A_3 and A_1 .

The number of directed arcs terminating on node A_3 of Figure 2 is less than the required number, 4, and node A_1 is missing a directed arc. Complete ranking was achieved at the last step with the ordered ranking A_2, A_4, A_5, A_1 and A_3 being the same as the solution using the logistic preference function.

The PROMETHEE algorithm was also applied to data provided in Villota (2009) as shown in Table A1 of the Appendix. We assigned minimization and maximization characteristics to the criteria of Villota (2009) data as shown in Table A2 of the Appendix. The Gaussian and logistic functions were used for the PROMETHEE calculation and the rankings were the same. Both reached the step of complete ranking and produced the same ranking order of A_5, A_2, A_4, A_1 and A_3.

Conclusion

The new Logistic preference function and the Gaussian preference function yielded the same ordered ranking for the five telecom networks in our case study. The ranking order was A_2, A_4, A_5, A_1 and A_3 under the PROMETHEE methodology. The logistic preference function was more efficient than the Gaussian preference function, in the sense that complete ordered ranking was achieved at step 5 (partial ranking step) for the logistic preference function, while complete ordered ranking was achieved at step 6 (complete ranking step) for the Gaussian preference function. For the decision problem presented in Villota (2009), the Gaussian and logistic functions produced the same ranking order. Both reached the step of complete ranking and produced the same ranking order of A_5, A_2, A_4, A_1 and A_3. The results suggest that for all continuous data for which the Gaussian preference function is applicable, our proposed logistic preference function is equally applicable and can be a perfect and more efficient substitute for the Gaussian preference function.

REFERENCES

Aburas A, Gardiner JG, Al Hokail Z (2010). Call Quality Measurement for Telecommunication Network and Proposition of Tariff Rates, pp. 1–5. http://www.iiis.org/CDs2010/CD2010IMC/CCCT_2010/PapersPdf/TA447AH.pdf (Accessed on 13th June, 2011).

Albadvi A (2004). Formulating National Information Technology Strategy: A Preference Ranking Model Using PROMETHEE Method. Eur. J. Oper. Res., 153: 290–296.

Anagnostopoulos KP, Petalas C, Pisinaras V (2005). Water Resources Planning Using the AHP and PROMETHEE Multicriteria Methods: the Case of NESTOS River –Greece. The 7th Balkans Conference on Operational Research "Bacor 05" Constanta. May 2005, Romania, pp. 1-12. http://fmi.unibuc.ro/balkan-conf/CD/Section4/ANAGNOSTOPOULOS-PETALAS-PISINARAS.pdf. (Accessed on 15th June, 2011)

Athawale VM, Chakraborty S (2010). Facility Location Selection Using PROMETHEE II Method. Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management (Jan. 9-10) Dhaka, Bangladesh, pp. 1-5.

Barton H, Beynon M (2009). A PROMETHEE Based Uncertainty Analysis of UK Police Force Performance Rank Improvement, 14: 1–29. http://www.ipes.info/WPS/WPS%20No%2014.pdf. (Accessed on 15th December, 2010).

Brans JP, Vincke PH, Mareschal B (1986). How to Select And How to Rank Projects: The PROMETHEE Method. Eur. J. Oper. Res., 24: 228–238.

Brans JP, Mareschal B (2010). PROMETHEE methods. https://www.inf.unideb. hu/valseg/dolgozok/anett.racz/docs/.../Promethee.pdf. (Accessed on 25th July, 2010).

Brans JP (1982). The engineering of decision: Elaboration instruments of decision support method PROMETHEE. Laval University, Quebec, Canada. http://www. dss.dpem.tuc.gr/pdf/How% 20to%20Decide

%20with%20PROMETHEE.pdf (Accessed on 10th January, 2011).
Covaliu Z (2001). Decision Analysis: Concepts, Tools and Promise. A Fair Isaac White Paper. 18009992955 from the US, 14154722211. http://www.edmblog.com/weblog/files/decision_analytics_white_paper.pdf. (Accessed on 18th August, 2010).
Ferreira JP, Cavalcante CAV, Almeida AT (2007). A Preventive Maintenance Decision Model Based on a MCDA Approach, pp. 1-6. http://old.ima.org.uk/Conferences/proceedings_mimar2007/proceedings_mimar2007_29.pdf. (Accessed on 11th August, 2010).
Figueira J, de Smet Y, Brans JP (2004). MCDA methods for Sorting and Clustering Problems: PROMETHEE Tri and PROMETHEE Cluster, Free University of Brussels. Department of Mathematics and Management Working Paper. http://www.ulb.ac.be/polytech/smg/indexpublications.htm. (Accessed on 18th July, 2010).
Manzano JIC, Quijada MTA, Nuño MMC (2011). Economic Evaluation of the Spanish Port System Using the PROMETHEE Multicriteria Decision Method. http://www-sre.wu-wien.ac.at/ersa/ersaconfs/ersa04/PDF/647.pdf. (Accessed on 12th May, 2011).
Maragoudaki R, Tsakiris G (2005). Flood Mitigation Planning Using PROMETHEE. Athens. Eur. Water 9/10 E.W. Publ., pp. 51–58. http://www.ewra.net/ew/pdf/EW_2005_9-10_06.pdf. (Accessed on 18th May, 2011)
Michailidis A, Chatzitheodoridis F (2006). Scenarios Analysis of Tourism Destinations. J. Soc. Sci., 2(2): 41-47.
Nemery P, Lamboray C (2007). Flowsort, A Flow-Based Sorting Method with Limiting or Central Profiles, Brussels, Belgium, pp. 1-24. http://code.ulb.ac.be/~pnemeryd/FlowSort_TOP.pdf. (Accessed on 19th July, 2010)
Podvezko V, Podviezko A (2010). Dependence of Multicriteria Evaluation Result on Choice of Preference Functions and their Parameters, Technological and Economic Development of Economy. Baltic J. Sustain., 16(1): 143–158.
Villota ALM (2009). Usability of Websites. Mphil Thesis. University of Birmingham, Birmingham.
Zhou Y, Vairavamoorthy K, Gramshaw F (2010). Development of a Fuzzy Based Pipe Condition Assessment Model Using PROMETHEE, pp. 1-10. http://www.delftcluster.nl/website/files/Development_of_a_Fuzzy_Based_Pipe_Condition_Assessment_Model_Using_PROMETHEE.pdf. (Accessed on 15th January, 2011).

APPENDIX

Table A1. Villota (2009) performance table of five websites (A1 - A5) and seven usability criteria (C1 to C7).

Characteristic	A_1	A_2	A_3	A_4	A_5
C_1	19	23	6	13	39
C_2	27	28	5	16	24
C_3	12	24	29	26	10
C_4	32	10	15	11	32
C_5	29	25	11	13	21
C_6	11	19	13	11	46
C_7	34	11	7	9	39

Table A2. Villota (2009) performance table with added maximization/minimization characteristics.

Characteristic	A_1	A_2	A_3	A_4	A_5
C_1 (max)	19	23	6	13	39
C_2 (max)	27	28	5	16	24
C_3 (max)	12	24	29	26	10
C_4 (min)	32	10	15	11	32
C_5 (min)	29	25	11	13	21
C_6 (max)	11	19	13	11	46
C_7 (max)	34	11	7	9	39