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RISK ANALYSIS IN FINANCE AND INSURANCE

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To my parents

Ivea and Victor Melnikov

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Preface

This book deals with the notion of 'risk' and is devoted to analysis of risks in finance and insurance. More precisely, we study risks associated with future repayments (contingent claims), where we understand *risks* as uncertainties that may result in financial loss and affect the ability to make repayments. Our approach to this analysis is based on the development of a methodology for estimating the present value of the future payments given current financial, insurance and other information. Using this approach, one can adequately define notions of *price* of a financial contract, of *premium* for insurance policy and of *reserve* of an insurance company. Historically, financial risks were subject to elementary mathematics of finance and they were treated separately from insurance risks, which were analyzed in actuarial science. The development of quantitative methods based on stochastic analysis is a key achievement of modern financial mathematics. These methods can be naturally extended and applied in the area of actuarial mathematics, which leads to unified methods of risk analysis and management.

The aim of this book is to give an accessible comprehensive introduction to the main ideas, methods and techniques that transform risk management into a quantitative science. Because of the interdisciplinary nature of our book, many important notions and facts from mathematics, finance and actuarial science are discussed in an appropriately simplified manner. Our goal is to present interconnections among these disciplines and to encourage our reader to further study of the subject. We indicate some initial directions in the Bibliographic remark.

The book contains many worked examples and exercises. It represents the content of the lecture courses 'Financial Mathematics', 'Risk Management' and 'Actuarial Mathematics' given by the author at Moscow State University and State University – Higher School of Economics (Moscow, Russia) in 1998-2001, and at University of Alberta (Edmonton, Canada) in 2002-2003.

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Introduction

Financial and insurance markets always operate under various types of uncertainties that can affect nancial positions of companies and individuals. In nancial and insurance theories these uncertainties are usually referred to as risks. Given certain states of the market, and the economy in general, one can talk about risk exposure. Any economic activities of individuals, companies and public establishments aiming for wealth accumulation assume studying risk exposure. The sequence of the corresponding actions over some period of time forms the process of risk management. Some of the main principles and ingredients of risk management are qualitative identi cation of risk; estimation of possible losses; choosing the appropriate strategies for avoiding losses and for shifting the risk to other parts of the nancial system, including analysis of the involved costs and using feedback for developing adequate controls.

The rst two chapters of the book are devoted to the (nancial) market risks. We aim to give an elementary and yet comprehensive introduction to main ideas, methods and (probabilistic) models of nancial mathematics. The probabilistic approach appears to be one of the most ef cient ways of modelling uncertainties in the nancial markets. Risks (or uncertainties of nancial market operations) are described in terms of statistically stable stochastic experiments and therefore estimation of risks is reduced to construction of nancial forecasts adapted to these experiments. Using conditional expectations, one can quantitatively describe these forecasts given the observable market prices (events). Thus, it can be possible to construct dynamic hedging strategies and those for optimal investment. The foundations of the modern methodology of quantitative nancial analysis are the main focus of Chapters 1 and 2. Probabilistic methods, rst used in nancial theory in the 1950s, have been developed extensively over the past three decades. The seminal papers in the area were published in 1973 by F. Black and M. Scholes [6] and R.C. Merton [32].

In the rst two sections, we introduce the basic notions and concepts of the theory of nance and the essential mathematical tools. Sections 1.3-1.7 are devoted to now-classical binomial model of a nancial market. In the framework of this simple model, we give a clear and accessible introduction to the essential methods used for solving the two fundamental problems of nancial mathematics: hedging contingent claims and optimal investment. In Section 2.1 we discuss the fundamental theorems on arbitrage and completeness of nancial markets. We also describe the general approach to pricing and hedging in complete and incomplete markets, which generalizes methods used in the binomial model. In Section 2.2 we investigate the structure of option prices in incomplete markets and in markets with constraints. Furthermore, we discuss various options-based investment strategies used in nancial engineering. Section 2.3 is devoted to hedging in the mean square. In Section 2.4 we study a discrete Gaussian model of a financial market, and in particular, we derive the discrete version of the celebrated Black-Scholes formula. In Section 2.5 we discuss the transition from a discrete model of a market to a classical Black-Scholes diffusion model. We also demonstrate that the Black-Scholes formula (and the equation) can be obtained from the classical Cox-Ross-Rubinstein formula by a limiting procedure. Section 2.6 contains the rigorous and systematic treatment of the Black-Scholes model, including discussions of perfect hedging, hedging constrained by dividends and budget, and construction of the optimal investment strategy (the Merton's point) when maximizing the logarithmic utility function. Here we also study a quantile-type strategy for an imperfect hedging under budget constraints. Section 2.7 is devoted to continuous term structure models. In Section 2.8 we give an explicit solution of one particular real options problem, that illustrates the potential of using stochastic analysis for pricing and hedging long-term investment projects. Section 2.9 is concerned with technical analysis in risk management, which is a useful qualitative complement to the quantitative risk analysis discussed in the previous sections. This combination of quantitative and qualitative methods constitutes the modern shape of financial engineering.

Insurance against possible financial losses is one of the key ingredients of risk management. On the other hand, the insurance business is an integral part of the financial system. The problems of managing the insurance risks are the focus of Chapter 3. In Sections 3.1 and 3.2 we describe the main approaches used to evaluate risk in both individual and collective insurance models. Furthermore, in Section 3.3 we discuss models that take into account an insurance company's financial investment strategies. Section 3.4 is devoted to risks in life insurance; we discuss both traditional and innovative flexible methods. In Section 3.5 we study risks in reinsurance and, in particular, redistribution of risks between insurance and reinsurance companies. It is also shown that for determining the optimal number of reinsurance companies one has to use the technique of branching processes. Section 3.6 is devoted to extended analysis of insurance risks in a generalized Cramér-Lundberg model.

The book also offers the Software Supplement: Computations in Finance and Insurance (see Appendix A), which can be downloaded from

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Finally, we note that our treatment of risk management in insurance demonstrates that methods of risk evaluation and management in insurance and finance are interrelated and can be treated using a single integrated approach. Estimations of future payments and of the corresponding risks are the key operational tasks of financial and insurance companies. Management of these risks requires an accurate evaluation of present values of future payments, and therefore adequate modelling of (financial and insurance) risk processes. Stochastic analysis is one of the most powerful tools for this purpose.