Addendum to the paper "Belnap's four-valued logic and De Morgan lattices"

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In my paper [3], recently published in this Journal, I study several aspects of the relations between Belnap's logic $\mathcal{B}$ and the class DM of De Morgan lattices. Some of these relations concern a Gentzen system for Belnap's logic, denoted by $G_{\mathcal{B}}$, which was introduced in 1988 by my colleague Ventura Verdú and me (see [5] and [6]); indeed, the study of this Gentzen system within the theory of full models developed in [4] is one of the main objects of my paper. In the final part of Section 4 I compare this Gentzen system with another, better known one denoted by $G_{\mathcal{BL}}$, specially in the framework of the theory of algebraizability of Gentzen systems developed in [9, 10].

The reader should take note that an algebraic study of a non-structural and multiple-conclusion version of the Gentzen system $G_{\mathcal{BL}}$, denoted by $\mathcal{G}_{\mathbf{B}}$, has been published by Mr. Alexej Pynko in [8] (already quoted in [3]). There, in comments following Corollary 3.6 on page 449, $\mathcal{G}_{\mathbf{B}}$ is compared to $G_{\mathcal{B}}$ (there denoted by $\mathcal{G}_{\mathcal{FV}}$) and results parallel to those in Proposition 4.10 of [3] are obtained.

Further, it is worth noticing that the results in Theorems 4.11 and 4.12 of [3] were first obtained by Pynko in his unpublished manuscript [7]. In it he develops a theory of algebraizability of sentential-like logical systems of the most general kind, which encompasses Willem Blok and Don Pigozzi's original theory [1], its extension to k-dimensional deductive systems [2], and also its extension to Gentzen systems by Jordi Rebagliato and Ventura Verdú [9, 10], on which [3] relies. In the final Section 4.5 of [7] the three above mentioned Gentzen systems are presented, together with the structural and multiple-conclusion versions $\widehat{\mathcal{G}}_{BC}$ and $\widetilde{\mathcal{G}}_{BC}$ of $G_{\mathcal{B}}$ and $G_{\mathcal{BL}}$, respectively. Then Theorem 4.88 of [7] states the same as Theorem 4.11 of [3], that is, the strong algebraizability of $G_{\mathcal{B}}$ (denoted by $\mathcal{G}_{\mathcal{BC}}$ in [7]), with the variety DM of De Morgan lattices as its equivalent algebraic semantics. And Theorem 4.98 of [7] states the non-algebraizability of $\widetilde{\mathcal{G}}_{BC}$, which is proved from the strong algebraizability of $\widehat{\mathcal{G}}_{BC}$; since it is obvious that the same fact and proof hold for their single-conclusion fragments, one can consider that this result also contains the non-algebraizability of $G_{\mathcal{BL}}$, which is the assertion in Theorem 4.12 of [3]. Moreover, the technique of the proof is also the same, that is, to use the strong algebraizability of $G_{\mathcal{B}}$ together with a key property of the general theories referred to above; this property (that an algebraizable logic cannot have a strongly algebraizable proper extension with the same theorems) is a consequence of several well-known facts that hold in any of these general theories, but as far as I know it was first explicitly noticed by Pynko in Theorem 3.16 of his [7].

The unpublished manuscript [7] was written in 1993, therefore any priority of discovery of the results mentioned in the preceding paragraph should be definitely credited to Mr. Pynko, to whom I express my apologies for not having noticed this before.

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Received 3 March 1998. Revised 9 March 1998