Card Trick Problem Set Solutions

Michael Allen

March 25, 2016

1-4 Lower Bounds

With the given procedure, we can establish a lower bound for the size of the deck for which we can perform the card trick for a given number of cards. These results are shown below.

hand	*deck*
2	3
3	6
4	15
5	52
6	245
n	$2(n-1)! + (n-1)$

5-6 Upper Bounds

Our procedure, however, is not optimal, and it is theoretically possible to perform the trick with a larger deck. We can set our upper bound at a size which makes maximal use of the information available to us.

hand	*deck*
2	3
3	8
4	24
5	124
6	725
n	$n! + (n-1)$

7

For the following, assume we have a deck of d cards from which the accomplice chooses a hand of size n.

a. The number of ways the accomplice can choose a hand is $C(d, n)$.

b. The number of different permutations of the n-1 cards the accomplice can show is $n!$.

c. The number of different strategies available to the accomplice and magician are $n!^{C(d,n)}$

8a

We can verify that a strategy is successful by showing that each possible hand maps to a different ordering chosen by the accomplice. The table below shows how this can be done for a deck with two suits (a and b) of three cards each where three cards are chosen .

hand			$\to$	1	2	*hide*	*hand*			$\to$	1	2	*hide*
$1a$	$2a$	$3a$	$\to$	$1a$	$3a$	$2a$	$2a$	$3a$	$1b$	$\to$	$2a$	$1b$	$3a$
$1a$	$2a$	$1b$	$\to$	$1a$	$1b$	$2a$	$2a$	$3a$	$2b$	$\to$	$2a$	$2b$	$3a$
$1a$	$2a$	$2b$	$\to$	$1a$	$2b$	$2a$	$2a$	$3a$	$3b$	$\to$	$2a$	$3b$	$3a$
$1a$	$2a$	$3b$	$\to$	$1a$	$3b$	$2a$	$2a$	$1b$	$2b$	$\to$	$1b$	$2a$	$2b$
$1a$	$3a$	$1b$	$\to$	$3a$	$1b$	$1a$	$2a$	$1b$	$3b$	$\to$	$3b$	$2a$	$1b$
$1a$	$3a$	$2b$	$\to$	$3a$	$2b$	$1a$	$2a$	$2b$	$3b$	$\to$	$2b$	$2a$	$3b$
$1a$	$3a$	$3b$	$\to$	$3a$	$3b$	$1a$	$3a$	$1b$	$2b$	$\to$	$1b$	$3a$	$2b$
$1a$	$1b$	$2b$	$\to$	$1b$	$1a$	$2b$	$3a$	$1b$	$3b$	$\to$	$3b$	$3a$	$1b$
$1a$	$1b$	$3b$	$\to$	$3b$	$1a$	$1b$	$3a$	$2b$	$3b$	$\to$	$2b$	$3a$	$3b$
$1a$	$2b$	$3b$	$\to$	$2b$	$1a$	$3b$	$1b$	$2b$	$3b$	$\to$	$1b$	$3b$	$2b$

8b

If we try to add another card to our deck, say $4a$, we find that there is no way to encode the hands, ($1a, 3a$,[any b]) and ($2a, 4a$,[any b]) with the rules we are given. Specifically, we cannot hide the higher of two consecutive cards (where the lowest is one greater than the highest) of the same suit because the two a cards are not consecutive.

9

We can represent strategies for the card trick in the form of an $(n-1)$-dimensional table which shows the magician how to decode the hidden card. Then, developing a strategy is just a matter of fitting one of the permutations of every possible hand into the table. The $n = 3, d = 7$ case can be computed by hand. The table for the $n = 3, d = 8$ case is shown below.

	0	1	2	3	4	5	6	7
0		3	4	5	5	2	2	2
1	2		4	5	6	6	3	3
2	3	3		5	6	7	7	4
3	4	4	4		6	7	7	4
4	1	5	5	5		7	7	1
5	1	2	6	6	6		7	1
6	1	2	3	0	0	0		1
7	1	2	3	0	0	0	0	

10

Generating strategies for each size of hand and deck can be accomplished by a brute force search with a computer, but there is also a clever closed form solution. It is possible (though difficult) to prove that for any hand size n, the upper bounds as specified in *problems 5-6* can be met.